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Improving Students' Mathematical Reasoning Ability and Self-Efficacy through DNR-Based Instruction

Iyon Maryono[1]*, Yaya S. Kusumah[2], Jarnawi A. Dahlan[3], Sunandar[4], and Asep Jihad[5]

[1, 2, 3] *Department of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Setiabudhi No. 299, Bandung, Indonesia*
[4, 5] *Department of Mathematics Education, UIN Sunan Gunung Djati Bandung, Jl. Soekarno-Hatta Cimincrang, Bandung, Indonesia*
**iyonmaryono@uinsgd.ac.id*

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Abstrak

Kemampuan Penalaran Matematis (KPM) dan efikasi-diri sangat penting bagi mahasiswa calon guru matematika. Namun, fakta menjunjukkan bahwa dua hal tersebut belum sesuai dengan harapan. Tujuan penelitian ini adalah untuk menganalisis dampak dari model DNR-*based Instructions* terhadap KPM dan efikasi-diri mahasiswa. Metode eksperimen semu digunakan pada penelitian ini dengan *nonequivalent control group design*. Sampel penelitian ini terdiri dari 39 mahasiswa kelompok model DNR-*based Instructions* dan 39 mahasiswa kelompok model konvensional. Penelitian ini menggunakan instrumen tes-KPM dengan validitas tinggi dan angket efikasi-diri. Hasil penelitian menunjukkan bahwa: KPM mahasiswa kelompok model DNR-*based Instructions* lebih baik dari pada KPM mahasiswa kelompok model konvensional, ditinjau secara keseluruhan maupun berdasarkan Pengetahuan Awal Matematis (PAM); Efikasi-diri mahasiswa kelompok model DNR-*based Instructions* berada pada kategori baik. Implikasi dari hasil penelitian ini adalah KPM dan efikasi-diri dapat ditingkatkan melalui aktivitas pembelajaran yang didasarkan pada cara memahami dan cara berpikir, kebutuhan intelektual dan penalaran berulang.

Kata kunci: DNR-*Based Instructions*, Efikasi-Diri, Kemampuan Penalaran Matematis

Abstract

Mathematical Reasoning Ability (MRA) and self-efficacy are very important for prospective mathematics teacher students. However, the facts show that these two things have not met expectations. The aim of this research is to analyze the impact of the DNR-based Instructions model on MRA and student self-efficacy. The quasi-experimental method was used in this research with a nonequivalent control group design. The sample for this study consisted of 39 students from the DNR-based Instructions model group and 39 students from the conventional model group. This research has used MRA-test instruments with high validity and self-efficacy questionnaires. The research results show that: MRA for students in the DNR-based Instructions model group is better than MRA for students in the conventional model group, both overall and based on Prior Mathematics Knowledge (PMK); The self-efficacy of students in the DNR-based Instructions model group is in the good category. The implication of the results of this research is that MRA and self-efficacy can be improved through learning activities that are based on ways of understanding and ways of thinking, intellectual needs and repetitive reasoning.

***Keywords**: DNR-Based Instructions, Self-Efficacy, Mathematical Reasoning Ability*

1. INTRODUCTION

Reasoning is the process of arriving at conclusions based on facts through a series of arguments. In mathematics education, the ability to reason or also called Mathematical Reasoning Ability (MRA) is an important ability, because through this ability a person can understand concepts, use appropriate ideas and procedures and recall forgotten understanding. As Ball & Bass (2003) stated that reasoning is a basic mathematical skill needed to understand concepts, use ideas and procedures flexibly and reconstruct forgotten understanding.

Mathematical reasoning is an important part of the content of the mathematics education curriculum. On various mathematics learning topics, reasoning can be used as one of the mathematical abilities that students need to achieve. As Jeannotte & Kieran (2017) stated that mathematical reasoning is an essential part of the mathematics education curriculum. Additionally, Hjelte et al. (2020) explained that empirical studies on mathematical reasoning are increasingly developing, because mathematical reasoning is an important part of mathematics education.

