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STUDY OF DIDACTIC TRANSPOSITION: EXPLORATION OF TAUGHT KNOWLEDGE IN REVEALING LEARNING OBSTACLES FOR MADRASAH STUDENTS

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ABSTRACT

Didactic transposition helps bridge the gap between formal mathematical knowledge and classroom instruction, ensuring more effective material delivery aligned with students' needs. The purpose of this study was to explore how linear equations of one variable are taught to eighth-grade students at Madrasah Tsanawiyah based on the didactic transposition approach. This study focuses on the knowledge transfer stage of a didactic transposition study on linear equations of one variable, using qualitative research with a descriptive method. The study's results showed that the presentation of material in the teaching and learning process led to a shift in understanding of linear equations of one variable from the concept of linear equations of one variable in a didactic context. Incomplete explanations of linear equations with one variable cause learning obstacles. Ontogenic obstacles arise from students' cognitive limitations, epistemic obstacles from conceptual complexity, and didactic obstacles from ineffective teaching strategies. This study is conducted because didactic transposition can bridge the gap between formal mathematical knowledge and classroom instruction, yet it may also create learning obstacles. The findings can help improve teaching strategies by addressing conceptual gaps and learning obstacles, making mathematics instruction more effective for students at Madrasah Tsanawiyah.

Keywords: Didactic Transposition, Learning Obstacle, Taught Knowledge

INTRODUCTION

Didactic transposition is closely linked to the theory of didactic situations, both contributing to the epistemological study of mathematical knowledge (Mello, 2020). These theories explore how scientific knowledge is adapted and simplified for classroom instruction. The study of didactic transposition underpins the theory of didactic anthropology, which investigates the legitimacy of didactic arrangements in mathematics education (Kluth & Almouloud, 2020). An integrated theory is proposed by combining the ideas of didactic transposition and didactic situations with concepts from cognitive science to analyze how academic knowledge is transformed into teachable material (de Mello, 2019). This integrated approach includes guidelines for effective didactic transposition in mathematics, physics, and chemistry, and considers the concepts of didactic contract and didactic situation.

Figure 1 illustrates the didactic transposition process, which explains how mathematical knowledge is transformed from scholarly understanding to knowledge that is taught and ultimately learned by students (Chevallard & Bosch, 2014). The theory of didactic transposition was developed to examine how mathematical concepts evolve and adapt as they transition from academia to classroom practice. This process involves several steps: first, scholarly knowledge is selected and reorganized into knowledge suitable for teaching, which is then put into classroom instruction as the actual material taught. Finally, through students' engagement with the content, it becomes learned knowledge. The didactic transposition process requires transforming abstract mathematical ideas into teaching-appropriate forms while maintaining their mathematical integrity. Understanding this process enables educators to design instructional strategies that effectively transform complex mathematical concepts into meaningful learning experiences for students (Chevallard & Bosch, 2019).

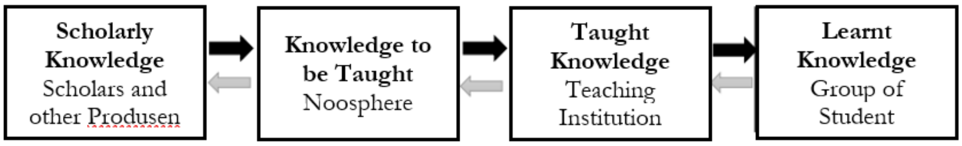

Figure 1. Didactic Transposition Process (Chevallard & Bosch, 2014)

Research indicates that effective mathematics teaching, particularly for linear equations in one variable, requires a focus on both procedural and conceptual understanding. The use of technology and visual aids can enhance students' comprehension when applied appropriately (Numonova, 2024). Specific teaching approaches, such as the balance model, have been shown to improve students' ability to solve linear equations and understand underlying principles (Mengistie, 2020; Atteh et al., 2018). This method facilitates the transition from arithmetic to algebraic thinking, which is crucial for beginners in algebra, while also supporting the development of algebraic reasoning by contextualizing symbols and symbolizing contexts. (Maudy et al., 2017). Overall, these studies emphasize the importance of employing diverse teaching strategies, including the integration of technology and the use of concrete-to-abstract representations, to enhance students' understanding of linear equations and algebraic concepts.

