

Analysis of Inverse Proportion in Mathematics Textbook Based on Praxeological Theory

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Abstrak

Buku teks yang memiliki konten isi memiliki peranan yang sangat penting dalam menunjang proses pengetahuan siswa secara utuh. Fakta menunjukkan bahwa sebagian besar guru bergantung pada buku teks, hingga ada keluhan pada isi dan sajian buku teks. Penelitian ini bertujuan untuk mendeskripsikan karakteristik sajian buku teks kelas tujuh pada materi perbandingan berbalik nilai ditinjau dari teori prakseologi yang memiliki empat elemen analisis, yaitu tugas, teknik, teknologi, teori. Penelitian ini adalah penelitian deskriptif kualitatif dengan objek penelitian adalah buku teks siswa kelas VII terbitan Kemendikbud, serta subjek penelitian melibatkan guru kelas tujuh. Temuan penelitian menunjukkan bahwa sajian buku teks materi perbandingan berbalik nilai sudah cukup baik. Namun, ada beberapa sajian yang perlu dilengkapi. Materi perbandingan berbalik nilai sudah bersifat kontekstual, tetapi terdapat ilustrasi tidak sesuai perkembangan kognitif siswa. Sajian teknik kurang lengkap dan tidak memberikan ruang bagi siswa untuk memilih cara menyelesaikan kasus perbandingan berbalik nilai, tidak ada kesempatan bagi siswa untuk memberikan justifikasi terhadap teknik yang dilakukan, serta beberapa tidak ada situasi untuk membuat siswa menyimpulkan terhadap hasil justifikasi yang dilakukan.

Kata kunci: Buku Teks, Perbandingan Berbalik Nilai, Teori Prakseologi

Abstract

Textbooks with content are very important in supporting the students' knowledge process. The facts show that most teachers depend on textbooks, so there are complaints about the content and presentation of textbooks. This study aims to describe the characteristics of a seventh-grade textbook presented on inverse proportion in terms of the praxeological theory, which has four elements of analysis, namely task, technique, technology, and theory. This research is a qualitative descriptive study with the object of research being a textbook of class VII students published by the Ministry of Education and Culture. The research subject involves seventh-grade teachers. The study results show that the presentation of inverse proportion in textbooks is good enough. However, there is some present that needs to be completed. The inverse proportion is already contextual, but some illustrations do not match students' cognitive development. The presentation of techniques is incomplete and does not provide space for students to choose how to solve cases of inverse proportion, there is no opportunity for students to justify the techniques used, and in some situations, there are no situations to make students conclude on the results of the justifications carried out.

Keywords: Textbook, Inverse Proportion, Praxeological Theory

1. INTRODUCTION

Textbooks are considered one of the fundamental materials as a way for teachers to teach and students to learn. In learning mathematics, mathematics textbooks are an integral part of life. Where students use mathematics textbooks to study and do homework, while teachers and professors/mathematicians each use textbooks to prepare classes and teach (Kajander & Lovric, 2009). Textbooks are considered to have the potential as a powerful tool to help students develop ideas and understanding of mathematical concepts. Reading textbooks provides opportunities for students to be involved in tasks that demand a high level of understanding, which is expected that students will have higher scores (Hadar, 2017; Weinberg & Wiesner, 2011). If the textbooks are different, students will get different learning opportunities, so learning opportunities affect student achievement (Yang & Sianturi, 2017). Thus, the quality of the books students uses plays an essential role in mathematics instruction, influencing learning opportunities and the quality of student learning and influencing student achievement to improve education (Johar et al., 2018; Murdaningsih & Murtiyasa, 2016; Sievert et al., 2019; Yang & Sianturi, 2017).

However, in contrast to a study by van Steenbrugge et al., (2013), that found no substantial impact of mathematics textbooks on achievement and student performance outcomes. In line with Kuecken and Valfort (2013) found no effect of the average textbook on student test scores. Regardless of the context regarding whether or not there is an influence between textbooks and student achievement, it should be noted that textbooks still contribute to student learning because as a source of learning, as in Permendikbud no. 8 Tahun 2016 pasal 1 ayat 1 concerning textbooks as the main source of learning (Kemendikbud, 2016a). Lockheed et al., (1986) explain that it is not whether or how many textbooks affect student achievement but rather what causes the effect. Knowing the relationship between the teacher and the textbooks is necessary. Therefore, the content in textbooks should support the students' complete knowledge process so that the learning process becomes intact and meaningful.

Facts show teachers are too dependent on textbooks (Wijayanti, 2019). Data shows that 70% of teachers use thematic books from publishers, while 30% design them themselves (Ain & Kurniawati, 2012), meaning that teachers prefer ready-made books rather than making them themselves. Teachers often teach in a way that is almost entirely dependent on textbooks (Sembiring et al., 2008). Other findings show that the majority of teachers 49–64% rely heavily on textbooks in terms of planning and preparing their lessons, and 79–92% of teachers use textbooks as the only source of practice in at least half of their lessons (Katrin van den Ham & Heinze, 2018). Suppose mathematics content is not included in the curriculum material, teachers can't present it (Reys et al., 2003), so it greatly influences how textbooks are used in the classroom (Herbel-Eisenmann & Wagner, 2007). This has led to many studies evaluating the contents of textbooks, implying that the development of textbooks is ideally a continuous process and will continue to change with the demands of the times for the field of education, including learning mathematics.