Reasoning is a tool for solving problems and integrating ideas so that they are well structured (Brodie, 2009). Therefore, mathematics learning practices need to focus on improving Mathematical Reasoning Ability (MRA). This is in line with the opinion of Mishra et al. (2022) which states that Mathematical Reasoning Ability is very important for a general intelligence system to carry out a task. Likewise, Saxton et al. (2019) stated that mathematical reasoning is a core ability in human intelligence.

Even though mathematical reasoning abilities are important to master, the facts show that these abilities still need to be improved. For example, Herman (2018) stated that reasoning is a mathematical ability that is not easily achieved by students, because it has high complexity. In addition, Bergqvist & Lithner (2012) found that most students completed assignments using algorithmic reasoning, not creative reasoning, so they tended to get stuck with rote memorization. Thus, learning innovations are needed to improve students' Mathematical Reasoning Abilities (MRA).

Prospective mathematics teacher students must have adequate Mathematical Reasoning Abilities (MRA), because they will teach mathematics in which there are many activities to convince students of the truth of a statement. However, the facts show that students' Mathematical Reasoning Abilities (MRA) still do not meet expectations. For example, Stylianides et al., (2013) concluded that many prospective mathematics teacher students are weak in mathematical reasoning. A similar thing was stated by Herman (2018) who stated that reasoning is a mathematical ability that is not easily achieved by students, because it has high complexity.

Students' success in achieving Mathematical Reasoning Abilities (MRA) is influenced by attitude. Among the attitudes that need to be developed is self-efficacy. According to Mukuka et al. (2021), in mathematics learning it is necessary to choose a learning approach that does not only focus on developing students' cognitive abilities such as mathematical reasoning but must foster students' affective attributes such as mathematics self-efficacy beliefs. Additionally, self-efficacy plays an important role in the context of achievement, and can influence the drive, direction, persistence, and outcomes of achievement-related actions (Schunk & Pajares, 2002). In the context of student performance, according to Zimmerman, (2000) self-efficacy beliefs have been shown to be sensitive to subtle changes in the context of student performance, interact with self-regulated learning processes, and mediate student academic achievement.

Several studies show that self-efficacy influences student learning outcomes. For example, the research results of Karunika et al. (2019) shows that students with good self-efficacy are able to explain the use of models, facts, properties, relations to use patterns and relationships to analyze mathematical situations; provide an explanation of existing models, images, facts, properties, relationships or patterns; and perform mathematical manipulations. This is in line with the opinion of Kingston & Lyddy (2013) which states that proportional reasoning is influenced by working memory capacity and self-efficacy. Apart from that, the strongest influence found was self-efficacy on teaching performance evaluation (Klassen & Tze, 2014). More specifically, Van Dinther et al. (2011) and Bartimote-Aufflick et al., (2016) revealed that educational programs have the possibility of increasing students' self-efficacy, and that educational programs based on social cognitive theory have proven to be very successful in this regard.

Based on the explanation in the previous paragraph, efforts need to be made to improve students' Mathematical Reasoning Abilities (MRA) and self-efficacy. Among the efforts that can be made is implementing a learning model that can explore reasoning abilities and foster self-efficacy. For example, Maryono et al. (2018) have researched the achievement of students' evidentiary abilities and self-confidence through the Moore learning method; Maryono et al. (2023) have researched the achievement of mathematical proof comprehension ability through the question strategy students have in online learning. The learning model based on Duality, Necessity and Repeated-reasoning (DNR) or also called DNR-based instruction has the characteristic of focusing on problem analysis, each problem is solved individually or in group work, presentations and discussions are carried out on the solutions obtained. The fundamental thing in DNR-based instruction is the premise of knowledge development, which states that problem solving should not only be an aim but also a means for learning mathematics (Harel, 2020).