Recent studies highlight the importance of effective teaching approaches for linear equations with one variable. Using a contextual approach significantly improved student learning outcomes, with post-test scores increasing to 86.42% (Hasina et al., 2021). The didactic transposition process from scholarly knowledge to classroom teaching requires careful consideration to avoid potential learning barriers (Suarsana et al., 2024). Teacher professionalism is influenced more by training experiences and educational background than by years of teaching experience (Harisman et al., 2019). Implementing a balancing model to teach linear equations enhanced students' understanding and problem-solving strategies, with most students showing improvement in post-test performance (Mengistie, 2020). These findings underscore the importance of contextual teaching methods, thoughtful curriculum design, and ongoing professional development for teachers to enhance the teaching and learning of linear equations with one variable.

One of the important topics taught at the Madrasah Tsanawiyah (MTs) level is linear equations of one variable. Linear equations of one variable are part of the mathematics curriculum for grade VIII and begin with an understanding of algebraic forms in grade VII. In general, a linear equation of one variable is defined as an equation that only has one variable with a power of one and uses an equal sign (=) to express the relationship between the two sides. The general form of linear equations of one variable is $ax + b = c$, where a, b, and c are constants, while x is the variable whose value must be found. Linear equations of one variable are often applied in everyday life, such as calculating the price of goods, determining average speed, and solving simple financial problems (Wicaksono et al., 2024).

Research on students' difficulties in learning algebra, particularly linear equations of one variable, reveals several common error types. Conceptual errors include misunderstandings of variables, difficulties in interpreting word problems into mathematical models, and misconceptions about algebraic concepts (Pramesti & Retnawati, 2019). Principal errors involve inaccurate use of equation forms and incorrect variable application (Kenney & Ntow, 2024). Students struggle with arithmetic operations, understanding algebraic statements, and the meaning of the equals sign (Asquith et al., 2007; Ntow et al., 2024). Difficulties also arise in identifying coefficients, performing operations with positive and negative terms, and applying the distributive property. Factors contributing to these challenges include a lack of understanding of basic algebraic definitions and concepts, as well as insufficient mastery of integer operations (Nurhamsiah & Dian, 2016). These findings underscore the need for targeted instructional strategies to address students' specific difficulties in learning algebra.

Students' difficulties in understanding linear equations of one variable can be caused by various internal and external factors (Septian & Monariska, 2021). Internal factors include a lack of interest in learning, poor learning habits, and learning experiences that do not support conceptual understanding. Meanwhile, external factors include a lack of appropriate textbooks, overly dense materials, and ineffective teaching methods. In this case, didactic transposition theory offers a perspective that can help overcome these difficulties.

Based on observations conducted at one of the State MTs in Palu City, it was found that there was a gap and inconsistency in the order of linear equation material of one variable between scholarly knowledge and curriculum, which had an impact on the making of lesson plans and the teaching process of teachers in the classroom. The researcher was interested in analyzing and exploring the problems that occurred in the school more deeply. The purpose of this study was to explore how linear equations of one variable were taught to grade VIII MTs students based on the didactic transposition approach. The implications of this study can help teachers develop more effective teaching strategies for conveying the concept of linear equations of one variable. By understanding the effective teaching process, it is hoped that this study can contribute to improving students' understanding and achievement in mathematics learning.