If you look at it starting from the elementary school level to senior high school, in the case the ratio and proportion material is a material that must be taught on an ongoing basis (Kemendikbud, 2016b). In mathematics, the concepts of ratios and proportions are foundational material for other sciences due to the fact many parts of the primary and secondary school curriculum draw on the concepts of ratios and proportions, such as item values, fractions, percentages, probabilities, measurements, transformations of shapes and figures (Ben-Chaim et al., 2012). Johar et al., (2018) Several textbooks and inverse proportion studies were also carried out. It was found that the distribution of proportional reasoning problems in textbooks for grades seven and grade 8 in the 2013 curriculum was not the same. Only three chapters contain proportional reasoning problems, namely ratios, rectangles and triangles, and the Pythagorean theory. In addition, there were complaints presented by teachers regarding the mathematics books for junior high school students in the 2013 curriculum, where the results of observations by Wahidah et al., (2021) in several schools

through interviews with mathematics teachers showed that mathematics books for junior high schools/MTs students of the same level were published by Ministry of Education and Culture, the use of vocabulary used is difficult to understand, both by students and the teacher himself. The researcher himself conducted interviews with several teachers that the BSE book used presented material with too many long narratives, so some students were too lazy to read long descriptions.

Therefore, the researcher wants to examine the in-depth textbooks, in the form of a recommended theory as a tool in analyzing textbooks, mainly focusing on items/tasks, namely the praxeological theory initiated by Yves Chevallard, because according to Glasnovic Gracin (2018) that tasks and the example listed on the textbook are the most essential element. The praxeological theory used to analyze textbooks focuses on the analysis of examples and questions in terms of the suitability of the series of tasks (T) that students will study, a collection of techniques (τ) that function to complete tasks (T), technology (θ) which is used to clarify and justify the technique (τ), and theory (Θ) to validate several technologies listed in the book. The theory of praxeology has been used in several studies analyzing mathematics textbooks, such as Wijayanti dan Winslow (2017) looking at how the idea of this praxeological reference model can explain analyzing the core mathematics of textbooks in a reasonably objective and detailed way. Praxeology can explain the comparison of communication in mathematics classes and textbooks so that textbooks can determine what mathematics should be taught and how (Pansell & Boistrup, 2018). Even praxeology can examine Japanese elementary school textbooks (Grades 1–6) and become a reasonable basis for whether the book can develop students' geometric thinking through problems, solutions, and activities in textbooks (Hayata & Amori, 2019).

Based on the background described above, this study aims to analyze textbooks on the inverse proportion, which is one of the essential materials in mathematics in terms of praxeological theory. The researcher will choose the BSE published by the Ministry of Education and Culture as the book to be analyzed. This is because the Ministry of Education and Culture issued a book that meets educational standards and can be accessed by the broader community by downloading it for free, namely the Electronic School Book (BSE), which many public schools use the book. Thus, it is hoped that the results of this research can serve as suggestions and input to all parties and imply that, ideally, the development of textbooks is a continuous process and will continue to change with the demands of the times for the field of education.

2. METHOD

The research used is descriptive qualitative research. The object of this research is the Mathematics Electronic School Book (BSE) for students of the 2013 revision of the 2017 curriculum published by the Ministry of Education and Culture, which focuses on material for inverse proportion in the form of introductory material and example descriptions which will be analyzed based on the praxeological theory of 6 assignments. In addition, the subject of research involved class VII teachers. Data collection techniques, namely content analysis, interviews, and documentation studies. The interview technique in the research aims to find out the teacher's opinion about the characteristics of the textbook presentation, investigate and analyze textbooks in facilitating students in acquiring knowledge, and confirm the findings obtained by researchers. At the same time, the documentation study technique aims to collect data on job descriptions/tasks, techniques, technology, and theory on the inverse proportion contained in the set material obtained from observations in textbooks. The instruments used were textbook analysis guidelines and interview guidelines. The data analysis technique was carried out in several steps, first by collecting and reducing data, namely collecting and having selected content to be analyzed with praxeological theory, namely examples of questions and exercises presented in textbooks deemed necessary to study in depth. The second is the presentation of the data. The presentation of the data is adapted to the analysis of praxeological theory in tabular form so that the data is presented following the provisions and the information is

organized in a concise and easy-to-understand manner. And the last is conclusion and data verification, namely assessing the suitability of the data findings obtained by researchers with the analysis results.