The first principle of DNR-based instruction is the principle of duality. The duality principle is a principle where during the learning process students improve their reasoning abilities based on how they understand and how they think (Harel, 2013). Ways of understanding refer to what is produced, such as definitions, conjectures, theorems, proof, problems, and solutions, while ways of thinking refer to the mathematical practices used to produce these products. Examples that include ways of thinking are empirical reasoning, deductive reasoning, structural reasoning, heuristics, and beliefs about the nature of mathematical knowledge and the acquisition process.

The second principle of DNR-based instruction is the principle of learning needs (necessity principle). Necessity principle is a learning principle that refers to the intellectual need to remove doubts (Harel, 2013). Intellectual needs are arised through the problems presented in learning. This situation motivates students to think about how and why mathematical knowledge arises. If intellectual needs are not stimulated, students will experience didactic obstacles. According to Bakar et al. (2019) didactic obstacles occur when teachers are unable to create learning that accommodates students' intellectual needs.

Furthermore, the third principle of DNR-based instruction is the principle of repeated reasoning. According to Harel (2008), the principle of iterative reasoning is that students must practice reasoning to internalize the desired way of understanding and way of thinking. Iterative reasoning is not just drill and routine problem solving, but an internalization process so that students can apply knowledge independently and spontaneously.

Furthermore, repeated deliberate practice is the third principle in DNR-based Instruction. Repeated reasoning is not only the practice of solving routine problems, but it is also very important for the process of internalizing ways of understanding and thinking, namely a conceptual state in which a person is able to apply knowledge independently, spontaneously and in an organized way. This is in line with the opinion of (Harel, 2008) who states that the principle of iterative reasoning is that students must practice reasoning to internalize the desired way of understanding and way of thinking.

Based on the description above, research is needed to test the improvement of Mathematical Reasoning Abilities (MRA) and self-efficacy through DNR-based instruction. There are three problem formulations in this research, namely: (1) Is the increase in mathematical reasoning abilities of students in the DNR-based Instruction model group better than students in the conventional learning model group in terms of overall analysis and based on the level of Initial Mathematical Knowledge (IMK) (high, medium and low)? (2) What is the quality of student self-efficacy in the DNR-based instruction model group and the conventional learning model group? (3) How is the development of students' Mathematical Reasoning Abilities (MRA) in the DNR-based instruction model group?

2. METHOD

Quasi-experimental methods have been used in this research. Nonequivalent Control Group Design has been chosen to conduct experiments implementing the DNR-based Instruction model. The research design is briefly presented in Table 1.

Table 1. Research Design

O	X	O
O		O

Information :
X : *DNR-based Instruction*
O : *Pretes*t and *Posttest*

Table 1 shows that the independent variable of this research is DNR-based instruction model, while the dependent variable is Mathematical Reasoning Ability (MRA). MRA data was taken from the pretest and posttest results. The data is analyzed to determine learning outcomes and improve students' MRA.

The research sample was taken from the population of students enrolled in basic geometry courses in the Mathematics Education Study Program at one of the universities in Bandung. The research sample was taken randomly and two groups were obtained, namely the experimental group of 39 students who studied through the DNR-based Instruction model, while the control class consisted of 39 students who studied through the conventional learning model.

Students' mathematical reasoning can be measured using test instruments developed from routine tasks and non-routine tasks (Iuculano & Menon, 2018). The main instrument used in this research is a five-item Mathematical Reasoning Ability (MRA) test. The MRA test indicators for each question item are: (1) Drawing logical conclusions, compiling evidence, providing reasons or evidence for the truth or solution; (2) Finding patterns or properties of mathematical phenomena to make generalizations; (3) Checking the validity of an argument; (4) Carrying out mathematical manipulations; (5) proposing conjectures. Based on the results of the validity test, the fifth item on the mathematical reasoning ability test has good validity. In detail, the validity of the items is presented in Table 2.

To measure self-efficacy, a 34-item self-efficacy scale instrument was used. There are six indicators of self-efficacy used, namely: (1) Able to overcome the problems faced; (2) Confident in his success; (3) Dare to face challenges; (4) Be aware of his own strengths and weaknesses; (5) Able to interact with other people; (6) Be firm or don't give up easily.