This study examines the didactic transposition of linear equations in one variable, focusing on the taught knowledge stage to understand how the material is delivered in class. Using a qualitative descriptive approach, data were collected from mathematics teachers and eighth-grade students at MTs Negeri 3, Palu City, through observations, interviews, and document analysis. Observations explored teaching strategies and classroom interactions, while interviews examined teachers' understanding and challenges in teaching linear equations in one variable. Document analysis assessed the alignment of lesson plans, textbooks, and student worksheets with didactic transposition theory. Data were analyzed through reduction, presentation, and drawing conclusions to identify patterns in linear equations in one-variable instruction. The findings aim to help educators refine their teaching strategies and bridge the gap between the knowledge to be taught and the knowledge learned, ultimately enhancing students' understanding of linear equations in one variable.

This study has several novelties: (1) the integration of didactic transposition theory to analyze the learning of linear equations in one variable at the Madrasah Tsanawiyah (MTs) level, (2) the exploration of how the concept of linear equations in one variable transitions from scientific knowledge to curriculum and classroom teaching, which is rarely examined in previous studies, (3) the identification of mismatches between scholarly knowledge and the curriculum, impacting lesson plan development and classroom instruction, (4) a new perspective on students' learning difficulties and the challenges teachers face in implementing didactic transposition, and (5) an alternative approach that bridges the gap between theory and practice, shifting the focus from student errors to improving teaching strategies.

METHOD

These studies examine the didactic transposition of mathematical concepts, specifically linear equations and fractions. Didactic transposition involves transforming scholarly knowledge into taught knowledge, with potential conceptual gaps emerging during this process (Unaenah & Suryadi, 2023). These findings underscore the importance of careful curriculum design and the need for enhancing teachers' knowledge and skills in effectively teaching mathematical concepts. However, in this article, the focus of the study is limited to the taught knowledge stage, which reflects how teachers teach linear equations in one variable material in class. Therefore, this study uses a qualitative approach with a descriptive method to provide an in-depth picture of the implementation of linear equations in one variable teaching in madrasahs.

The subjects of this study were mathematics teachers and eighth-grade students at MTs Negeri 3, Palu City, Central Sulawesi. Data were collected through classroom observations, interviews with teachers, and analysis of learning documents. Observations were conducted to examine the teaching strategies employed by teachers, teacher-student interactions, and the delivery of material on linear equations in one variable in class. Interviews were conducted to explore teachers' understanding of linear equations in one variable material and the challenges faced in the teaching process (Ball et al., 2008). Additionally, a document analysis was conducted on lesson plans to assess the suitability of the material taught in relation to the didactic transposition theory (Chevallard & Bosch, 2014).

The data analysis techniques in this study encompass data reduction, data presentation, and conclusion (Malmqvist et al., 2019). Data from observations, interviews, and document analysis were analyzed to identify patterns related to how the concept of linear equations in one variable is taught in the classroom. Furthermore, the reduced data is presented in the form of descriptive narratives, allowing for the identification of didactic transposition patterns in the taught knowledge stage. Finally, conclusions are drawn based on the interpretation of the findings to provide an overview of how didactic transposition occurs in one-variable linear equation teaching and the challenges faced by teachers in the process (Casi & Garzetti, 2024).

RESULTS AND DISCUSSION

The analysis of internal didactic transposition of teachers aims to identify the facts and phenomena behind the transition of knowledge on linear equations of one variable from knowledge to be taught (knowledge to be taught) to knowledge of linear equations of one variable as taught knowledge (taught knowledge) in class. Finally, the transition of linear equations of one variable as knowledge to be taught (taught knowledge) in class into linear equations of one variable as knowledge learned by students (learnt knowledge). The material of linear equations of one variable studied in the analysis of internal didactic transposition of teachers includes the concept of linear equations of one variable and the solution of linear equations of one variable. The sources analyzed are the lesson plan documents created by teachers and the learning process conducted in class.

Analysis of the material of linear equations of one variable in the Lesson Plan

The Lesson Plan for the mathematics subject of linear equations of one variable, which is the subject of the researcher's study, was prepared by an MTs mathematics teacher. In this lesson plan, three reading sources and references are mentioned, namely 1) the Grade VIII Teacher's Guidebook and the Grade VIII Student Book by Independent Curriculum Mathematics Book, Jakarta: Ministry of Education, Culture, Research and Technology of the Republic of Indonesia 2022, 2) YouTube, Google, and sites, and 3) other relevant books. The lesson plan that was prepared was carried out in two meetings.