In the analysis of textbooks, the praxeological theory will be used by examining the presentation of inverse proportion in terms of the four praxeological organizations. In addition, due to this praxeological theory, the development of the project from the Theory of Didactical Situations (TDS) was initiated by Brousseau, where TDS adopted that creates situations that provide opportunities for students to form their knowledge in an environment (Selman & Tapan-Broutin, 2018), as well as giving rise to the process of constructing knowledge (knowledge epistemization), namely action situations, formulation situations, validation situations, and institutionalization situations (Artigue et al., 2014). Thus, in analyzing textbooks, a first step is needed, which is guidelines or signs for analysis of mathematics textbooks based on the eligibility criteria for books by the Badan Standar Nasional Pendidikan (BSNP), which are considered to meet the eligibility requirements for use in learning through the Minister of National Education Regulation, element -elements of praxeology, as well as Theory of Didactical Situations (TDS). The following are guidelines for analyzing material presentation in textbooks:

Table 1. Guidelines for Analysis of Presentation in Student Textbooks


No	Element Praxeology	Indicator
1	Type of Task (T)	<ul style="list-style-type: none"> a. The series is presented mathematically and coherently and can develop students' logical thinking skills/logical order b. Using pictures or other graphic illustrations adapted to students' cognitive development or close to the student's environment. c. The tasks presented are tasks that contain contextual, concrete problems and use specific models d. The tasks presented have a functional relationship, where there is coherence between the first task and the second and subsequent tasks.
2	Technique (τ)	Students have space/opportunity to be actively involved (action situation) in the process of understanding the material, where they try to solve problems by finding their own way. That is, there is an opportunity to determine the method desired by students (formulation situation).
3	Technology (θ)	Students have room/opportunity to carry out the verification process, justification of assignments and techniques (validation situation)
4	Theory (Θ)	Students have the space/opportunity to carry out institutional situations, which is essential because it supports students in applying conclusions (theories) obtained in validation situations to different contexts and problems before. An example is giving a problem to students in a different context before. Thus, the theory conveyed must be clear and straightforward.

3. RESULTS AND DISCUSSION

After making guidelines for analyzing textbooks, this study analyzed proportion presentations based on praxeology, namely inverse proportion. Based on the results of the analysis, study, and elaboration carried out by researchers on the presentation of inverse proportion in textbooks. The presentation of the material is converted to a form of praxeology, where the presentation of the material is viewed from the four elements of praxeology, namely *type of task* (T), *technique* (τ), *technology* (θ), and *theory* (Θ). After converting the presentation of the material into an analysis of the four elements of praxeology, the researcher conducted an in-depth analysis based on the analysis guidelines that the researcher had compiled. Thus, the researcher can determine whether there are identified problems in the presentation of inverse proportion in grade 7 students' mathematics textbooks. The following is an analysis of the presentation of student textbooks in terms of Praxeology theory:

a) Task 1

Table 2. Description of Task 1 (T_1) in students' book

Type of Task (Tugas)	<p>Dalam Kegiatan 5.3, kalian telah mempelajari perbandingan senilai dengan rasio kedua variabel adalah konstan. Hubungan lain antar dua variabel adalah perbandingan berbalik nilai. Hubungan antara ukuran dari gigi dengan kecepatannya adalah perbandingan berbalik nilai. Dari gambar di di samping, gir A memiliki banyak gigi dua kali lipat dari gigi yang dimiliki oleh gir B. Sehingga, jika gir A berputar satu kali, gir B akan berputar dua kali. Misalkan jika gir A berputar empat putaran, maka gir B berputar delapan kali putaran.</p>	 <p style="font-size: small;">Sumber: Kemdikbud Gambar 5.15 Gir</p>
Technique (Teknik)	<p>Misalkan Pak Fatkhur adalah seorang penyedia jasa tukang bangunan (kuli bangunan). Beliau berpengalaman dalam proyek-proyek pembangunan rumah tinggal, karena beliau sendiri juga seorang tukang bangunan. Beliau menjelaskan bahwa dalam menyelesaikan sebuah rumah yang berukuran $12,5 \text{ m} \times 7 \text{ m}$ diselesaikan oleh 5 tukang, termasuk Pak Fatkhur sendiri, selama 2 bulan sampai selesai. Untuk mempercepat penyelesaian bangunan, Pak Fatkhur sanggup menyediakan tukang tambahan sesuai dengan permintaan pelanggan. Pak Fatkhur dan 9 temannya pernah membangun rumah selama 1 bulan. Nah, coba kalian duga, berapa lama waktu yang dibutuhkan oleh Pak Fatkhur dan 5 orang temannya untuk menyelesaikan sebuah rumah dengan ukurannya yang sama seperti cerita di atas? Jika pelanggan Pak Fatkhur ingin memiliki rumah yang bisa diselesaikan selama 25 hari, berapa pekerja yang dibutuhkan untuk menyelesaikan pembangunan rumah? Bagaimana strategi untuk menyelesaikannya?</p> <p>Source: BSE Student Book p. 41 <i>There is no way presented in the textbook</i></p>	
Technology (Teknologi)	<p><i>There is no opportunity for students to justify tasks and techniques</i></p>	
Theory (Teori)	<p><i>There is no opportunity for students to be able to conclude firmly and straightforwardly about the concept of inverse proportion</i></p>	