Table 2. Validity of Mathematical Reasoning Items

Question Number	Indicator of MRA	Validity	
		Score	Interpretation
1	Drawing logical conclusions, compiling evidence, providing reasons or evidence for the truth or solution	0.72	High
2	Finding patterns or properties of mathematical phenomena to make generalizations	0.61	High
3	Checking the validity of an argument	0.82	Very high
4	Carrying out mathematical manipulations	0.83	Very high
5	Proposing conjectures	0.71	High

Apart from that, to determine the development of students' mathematical reasoning abilities during learning, an analysis of student answers (student answer sheet) was carried out. There are four types of Student Answer Sheets (SAS) that have been analyzed, namely: SAS-1, SAS-2, SAS-3, and posttest answer sheets. Each type of student answer sheet is taken based on the student's initial mathematics knowledge category (low, medium and high).

Pretest, posttest and self-efficacy score data were analyzed using descriptive statistics. To determine the increase in students' mathematical reasoning abilities (MRA), the normalized gain (N-gain) formula from Hake (1998) was used based on MRA pre-test and post-test score data. Next, the N-gain data on students' mathematical reasoning abilities was analyzed using inferential statistics, namely the two-way ANOVA test. Apart from that, the development of students' mathematical reasoning abilities was also analyzed based on problem solving data during the implementation of DNR-based instruction.

3. RESULT AND DISCUSSION

A. Improving Mathematical Reasoning Ability

The increase in students' Mathematical Reasoning Ability (MRA) was analyzed based on pre-test and post-test score data by determining the normalized gain score (N-gain) for the DNR-based instruction model group and the conventional model group. Descriptive statistical data on the average N-Gain score of students for each group is presented in Table 3.

Table 3. Descriptive Statistics of N-Gain Values

Learning model	Average N-Gain	Criteria
DNR-based instructions	0,44	Medium
Conventional	0,36	Medium

Based on Table 3, the average increase in students' mathematical reasoning abilities in the DNR-based instructions class was 0.44 and in the conventional class was 0.36. The average N-Gain for both is in the medium category. However, the results of students' mathematical reasoning abilities in the DNR-based instructions class were higher than those in the conventional class with an average difference in N-Gain score of 0.08.

To find out whether the increase in mathematical reasoning abilities of students in the DNR-based instructions group was significantly different from the conventional group based on the level of Initial Mathematical Knowledge (IMK) (high, medium and low), a two-way ANOVA test was carried out. The assumptions that must be met in carrying out the two-way ANOVA test have been fulfilled, namely that the data is normally distributed and has a homogeneous variance. The results of the two-way ANOVA test are presented in Table 4.

Table 4 presents the results of the two-way ANOVA test on data on increasing mathematical reasoning abilities with a significance level of a = 0.05. The difference in overall MRA improvement between the DNR-based instruction model group and the conventional learning model group is determined based on the Sig value in the first row (Learning). Sig. value was recorded as 0.047 < 0.05, this shows that the overall increase in students' mathematical reasoning abilities in the DNR-based instruction model group was better than in the conventional learning model group. This fact shows that through the DNR-based instruction model developed by Harel (2020) MRA has been successfully improved. These results are in accordance with Harel (2020) claim which states that DNR-based instructions are the conditions for achieving important goals in stimulating students' intellectual needs to learn mathematics, helping them acquire ways of understanding and thinking mathematically, and ensuring that they internalize and retain mathematical knowledge they learn. This shows that learning activities are based on ways of understanding and ways of thinking; intellectual needs and repeated-reasoning succeeded in increasing students' MRA.