Table 1 outlines the initial competencies that students should possess and the learning objectives expected to be achieved in the study of linear equations with one variable. These competencies include fundamental arithmetic operations, basic algebraic manipulations, and an understanding of equality and inequality, which serve as prerequisites for comprehending linear equations (Abdul-Karim et.al, 2023). The learning objectives outlined in the table emphasize students' ability to identify, formulate, and solve linear equations in various contexts, including real-world problem-solving scenarios. Additionally, the objectives aim to enhance students' critical thinking and reasoning skills by encouraging them to explore different representations of linear equations, such as symbolic, graphical, and numerical forms. Through structured learning activities, students are expected to develop a conceptual understanding of linear equations, recognize patterns in algebraic relationships, and apply appropriate strategies to solve problems efficiently. The table also highlights progressive stages of learning, ensuring that students move from basic understanding to more advanced applications, thereby fostering deeper mathematical thinking.

Table 1. Initial Competencies and Learning Objectives

No.	Initial Competence	Learning objectives
1	• Determine the truth value of a statement • Create a modelling simulation of an open and closed statement • Change problems related to linear equations of one variable into a mathematical model	• Students are able to determine the truth value of a statement • Students are able to create modelling simulations of open and closed statements • Students are able to change problems related to linear equations of one variable into mathematical models
2	• Apply the rules of addition and subtraction to solve problems of linear equations in one variable. • Transform problems related to linear equations in one variable into mathematical models. • Apply the rules of multiplication and division to solve problems of linear equations in one variable.	• Students are able to apply the rules of addition and subtraction to solve problems involving linear equations of one variable. • Students are able to change problems related to linear equations of one variable into mathematical models. • Students are able to apply the rules of multiplication and division to solve problems involving linear equations of one variable.

Source: MTs Teacher Lesson Plan

Based on interviews with teachers, the lesson plan is used for two meetings. However, in the learning activities outlined in the lesson plan, there is no visible division of materials between Meeting 1 and Meeting 2. In this lesson plan, it is stated that Blended Learning is employed through a learning model that incorporates Project-Based Learning (PBL) and differentiated learning based on Social-Emotional Learning (SEL). Learning activities consist of preliminary activities, core activities, and closing activities. First, the preliminary activity takes the form of apperception, which involves checking student attendance and motivation. The core activity is learning through various activities, including *"Let's observe," "Let's explore," "Let's* *work together," "Let's try," technology, and thinking creatively and critically*. The closing activity is filled with reinforcement activities and evaluation of the learning that has been carried out.

In the first meeting, the lesson plan is designed to guide students in understanding the concept of linear equations in one variable through an exploration and discussion-based approach. Learning begins by observing a conversation between two characters, Humam and Aldo, which contains statements that can be assessed for their truth. This activity aims to help students recognize the difference between closed and open sentences, as well as understand basic concepts in mathematical logic. Next, students are invited to identify open and closed sentences through small-group discussions. They will analyze several statements and identify the truth value of each statement. Through this activity, students are trained to think critically and understand how mathematical statements can be categorized based on their nature. In the next stage, students will find the general form of a linear equation of one variable through exploration activities. They are asked to pair mathematical models with the appropriate category, namely, whether they are linear equations of one variable or not. This activity is designed to provide direct experience in recognizing the structure of linear equations and distinguishing them from other mathematical forms, such as inequalities.