The initial concept material presents the inverse proportion in long descriptive paragraphs. In line with interviews conducted with teachers, the following is an excerpt of the dialogue: "The results of the sample questions are long-winded. So teacher sometimes look for other ways, the students find it difficult, because when they read the questions they don't understand. Because reading mathematics is different from reading Indonesian language material. What difficulties do your children have? Known in the problems, recognize symbols and so on." Task T_1 contains descriptions in the form of examples of inverse proportion, such as the relationship between the size of the gear and its speed and the relationship between the number of workers and the time needed, which seems too long in the paragraph and makes students not understand the meaning of the paragraph. In the example in the introductory material, namely, the relationship between the size of the gear and its speed can cause problems in students' minds. Even though the book already uses pictures or graphic illustrations of gears, these examples are not yet in accordance with students' cognitive development or close to the student's environment. In fact, according to Piaget, cognitive development is the progressive reorganization of mental processes as a result of biological maturation and

environmental experiences (McLeod, 2018). Thus, children will build an understanding of the world around them, then experience the difference between what they already know and what children find in their environment.

Therefore, in the introductory material, it is better to use examples of cases that are close to the student's environment, such as the relationship between the number of farm animals and the length of time used by livestock in consuming the feed provided by breeders, or other cases that are following students' cognitive development. In addition to content adapted to the student's environment, the presentation of introductory material should not be made in a narrative that is too long, even assisted by presenting material in the form of illustrations/graphics and tables.

In the introductory material or assignment 1, there is no method presented in the textbook, there is no room for students to justify tasks and techniques, and it does not provide opportunities for students to draw firm and straightforward conclusions about the concept of inverse proportion. Therefore, task 1 only presented contextual examples in inverse proportion. Then, according to the results of the related teacher interviews, the introductory material should also be added by reviewing students' knowledge in drawing and identifying from graphs, which form the basis of knowledge in interpreting inverse proportion graphs.

b) Task 2

Table 3. Description of Task 2 (T₂) in students' book

Type of Task (Tugas)	Speed and travel time Alan rides a motorcycle and covers a distance of 480 km when going home. Every time he goes home, he tries a different average speed and records the travel time. The table below shows the average speed of the motor and the time taken.										
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="background-color: #0056b3; color: white;">Kecepatan Rata-rata (x) (km/jam)</td> <td style="text-align: center;">80</td> <td style="text-align: center;">75</td> <td style="text-align: center;">60</td> <td style="text-align: center;">40</td> </tr> <tr> <td style="background-color: #0056b3; color: white;">Waktu (y) (jam)</td> <td style="text-align: center;">6</td> <td style="text-align: center;">6,4</td> <td style="text-align: center;">8</td> <td style="text-align: center;">12</td> </tr> </table>	Kecepatan Rata-rata (x) (km/jam)	80	75	60	40	Waktu (y) (jam)	6	6,4	8	12
Kecepatan Rata-rata (x) (km/jam)	80	75	60	40							
Waktu (y) (jam)	6	6,4	8	12							
Technique (Teknik)	Alan wants to know how long it will take him to ride his motorcycle at an average speed of 50 km/hour Source: BSE Student Book p. 42 By multiplying each column between average speed rata (x) and time (y), $80 \times 6 = 480$; $75 \times 6,4 = 480$; ... It is found that 480 is a constant, and $xy = 480$ or $y = \frac{480}{x}$, where is the average speed (x) and time (y). So it is substituted with the value of x, which is 50 km/hour. Give a conclusion										
Technology (Teknologi)	It can be seen that the ratio between the columns is different (not a comparison), but the product of the two shows the same or constant result. This is the nature of inverse proportion.										
Theory (Teori)	Inverse Proportion is the product of two variables or quantities (x and y) is constant, which can be shown by the equation: $xy = k$, or $y = \frac{k}{x}$ Where k is constant										

On task T₂ a table shows the relationship between average speed and time, and the distance traveled is known. Students determine the time taken in this task if the speed is known. The technique carried out complies with the guidelines because students check by multiplying the velocity and time in the table, thus formulating an equation $xy = 480$, or $y = \frac{480}{x}$, where x is the average speed and y is time.