Table 4. Two Way Anova Test of N-Gain Data Based on IMK Level

Tests of Between-Subjects Effects

Dependent Variable: N-Gain

Source	*Type III Sum of Squares*	Df	*Mean Square*	F	Sig.
Learning	.137	1	.137	4.102	.047
IMK	.275	2	.137	4.109	.021
Learning * IMK	.014	2	.007	.215	.807

To see in more detail the influence of the DNR-based instruction model on increasing MRA, the results of the two-way ANOVA test based on IMK were analyzed. By paying attention to Table 4 second row (IMK). Sig. value was shown as 0.021 < 0.05, this shows that based on IMK level (low, medium, high) the increase in students' mathematical reasoning abilities in the DNR-based instruction model group is better than in the conventional learning model group. This fact further strengthens the previous fact, that students at low, medium and high IMK levels succeeded in increasing MRA through the DNR-based instruction model. As Harel (2008) stated, repetitive reasoning functions to internalize knowledge, especially for low-level students..

The interaction effect between the learning model groups (DNR-based instructions and conventional learning models) applied and the IMK level (High, Medium, Low) on increasing student MRA is presented in Table 4, third row (Learning*IMK). Sig value. was recorded as 0.807 ≥ 0.05, this shows that there is no interaction effect between the learning model groups (DNR-based instructions and conventional learning models) applied and the IMK level (High, Medium, Low) on increasing students' reasoning abilities. In other words, the interaction between learning model groups and IMK levels does not have a significant effect on increasing student MRA.

To determine the differences in increasing mathematical reasoning abilities between students with high, medium and low levels of IMK, a follow-up test was carried out, namely the Post Hoc Tukey test, the results of which are presented in Table 5.

Based on the Sig. value In Table 5, at a significance level of a = 0.05, several conclusions are obtained. The increase in mathematical reasoning abilities of students with high IMK levels is better than students with medium IMK levels and low IMK levels. This can be seen based on the sig. value, which is less than 0.05. On the other hand, the improvement in mathematical reasoning abilities of students with medium IMK levels is no better than students with low IMK levels. This can be seen based on the sig. value, which is more than 0.05.

Table 5. Post Hoc Tukey Test Based on Initial Mathematical Knowledge (IMK)

Multiple Comparisons ***Dependent Variable: N-Gain* Tukey HSD**				
(I) IMK	(J) IMK	*Mean Difference (I-J)*	*Std. Error*	Sig.
High	Medium	0.1502	0.05705	0.028
	Low	0.1717	0.07044	0.045
Medium	High	-0.1502	0.05705	0.028
	Low	0.0215	0.05542	0.920
Low	High	-0.1717	0.07044	0.045
	Medium	-0.0215	0.05542	0.920

The following shows an interaction plot of differences in the increase in students' mathematical reasoning abilities in Figure 1.