Based on the analysis of the lesson plan's contents at the first meeting above, the presentation of the material in the lesson plan raises several learning obstacles that can be categorized as ontogenic, epistemic, and didactic obstacles in learning the material on linear equations of one variable. Ontogenic obstacles occur due to the limitations of students' cognitive development in understanding the concepts taught. Some students may struggle to understand the concept of closed and open sentences due to their high level of abstraction. Research has identified several common difficulties students face in learning algebra. These include misconceptions about variables, particularly in understanding their role in different contexts (Jupri et al., 2014). Students also struggle with algebraic expressions, often failing to manipulate them according to accepted rules (Nadira et al., 2023). The concept of equality and proper use of the equal sign pose significant challenges (Asquith et al., 2007). Translating word problems into algebraic language is another major hurdle. Additionally, students often struggle to distinguish between equations and inequalities, particularly when solving them graphically (Tsamir & Almog, 2001). Interestingly, while teachers can generally predict students' understanding of variables, they often misjudge students' grasp of the concept of the equal sign (Asquith et al., 2007). These findings highlight the need for targeted interventions and professional development to address these persistent challenges in algebra education.

Meanwhile, epistemic obstacles arise due to the gap between students' initial understanding and formal mathematical concepts. One potential obstacle is a misunderstanding of the concepts of true and false statements. Students face various obstacles in understanding linear equations with one variable. These include ontogenic obstacles, where students struggle to transition from arithmetic to algebraic thinking (Fardian et al., 2024). Epistemological obstacles arise from limited context, leading to errors in concept application and problem-solving (Wicaksono et al., 2024). Students often have misconceptions about basic algebraic concepts such as variables, coefficients, and constants (Yansa et al., 2021). Didactical obstacles occur when teachers use procedural approaches, hindering students' conceptual understanding (Wicaksono et al., 2024). Additionally, students struggle with translating verbal situations into mathematical models and understanding the meaning of "solution" in equations. These difficulties are compounded by teachers' tendency to start with formal definitions and provide examples with inconsistent difficulty levels (Nurjanah et al., 2024). Addressing these challenges requires innovative teaching strategies and targeted assistance for students.

On the other hand, didactic obstacles arise due to learning strategies or teaching methods that are less appropriate to students' needs. One of the obstacles seen in this lesson plan is the lack of connection between the initial activity and the core concept of one-variable linear equations. Research indicates that students face significant challenges in understanding mathematical concepts, particularly linear equations and functions. Common issues include difficulties in defining concepts, applying them to real-world situations, and understanding variables (Panjaitan & Juandi, 2024). Traditional teaching approaches often focus on abstraction rather than practical applications, resulting in student confusion and a disconnection from everyday experiences. To address these challenges, studies suggest incorporating ethnomathematics and contextual learning to improve comprehension (Sari et al., 2023). Visual representation approaches are also proposed as an alternative teaching method to reduce barriers to understanding (Hamash et al., 2024). Additionally, research highlights the importance of teachers maximizing their role in the learning process and utilizing diverse learning models to stimulate students' abilities (Lisnawati & Nirmala, 2024). These findings underscore the need for more contextually oriented, visually focused, and student-centred approaches to teaching linear equations and functions in mathematics education.

Next, in the second meeting, the lesson plan was designed to help students solve linear equations in one variable through various exploratory approaches, technology, creativity, and critical thinking. Learning begins with the "*Let's Explore*" activity, where students observe the illustration of scales in their student's book. This illustration aims to help students understand the concept of balance in linear equations, with teacher guidance in drawing conclusions from the image and connecting it to the form of a mathematical equation. In the Let's Try stage, students are allowed to apply their understanding by answering questions related to the illustration of the scales. They are asked to create a mathematical model from the illustration and determine its solution. This activity aims to train students in changing visual representations into linear equations and applying appropriate algebraic operations. In the Let's Technology stage, students are invited to deepen their understanding by using technology. They will access interactive media by scanning the QR code or clicking the provided link. This media features various elements that enable students to explore the concept of linear equations in a dynamic manner. The teacher plays a crucial role in guiding students to use media effectively, thereby providing a more engaging and interactive learning experience.