The technology in this task verifies that the ratio between the columns is different, but the product of the two is the same. It's just that technology has not stated the meaning of the relationship between

speed and time "If the speed decreases, the time taken will increase", and vice versa. As for the basic concept (theory) that is produced is the characteristic inverse proportion in the table and the equation form.

c) Task 3

Table 4. Description of Task 3 (T_3) in students' book

Type of Task (Tugas)	Based on Problem 5.2, draw a graph of the equation that expresses the ratio between the average speed and the time traveled. Source: BSE Student Book p. 43 Note: The writing on the sample questions is wrong, so students are suspected of getting the wrong information
Technique (Teknik)	Based on the table, coordinate points can be obtained (x, y) , then entered into Cartsius coordinates as follows: <i>Grafik yang terbentuk adalah sebagai berikut.</i>
Technology (Teknologi)	It can be seen that the graph formed from the inverse proportion equation does not pass through the origin $(0, 0)$ and does not intersect the coordinate axes
Theory (Teori)	The inverse proportion graph has the following characteristics: Does not pass through the origin $(0, 0)$ and does not intersect the coordinate axes

On task T_3 ask students to draw a graph of the data T_2 . However, on T_3 , there was a writing error in the question instructions, so it was assumed that students and teachers would get incorrect information, namely in the following figure:

Berdasarkan Masalah 5.2, gambarkan grafik persamaan yang menyatakan perbandingan antara kecepatan rata-rata dan waktu yang ditempuh.

Figure 1. Task 3 on the textbook

Based on Figure 1, there is a sentence "based on Problem 5.2..." when viewed in a textbook, it turns out that Problem 5.2 has no relationship with the context of the average speed and time taken, resulting in a writing error in this third task. Therefore, it is better to write "Problem 5.2" to be replaced with "Based on Ayo Amati, the comparison between the average speed and the time traveled, draw a graph of the equation based on this information."

In addition, the technique in task 3 was complex because the coordinate points that had to be drawn were decimal numbers, so it was better if the numbers were presented in integer form. Moreover, according to the results of interviews with teachers, sometimes students also have difficulty drawing

graphs because they forget or when the teacher had not delivered the previous education (Elementary School). the following excerpts from the dialogue:

Teacher: "yes its' a problem, because some say it hasn't been taught yet and forget, because of how to draw graphs students experience confusion, so you have to explain it first. And the material for drawing graphs in detail is actually in grade eight, whereas even in grade seven, they have been asked to draw graphs. This is the problem."

Thus, the teacher anticipates learning to insert explanations about graphs in terms of supporting the learning process of inverse proportion.

Also, in task 3, there is no institutional situation, where after the conclusions presented in the technique of task 3, namely: "Note that the graph formed from the inverse proportion equation does not pass through the origin (0, 0) and does not intersect the coordinate axes" because there is no opportunity for students to do an analysis comparing the form of comparison graphs of proportion and inverse proportion. In fact, this situation is critical because it supports students in applying the conclusions obtained in validation situations (inverse proportion graphs) to different contexts and problems before, such as being given several forms of graphs and doing graph analysis, which includes inverse proportion graphs.

There is no justification process in task 3 because it does not ask students to verify the shape of the graph that has been made. The theory presented meets the guidelines prepared by the researcher, where the textbook clearly presents the conclusions of the graphs drawn, namely the inverse proportion graph. However, the conclusions obtained can be applied to other contexts, so it is advisable to add questions at the end of the assignment. Then, after the inverse proportion graphs are obtained, what is the difference between the proportion and inverse proportion graphs?

d) Task 4

Table 5. Description of Task 4 (T₄) in students' book

Type of Task (Tugas)	12 people can complete a piece of work in 20 days. How long will it take to complete the work if six people do it? Source: BSE Student Book p. 46						
Technique (Teknik)	Using writing a comparison of two quantities between many workers and time, for example: Comparison can be obtained from:						
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <th>Worker</th> <th>Time</th> </tr> <tr> <td style="text-align: center;">12</td> <td style="text-align: center;">20</td> </tr> <tr> <td style="text-align: center;">6</td> <td style="text-align: center;">x</td> </tr> </table>	Worker	Time	12	20	6	x
Worker	Time						
12	20						
6	x						
	$\frac{\text{many worker A}}{\text{many worker B}} = \frac{\text{time B}}{\text{time A}}$ $\frac{12 \text{ orang}}{6 \text{ orang}} = \frac{x}{20 \text{ hari}}$						
	Then find x in various ways. Give a conclusion (obtained x = 40) <i>There is no opportunity for students to seek alternative ways</i>						
Technology (Teknologi)	<i>There is no opportunity for students to justify the technique about the reasons one of the ratios is written upside down or to encourage students to find alternative solutions</i>						
Theory (Teori)	<i>There is no opportunity for students to be able to draw firm and straightforward conclusions about how to compare two quantities in the case of inverse proportion</i>						

On Task T_4 presented a case of comparison of the reverse value between workers and time. The technique presented in textbooks only uses a comparison of two ratios, and one of them is reversed to find a value x so as not to provide opportunities for students to find their way. Even though in the previous task, namely task 2, the basic concept was given that two quantities that change in value when multiplied by the two are constant, there is another way. There is no justification in textbooks why one of the ratios in the comparison case reverses the value, and the writing is reversed when calculating. There is no conclusion (theory) about the general way of applying value-reverse comparisons in everyday life. An explanation of the characteristics of cases/problems in the application of rinverse proportion must be fully detailed to students, because students will find it difficult to solve inverse proportion questions. After all, they do not know the differences in comparison cases of direct or inverse proportion. As the results of interviews with teachers, the following excerpts from the dialogue:

Teacher : “More detailed is the material, where the comparative material reverses the value, meaning that the writing is also reversed. Then the cross multiplication is also reversed, it is not the same as the direct proportion, so it must be confirmed again in the book”.