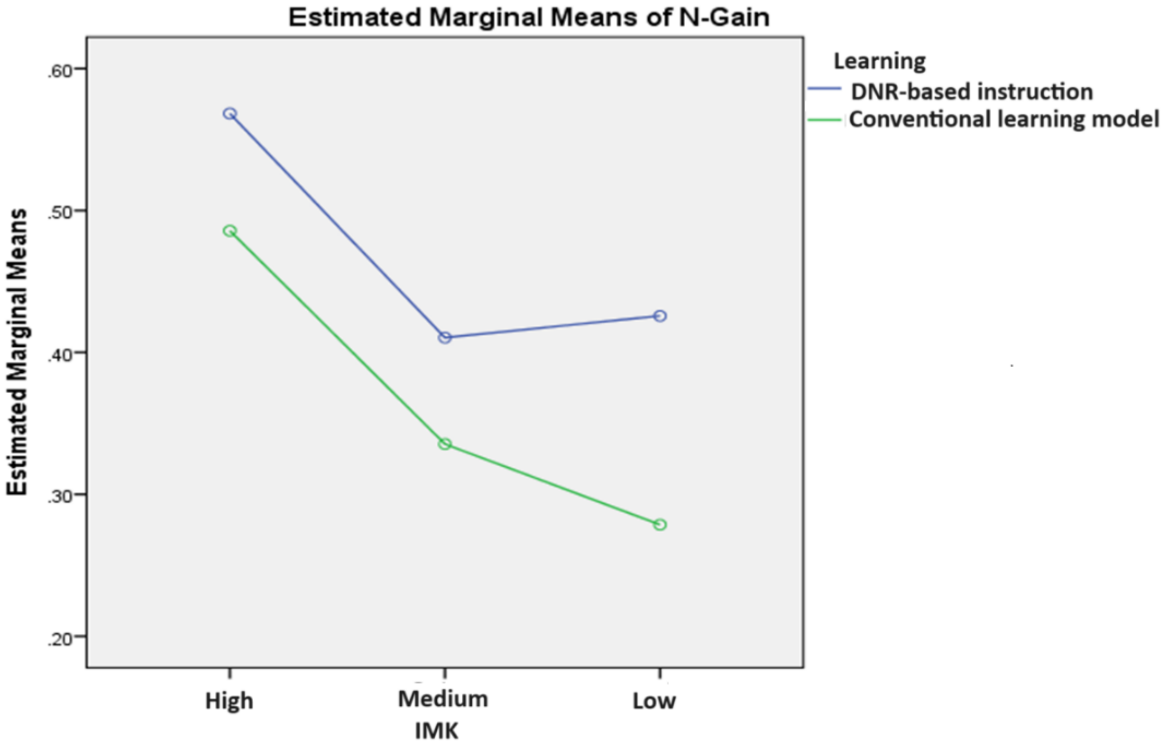

Figure 1. Effect of Interaction between Learning Model Group and IMK Level on Increasing MRA

Based on Figure 1, the increase in MRA for students with high IMK levels in the DNR-based instructions model group outperforms the conventional learning model group with medium and low IMK levels. The increase in MRA for students with medium IMK levels in the DNR-based instructions model group outperformed the conventional learning model group with medium and low IMK levels. The increase in MRA for students with low IMK levels in the DNR-based instructions model group outperformed the conventional learning model group with medium and low IMK levels. According to Basir & Wijayanti, (2020) to improve mathematical reasoning abilities, a scaffolding strategy is needed which includes (1) Providing problem stimuli; (2) Explanation of the problem formulation; and (3) Question and answer dialogue. This strategy appears in DNR-based instruction, especially at the problem solving and internalization of reasoning processes. The ability to generalize and justify will emerge if teachers design challenging learning for students followed by activities to guide students (Marasabessy, 2021). This is in accordance with the principle of necessity in the DNR-based instruction model, namely that students must be given challenges so that their intellectual needs arise.

B. Student Self-efficacy after Implementing DNR-Based Instructions

To determine the quality of student self-efficacy in the DNR-Based Instructions model group and the conventional model group, a self-efficacy questionnaire with a Likert scale was used. The calculation of the average score of student self-efficacy in the two learning model groups is presented in Table 6.

Table 6. Average Student Self-Efficacy Score

Group	Average Score	Neutral Score
DNR-Based Instructions	2.71	2.5
Konvensional	2.59	

Based on the data in Table 6, the average student self-efficacy score in the DNR-based instruction model group is 2.71 and the average student self-efficacy score in the conventional learning model group is 2.59. The average score of student self-efficacy in the two learning model groups exceeded the neutral score of 2.5. This shows that student self-efficacy in both learning model groups is relatively positive. However, the average score of student self-efficacy in the DNR-based instruction group exceeded that of the conventional model group. This finding is in accordance with the findings of Ozkal (2019) and Olivier et al. (2019)O, namely that self-efficacy has a significant effect on learning and mathematical performance. The DNR-based instruction model facilitates students for discussion, group study and feedback from the teacher. Learning like this is positively correlated with student self-efficacy Sökmen (2021).

C. Development of Students' Mathematical Reasoning Ability in the DNR-Based Instructions Model

The development of students' mathematical reasoning abilities in the DNR-based instructions model was analyzed based on student worksheets that have been answered, namely Student Answer Sheet (SAS) during three lessons. There are four Student Answer Sheets (SAS) that have been analyzed, namely: SAS-1, SAS-2, SAS-3, and the Posttest Answer Sheet (PAS). Each Student's Answer Sheet is taken based on the level of Initial Mathematical Knowledge (IMK) (low, medium and high), as presented in Table 7, Table 8, and Table 9.