Based on the analysis of the lesson plan content at the second meeting, the presentation of the material in the lesson plan created several learning obstacles that can be categorized as ontogenic, epistemic, and didactic obstacles in learning the material on linear equations of one variable. Ontogenic obstacles arise due to the limitations of students' cognitive development in understanding the concept of linear equations of one variable. Some students may struggle to understand the concept of balance, as illustrated by scales, particularly in connecting it to the form of the equation (Beswick, 2012; Goos et al., 2023). Additionally, students who lack a strong understanding of algebraic operations may struggle when they need to model a situation as an equation and solve it. Another obstacle is the level of abstract thinking required in the 'Let's Think Critically' activity, where students are asked to analyze unusual forms of equations, which may be too complex for those who are still at the bare understanding stage.

Research on learning obstacles in linear equations reveals several challenges that students face. Epistemological obstacles arise due to limited context and difficulties in translating problems into mathematical models (Gupta & Elby, 2011). Students struggle with understanding equations as a balance, often viewing them merely as calculation procedures (Lhachimi et al., 2022). Ontogenic obstacles occur when students lack basic mathematical knowledge, while didactical obstacles stem from teachers' inability to accommodate students' intellectual needs (Bakar et al., 2024). Common difficulties include determining equations with unknown gradients, transitioning from arithmetic to algebraic thinking, and grasping concepts of coefficients, variables, and constants (Kadarisma & Amelia, 2018). To address these challenges, researchers suggest strengthening initial understanding, improving teaching methods, and utilizing didactic tools, such as balance models, to enhance conceptual understanding and mathematical reasoning (Lithner, 2017).

Learning obstacles in linear equations can be categorized into ontogenic, epistemological, and didactical types. Ontogenic obstacles arise from students' limited basic mathematical knowledge and the transition from arithmetic to algebraic thinking (Sidik et al., 2021). Epistemological obstacles include difficulties in translating problems into mathematical models, performing calculations, and explaining answers (Moru, 2007). Students also struggle with understanding coefficients, variables, and constants. Didactical obstacles stem from teachers' inability to accommodate students' intellectual needs and procedural teaching approaches (Ishak et al., 2025). To address these challenges, researchers suggest strengthening fundamental algebraic concepts, using visual aids, and fostering motivation. Innovative approaches, such as didactic balance games, can help students master equation-solving techniques while developing mathematical reasoning skills (Elkjær & Thomsen, 2022).

Analysis of the material on linear equations of one variable that is taught (Taught Knowledge)

The material on linear equations of one variable presented in school mathematics textbooks serves as a guideline for teachers in presenting the material on linear equations of one variable as knowledge that is actually taught in class (i.e., taught knowledge). In general, before presenting material as taught knowledge, teachers carry out a series of material transition processes to enable students to teach it. The transition of material certainly takes into account the characteristics and learning environment of students. This transition process occurs through the internal didactic transposition.

Based on observations of the teaching and learning process, an analysis was conducted on the material of linear equations in one variable as taught in the classroom. This analysis examines how the concept is presented and structured during lessons, focusing on two key aspects: (1) the teaching and learning process of introducing the concept of linear equations in one variable and (2) the teaching and learning process of solving linear equations in one variable. Each aspect is described based on classroom interactions, instructional strategies, and student engagement, providing a comprehensive understanding of how the material is conveyed and understood.

The first meeting focused on determining the truth value of a statement, creating modelling simulations for open and closed statements, and transforming problems related to linear equations of one variable into mathematical models. Based on the results of the observations, learning occurred almost in accordance with the objectives outlined in the lesson plan, including determining the truth value of a statement, creating modeling simulations of open and closed statements, and transforming problems related to linear equations of one variable into mathematical models. Overall, students were quite active in participating in learning activities, although there were several obstacles in understanding the concepts taught.

In determining the truth value of a statement, students enthusiastically observe a conversation between Humam and Aldo that involves guesses, helping them identify true and false statements in concrete examples. However, some struggle to distinguish between closed sentences with absolute truth values and open sentences that depend on variables, especially when dealing with concepts like prime numbers or algebraic forms. To reinforce understanding, students work in groups to classify statements, yet some still face difficulties due to a lack of conceptual grasp of variables in mathematical statements. Similarly, when translating verbal problems related to linear equations into mathematical models, some students successfully construct equations. In contrast, others encounter challenges due to limited experience in modelling real-world situations and gaps in their understanding of algebraic operations.