Thus, the teacher is important in constructing students' knowledge about comparisons, so students can distinguish comparisons of direct or inverse to a contextual problem.

e) Task 5

Table 6. Description of Task 5 (T_5) in students' book

Type of Task (Tugas)	Mr. Fatkhur is a builder (builder) service, provider. He is experienced in residential construction projects and is also a builder. He explained that in completing a house, it could be completed by five builders, including Mr. Fatkhur himself, for two months until the finishing was complete. To speed up the completion of the building, Mr. Fatkhur can provide additional workers according to customer requests. Mr. Fatkhur and 9 of his friends built a house in 1 month. So, now try to guess how long it took Mr. Fatkhur and 5 of his friends to complete a house the same size as described above. If Mr. Fatkhur's customer wants a house that can be completed in 25 days, how many workers are needed to finish building the house? Source: BSE Student Book p. 48										
Technique (Teknik)	Using writing a comparison of two quantities between the number of workers and time, for example, it can be written in tabular form to make it easier:										
	<table border="1" style="margin: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Worker</th> <th style="padding: 5px;">Time</th> </tr> </thead> <tbody> <tr> <td style="text-align: center; padding: 5px;">5</td> <td style="padding: 5px;">2 bulan = 60 hari</td> </tr> <tr> <td style="text-align: center; padding: 5px;">10</td> <td style="padding: 5px;">1 bulan = 30 hari</td> </tr> <tr> <td style="text-align: center; padding: 5px;">6</td> <td style="text-align: center; padding: 5px;">x</td> </tr> <tr> <td style="text-align: center; padding: 5px;">y</td> <td style="padding: 5px;">25 hari</td> </tr> </tbody> </table>	Worker	Time	5	2 bulan = 60 hari	10	1 bulan = 30 hari	6	x	y	25 hari
Worker	Time										
5	2 bulan = 60 hari										
10	1 bulan = 30 hari										
6	x										
y	25 hari										
	Then find the value of x first by making a comparison: $\frac{10}{6} = \frac{x}{30}$										
	Then find x in various ways. Give a conclusion ($x = 50$) Then, find the value of y by making a comparison: $\frac{6}{y} = \frac{25}{x}$										
	$\frac{6}{y} = \frac{25}{50}$, with $x = 50$										
	find x in various ways. Give a conclusion <i>There is no opportunity for students to seek alternative ways</i>										

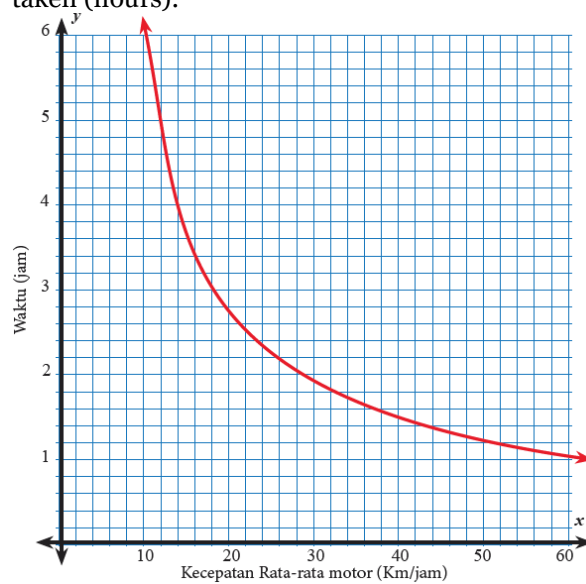
Technology (Teknologi)	It can be seen that the ratio between the columns is different (not proportion), but the product of the two shows the same or constant result. This is the nature of inverse proportion.
Theory (Teori)	<i>There is no opportunity for students to be able to draw firm and straightforward conclusions about how to compare two quantities in the case of inverse proportion</i>

On Task T₅A comparison of inverse values between workers and time is also presented, but the writing is in the form of long paragraphs. The technique presented in textbooks only compares two ratios; one is reversed to find the value of x., not to provide opportunities for students to find their way. Even though in the previous task, namely task 2, the basic concept was given that two quantities that change in value when multiplied by the two are constant, there is another way. There is no justification in textbooks why one of the ratios in the comparison case reverses the value, and the writing is reversed when calculating. There is no clear theory about the general way of applying value-reversal comparisons in everyday life. In theory, there is also no conclusion, so it is hoped that new questions will arise about "based on the table above, what is the relationship between workers and days?"

f) Task 6

Table 7. Description of Task 6 (T₆) in students' book

Type of Task (Tugas) The distance from city P to city Q is 60 km. The graph below shows the relationship between motorcycle speed (km/h) and time taken (hours).