Tabel 7. Development of MRA in Students with Low Levels of Initial Mathematical Knowledge

SAS-1	SAS-2	SAS-3	PAS
Students are unable to identify and organize evidence according to problem solving	Students are able to identify information in preparing proof steps but are not yet in accordance with the correct proof steps	Students were able to manipulate geometry as a solution step, but in the next proof step a conceptual error occurred.	Students are able to fulfill the four reasoning indicators even though overall there are still many shortcomings. The indicators that are not met are geometric manipulation

Based on Table 7, a student at a low IMK level was initially unable to identify the data and facts of the problem given. In the second lesson, the student was able to identify data and facts, he tried to formulate a proof argument, but the steps were not correct. In the third lesson, the student was able to carry out geometric manipulation as a problem solving step, but a conceptual error occurred. The student's answer to the post-test problem shows the ability to understand the problem, but there are still errors in the solution step.

Tabel 8. Development of MRA in Students with Medium Levels of Initial Mathematical Knowledge

SAS-1	**SAS-2**	**SAS-3**	**PAS**
Students can identify and organize evidence well	Students are able to compile evidence but are not complete in identifying	Students do not manipulate geometry to complete the	Students are able to fulfill all reasoning indicators even though overall there

SAS-1	SAS-2	SAS-3	PAS
according to problem solving.	it, so there are proof steps that are missed.	information needed to solve proof problems.	are still many shortcomings that need to be corrected.

A student with a medium IMK level, as presented in Table 8, shows the ability to organize evidence well in the first lesson. In the second lesson he was able to organize the evidence, but it was incomplete (there were steps missing). Furthermore, in the third lesson he did not carry out geometric manipulation so he could not solve the proof problem. His answer to the post-test problem showed his ability to understand the problem as a whole, but his answer still contained errors.

Tabel 9. Development of MRA in Students with High Levels of Initial Mathematical Knowledge

SAS-1	SAS-2	SAS-3	PAS
Students can identify and organize evidence well even though there are still errors in giving reasons for a statement	Students can identify and organize evidence according to the solution of the problem even though there are still errors in one proof step	Students are able to manipulate geometry to complete the information needed to solve proof problems	Students were able to fulfill all the reasoning indicators well even though overall there were still errors in solving questions using geometric manipulation indicators.

A student with a high IMK level, as presented in Table 9, in the first lesson he was able to organize evidence, but there were still a few errors, as well as in the second lesson. In the third lesson, he was able to carry out geometric manipulations and solve proof problems correctly. His answers to the post-test problems showed his ability to understand the problem well, but there were still errors when carrying out geometric manipulations.

The description of the development of Mathematical Reasoning Abilities (MRA) of three students representing three IMK levels in the DNR-based instruction model group shows good changes. The development of MRA at several learning times is also influenced by the level of difficulty of the questions given, so that sometimes there is a stagnation or even a decline as occurred in a student at a medium IMK level in Table 8. According to Harel (2020) providing challenges can foster students' intellectual needs and Giving problems repeatedly (repeated reasoning) is an effort to internalize knowledge. Thus, through the application of the DNR-based instruction model, students' MRA can logically develop well.

4. CONCLUSION

Students' Mathematical Reasoning Abilities (MRA) and students' self-efficacy can be developed through learning activities that consider students' ways of understanding and ways of thinking; students' intellectual needs and repetitive reasoning, namely through the DNR-based instruction model. The results of this research show that through the DNR-based instruction model, students' MRA can be improved both overall and based on high, medium and low Initial Mathematical Knowledge (IMK) levels. The quality of self-efficacy of students who learn through the DNR-based instruction model is categorized as good. In addition, through the DNR-based instruction model, student MRA develops at various IMK levels (high, medium and low).

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