In the second meeting, they discussed changing problems related to linear equations of one variable into mathematical models and applying the rules of addition, subtraction, multiplication, and division to solve the problems. Based on observations, the presentation of the material began with the teacher's introduction to *the Let's Explore activity*, where students were invited to observe an illustration of a two-sided balance. The teacher used a guided discussion method to encourage students to identify the relationship between the loads on both sides and how this concept relates to linear equations of one variable. Students were invited to formulate critical questions, such as how changes on one side will affect the value on the other side. In this process, students not only understand that linear equations reflect balance but also begin to realize that the algebraic operations they have learned can be used to maintain this balance. With the teacher's guidance, they attempted to write the balance relationship in the form of a simple equation, compared their answers with those of their deskmates, and discussed possible methods for solving it.

After understanding the basic concept of equations, students participate in the "Let's Try" activity, where they analyze scale illustrations with varying load values. Working in small groups, they write corresponding equations and solve them using addition, subtraction, multiplication, and division. The teacher facilitates learning by guiding discussions and providing assistance to those who struggle. Students then access interactive media via a QR code to experiment with digital simulations, reinforcing their understanding through visual representations of algebraic operations. In the final stage, students participate in *"Let's Think Creatively" and "Let's Think Critically"* activities. In *"Let's Think Creatively," they pair up to create and exchange story problems, thereby enhancing their ability to apply mathematical concepts in real-world* contexts. Meanwhile, in *"Let's Think Critically," they analyz*e and solve unconventional equations, exploring efficient problem-solving strategies through discussion. The session concludes with a reflection on learning experiences, challenges faced, and practical strategies for solving one-variable linear equations.

Based on observations from the first and second meetings, the teaching of linear equations in one variable shows a shift from scholarly knowledge to taught knowledge, creating a conceptual gap. This gap appears between the formal definition in scholarly knowledge, the definition in school textbooks, and the way the teacher presents it in class, particularly in explaining the whole meaning of linear equations in one variable. Weaknesses in material presentation may lead to three types of obstacles: (1) Ontogenic obstacles, caused by students' cognitive limitations, such as difficulty understanding variables and forming equation models from verbal problems; (2) Epistemic obstacles, arising from the complexity of translating verbal statements into equations and distinguishing linear equations from other types; and (3) Didactic obstacles, due to ineffective teaching strategies, such as overly abstract explanations, lack of visual aids, or a teaching pace that does not match students' needs.

Table 2 presents the results of the analysis of the material on linear equations with one variable, highlighting how the concept is transformed from the intended curriculum into actual classroom instruction. This analysis examines the alignment between the prescribed learning objectives, instructional strategies, and students' learning experiences. The findings reveal how the didactic transposition process influences the way linear equations in one variable are structured and delivered, ensuring that complex mathematical ideas are made accessible to students. Key aspects analyzed include the types of problems presented, the use of representations such as equations and graphs, and the methods employed by teachers to facilitate understanding. Additionally, the table outlines the challenges encountered in teaching linear equations in one variable, including students' difficulties in conceptualizing variable relationships and applying appropriate problem-solving strategies. The results offer valuable insights into how the material is adapted to meet pedagogical needs while maintaining mathematical rigour, enabling educators to refine their approaches and enhance student comprehension and engagement.