- Using the graph above, determine the vehicle's speed if it takes to travel from city P to Q is 1.5 hours. Explain how you got the answer.
- Can you determine the equation of the graph above? Explain.
- Open-Ended Question
Can you determine the speed the driver needs to cover the total outbound and return journey time of 3 hours? How do you determine it?

Technique (Teknik) Source: BSE Student Book p. 49
When asked how fast, if it takes 1.5 hours, you can use the equation $v = \frac{60}{t} = \frac{60}{1,5} = 40$ km/hour
(τ_b) So the graph equation is obtained:

	$vt = 60$ or $v = \frac{60}{t}$, v is velocity, and t is time
	(τ_c) the answers varied , where for a 3-hour return trip, it could travel as fast as $v = \frac{60}{t} = \frac{60}{3} = 20$ km/hour, or
	The trip goes for 1 hour, so velocity $v = \frac{60}{t} = \frac{60}{1} = 60$ km/hour and the return trip for 2 hours, so velocity $v = \frac{60}{t} = \frac{60}{2} = 30$ km/hour
	And other variations of the answer
Technology (Teknologi)	Connecting distance, time, and speed, to obtain the equation between the three, namely: space = velocity \times time, or velocity = $\frac{\text{space}}{\text{time}}$
Theory (Teori)	It can be seen that the equation between velocity and time shows the equation is inverse proportion. <i>There is no opportunity for students to be able to draw firm and straightforward conclusions about how to compare two quantities in the case of reverse value comparisons</i>

On Task T_6 there is an inverse proportion graph presented, where the information on task 6 does not hone students' knowledge in remembering the relationship between space, velocity, and time. This is because not all students still remember the material on the relationship between space, velocity, and time. Therefore, this task should be given additional information, such as finding equations first from what is known, so that task 6 can be done by students mathematically and coherently. The technique is carried out by formulating velocity and time, where the space is constant. The justification was also carried out because there were open questions so that students assessed the techniques they worked on themselves in question point c. However, the theory at the conclusion does not yet exist, meaning that there is a need for additional questions in task 6, namely "What is the relationship between velocity and time? If the velocity is getting higher, then the time..." or vice versa. This is done so that the conclusions obtained by students become basic knowledge, not just memorizing formulas. It can also be concluded by giving examples of other cases related to inverse proportion.

Recapitulation of Praxeological Analysis in Student Textbooks

After the per-task praxeological analysis described above, the following is a recapitulation table of praxeological analysis in student textbooks of inverse proportion as follows:

Table 8. *Praxeological Analysis Recapitulation at Inverse Proportion in Student Textbooks*

Task	τ Technique	θ Technology	Θ Theory
T_1	×	×	×
T_2	√	√	√
T_3	×	×	√
T_4	×	×	×
T_5	×	×	×
T_6	√	√	×

Based on table 8, it can be seen that First, only T_2 which are assumed to have techniques, technologies, and theories guided by praxeology because the tasks are presented in a tabular form

about contextual problems, where the techniques are carried out to make students carry out verification and formulation so that an equation is obtained. The technology for this task is also available where there is a justification process that the ratios between columns are different. Still, the product of the two is the same. The basic concept (theory) presented in the textbook describes the characteristics of the inverse proportion in the table and the form of the equation. However, it would be better if the task contained questions about the meaning of the relationship between velocity and travel time "If the velocity decreases, the time taken will increase," and vice versa. This is so that students can understand the concept of inverse proportion in a context/problem.

Second, that T_1 , T_4 and T_5 which is assumed not to have all met the elements of technique, technology, and theory guided by praxeology because the techniques presented in T_1 not served, then on T_4 and T_5 students are not allowed to look for ways to choose because they only do technical elements (τ) that are procedural, namely finding the x value of the two ratios in the ratio of inverse proportion (between workers and time) with the version of the method described in the textbook, namely making a table of the two quantities which are compared with the concept of inverse proportion, then applying the cross product rule. Even though the students in the previous task, namely task 2, have been given the conclusion that the comparison of inverse proportion is the product of two variables or quantities (x and y) that are constant, which can be shown by the equation:

$$xy = k, \text{ or } y = \frac{k}{x} \text{ Where } k \text{ is constant}$$

That is, apart from using the rules for forming comparisons of two ratios presented in textbooks, it turns out that there is another alternative that was presented in the previous task, namely multiplying two known quantities (with the concept of multiplication and division) with the principle that the result of both must be the same, there is no room for students. To look for other options in student textbooks, for example, as follows:

Perbandingan dapat diperoleh dari tugas 4 :