Table 2. The results of the analysis of the material on linear equations of one variable as Taught Knowledge

No.	Sub Material	Conceptual Gap between Taught Knowledge and Scholarly Knowledge and Knowledge to be Taught	Prediction of conceptual gaps between Learnt Knowledge and Scholarly Knowledge	Prediction of learning obstacles
1	Teaching and learning the concept of linear equations in one variable	There is a conceptual gap between Taught Knowledge and Scholarly Knowledge, as well as between Taught Knowledge and Knowledge to be Taught.	There is a conceptual gap between Learned Knowledge and Scholarly Knowledge.	Didactic obstacles, epistemological obstacles, and ontogenic obstacles
2	The teaching and learning process of solving linear equations in one variable	There is a conceptual gap between Taught Knowledge and Scholarly Knowledge, as well as between Taught Knowledge and Knowledge to be Taught.	There is a conceptual gap between Learned Knowledge and Scholarly Knowledge.	Didactic obstacles, epistemological obstacles, and ontogenic obstacles

Based on the table of results of the analysis of the material of linear equations of one variable as Taught Knowledge, then in Table 2, teaching and learning processes (carried out by the teacher there is a conceptual gap between Taught Knowledge and Scholarly Knowledge, as well as between Taught Knowledge and Knowledge To be taught. This occurs because the interpretation of the linear equation of one variable taught to students does not align with Scholastic Knowledge, resulting in a gap between the knowledge learned and Scholastic Knowledge.

At the first and second meetings, based on the results of observations, several learning obstacles were identified, which can be categorized into three types: ontogenic, epistemic, and didactic obstacles. Research on learning obstacles in mathematics education reveals several challenges that students face, particularly in algebra and geometry. Ontogenic obstacles arise from students' lack of basic mathematical knowledge and readiness to learn (Nansiana et al., 2024). Epistemological obstacles arise when students struggle to translate problems into mathematical models, make calculation errors, or fail to provide clear explanations for their answers (Alvidrez et al., 2024). These obstacles are often related to students' limited understanding of concepts and contexts (Pauji et al., 2023). Didactical obstacles stem from ineffective teaching methods that fail to accommodate students' intellectual needs. Studies have identified these obstacles across various educational levels, with geometry being a particularly challenging subject (Fauzi et al., 2024). Understanding these learning obstacles is crucial for developing effective didactic designs and improving students' mathematical literacy and algebraic thinking skills (Soboleva et al., 2020).

Recent studies have identified various learning obstacles in algebra, particularly with linear equations. These include ontogenic obstacles resulting from cognitive development differences, epistemological obstacles stemming from limited contextual understanding, and didactical obstacles arising from ineffective teaching methods (Hendriyanto et al., 2024; Fardian et al., 2024). Common challenges include difficulties transitioning from arithmetic to algebraic thinking, understanding variables and coefficients, and applying equations to real-world situations (Namkung & Bricko, 2021). Students often struggle with visualizing the relationships between equations and graphs, as well as applying appropriate problem-solving strategies. Specific issues arise in determining equations when gradients are unknown, indicating a reliance on memorized formulas rather than conceptual understanding (Sehole et al., 2023).

CONCLUSION

The knowledge taught by teachers in classroom learning (taught knowledge) on the material of linear equations of one variable is studied and analyzed based on several sources, including lesson plans created by teachers and the learning process carried out in the classroom. The presentation of material in the lesson plans creates learning obstacles in understanding the concept and solving linear equations of one variable. This is because the material studied and discussed in learning linear equations of one variable is directly related to questions involving numbers, without incorporating story problems that relate to students' lives. This shift reveals a conceptual gap, both between the concept definition presented by teachers during instruction, referred to as taught knowledge, and the formal concept definition found in scholarly knowledge, and also between the concept definition presented in teaching and learning and the one presented in school mathematics textbooks as authoritative knowledge. Weaknesses in material presentation can lead to learning obstacles: (1) ontogenic obstacles due to students' cognitive limitations and difficulties understanding variables and equations, (2) epistemic obstacles caused by the complexity of mathematical concepts and challenges in translating verbal statements into equations, and (3) didactic obstacles resulting from ineffective teaching strategies, such as abstract explanations, lack of visual aids, and inappropriate pacing. These findings can help improve teaching strategies by addressing conceptual gaps and learning obstacles, thereby making mathematics instruction more effective for Madrasah Tsanawiyah students.

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