Pekerja		Waktu	
12	×	20	→ $12 \times 20 = 240$
6	×	x	→ $6 \times (x) = 240$

Figure 2. An alternative way to task 4

It can be seen that finding the value of x can be done in an alternative way, namely, the multiplication rule. Therefore, there is no justification process for the three tasks because T_1 just observe the inverse proportion situation, then T_4 and T_5 the justification process was also not carried out because there was no room for students to justify the assignments and techniques presented in the student textbooks, namely justification for the reasons why writing one of the ratios in the ratio of reversed values had to be written upside down. This will affect the knowledge aspect of students whether it becomes meaningful or not, becomes long-term memory or not, because based on TDS which contains one of them is a validation situation, where students carry out the process of proving and developing what is obtained so that the technique carried out is said to be valid. Where using seeking a more objective justification, such as an explanation, evidence, or demonstration, so that the model that has been created is relevant to another (Alves et al., 2021). Thus, the methodological principle of TDS involves situations that apply knowledge and focus on meaning, not just giving assignments in the hope that students can complete them efficiently without knowing why they use the technique. Even though using the multiplication rule can also be done, provided that they understand the concept of inverse proportion, the product of two variables or quantities (x and y) is constant. Thus, students do not have the opportunity to use and develop perceptual, memorial, and reflective that have been obtained in previous assignments in constructing new knowledge, namely the application of inverse proportion to a problem.

The theory presented in T_1 , T_4 , and T_5 not presented clearly and straightforwardly so that students cannot conclude the three tasks. Even though the theory is critical because it supports students in applying the conclusions (theory) obtained in validation situations to different contexts and problems before. Even though the theory is important because it supports students in applying the conclusions (theory) obtained in validation situations to different contexts before, this is related to how the teacher knows the expected regularity and is developed to identify possible steps taken by students in developing initial mathematical ideas to become formal concepts (Sztajn et al., 2012). This formal concept is called theory so that when students get the theory, students can complete different contexts. Thus, if students are involved in situations that apply knowledge and focus on meaning, it will provide a purpose for students to be involved in situations, and knowledge becomes meaningful and useful (Mangiante-Orsola, Christine; Perrin-Glorian, Marie-Jeanne; Strømskag, 2018). Thus, it is hoped that it will build knowledge in students that is epistemic and meaningful, not just memorizing formulas or mastering procedural solutions.

Finally, when viewed in terms of the arrangement of the task sequence (T), namely T_1, T_2, T_3, T_4, T_5 , and T_6 has been presented not coherently, the following is the description of the task flow and the objectives of the task:

T_1 → the concept of introducing inverse proportion situations

T_2 → Inverse Proportion form tables and equations

T_3 → comparison of turned values form graph

T_4 → inverse proportion implementation (finding x)

T_5 → inverse proportion implementation (finding x)

T_6 → Inverse Proportion form graphs and equations

Note:

→: discuss

Can be seen between T_3, T_4, T_5 , and T_6 not related to each other, where T_6 placed at the end of the task flow, whereas T_6 still discussing the comparison of inverse values in various forms, namely tables, graphs, and equations. As for the discussion of the form of comparison of inverse, values are in T_2 and T_3 , so that the tasks have not formed good and correct mathematization. Therefore, it is best if the task flow is rendered from T_1, T_2, T_3, T_6, T_4 , and T_5 , because the process of building correct knowledge, according to TDS:

1. An action situation in the form of a dish that is closest to the student (illustration on T_1)
2. In formulation situations, students try to find ways to increase student action so that it is easier to achieve the expected goals in the learning process. So available on assignment T_2, T_3 , and T_6 because there are activities where students try to formulate equations and draw and read graphs.
3. Validation and institutional situations, where students are expected to verify the conclusions that students have obtained, namely the equations obtained from T_2 to complete T_4 and T_5

4. CONCLUSIONS

Based on the results of the analysis and in-depth study of student textbooks on inverse proportion, and the guidelines that have been prepared by researchers on the four elements of praxeology, as well as interviews with teachers as material for consideration and confirmation of the results of the analysis of textbooks by researchers, several things were found that shows that the textbooks of students and teachers in this study have advantages and disadvantages. The presentation of inverse proportion in textbooks is quite excellent and contextual. However, several presentations need to be completed, such as there are illustrations that are not following students' cognitive development, the composition of the tasks presented has not formed good and correct mathematization, and some

technical presentations are incomplete and do not provide space for students to choose how to solve cases of inverse proportion, there is no opportunity for students to justify the techniques used, and in some situations, there are no situations to make students conclude the results of the justifications carried out.

The analysis of the presentation of inverse proportion in this study becomes suggestions, input, and recommendations for policymakers, especially the mathematics textbook drafting team on inverse proportion in the upcoming new curriculum. Then, the results of research on the contents of the inverse proportion can become a basis for teachers as users of textbooks. Therefore, teachers are expected not only to rely on textbooks but also to look for other sources of teaching materials as material for consideration in the learning process so that the process of forming students' knowledge about inverse proportion turns into an absolute value, not just for students to memorize formulas and students quickly forget. Besides that, there are also some limitations in this study, where the researcher only examined one book and material. Therefore it is hoped that future researchers can analyze other types of textbooks or analyze them with other essential materials, both at the elementary to high school levels.

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