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INTEGRATING REALISTIC MATHEMATICS EDUCATION AND ISLAMIC VALUES: A HYPOTHETICAL LEARNING TRAJECTORY FOR TEACHING CUBOID GEOMETRY

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ABSTRACT
Integrating Islamic contexts into mathematics learning has recently gained increasing scholarly attention. This study aims to develop and test a Hypothetical Learning Trajectory (HLT) for learning cuboid geometry through realistic contexts embedded with Islamic values. Employing a design research approach, the study followed three key stages: (1) preparing for the experiment, (2) conducting the experimental design, and (3) conducting retrospective analysis. The HLT was implemented with Grade VIII students from three state-run Madrasah Tsanawiyah in different provinces of Indonesia (Padang, Malang, and Bandung) to capture contextual diversity and ensure the transferability of the findings. Data were collected through analysis questionnaires, students' written work, video transcripts, and observation sheets, then analyzed using thematic coding and descriptive analysis aligned with the RME framework. The findings indicate that integrating Islamic values within realistic contexts supported students in constructing geometric concepts meaningfully while internalizing the moral and cultural dimensions of learning. Empirically, the study is limited by its small-scale implementation and focus on a single mathematical topic; Therefore, future research should expand to broader mathematical domains and explore longitudinal impacts to strengthen evidence-based recommendations for culturally responsive mathematics education.

Keywords: Islamic Values, Learning Trajectory, RME

INTRODUCTION
Geometric shapes, particularly cuboids, are among the most vital topics in the mathematics curriculum (Alex & Mammen, 2012; Clements & Sarama, 2011; Doğruer, 2018; Ibili et al., 2020; Kuzle, 2021; Sunzuma & Maharaj, 2022; Uriarte-Portillo et al., 2023). Classroom geometry instruction typically focuses on relationships between shapes, their structures, and associated theorems and formulas (Gargrish et al., 2021; Novak & Tassell, 2017; Nugraha, 2019; Widada et al., 2021). Students' mastery of geometry is essential, as geometric thinking enhances their ability to solve a wide range of real-life problems (Pradibta et al., 2023; Verner et al., 2019). Moreover, engaging in geometric thinking improves students' visualization and spatial reasoning skills, which are crucial for grasping abstract mathematical concepts (Naufal et al., 2025). According to Couto and Vale (2014), these abilities are instrumental in addressing everyday challenges.

However, despite its importance, many students still struggle to develop higher-order geometric thinking skills, particularly when learning three-dimensional shapes such as cuboids.

Prior studies have emphasized conceptual understanding and formula application, yet few have explored how instructional strategies can effectively foster creative and flexible problem-solving in geometric contexts (Fauzan et al., 2018). This gap highlights the need to investigate approaches that not only strengthen students' geometric understanding but also enhance their cognitive flexibility and creative reasoning in solving geometry-related problems.

Despite extensive investigations, students' geometric thinking levels across elementary and secondary education remain below expectations (Couto & Vale, 2014; Naufal et al., 2025; Pradibta et al., 2023; Verner et al., 2019). Prior studies consistently attribute this issue to the abstract nature of geometric concepts, which are more challenging to comprehend than other mathematical domains (Owens, 2014; Serin, 2018; Shen, 2021). Students often struggle to interpret abstract visual representations, which can hinder the development of a deep understanding of geometry (Sari et al., 2017; Sinclair & Bruce, 2015; Utami et al., 2019; Widada et al., 2019). As a result, geometry remains one of the most challenging topics in mathematics education (Bhagat & Chang, 2015; Yavuz et al., 2016). Several contributing factors have been identified, including ineffective instructional approaches, limited use of concrete or visual learning media, and a lack of meaningful learning experiences (Dwidayati et al., 2019; Haryanto, 2020; Jelatu et al., 2018; Nindiasari et al., 2024; Novita et al., 2018; Nurwijayanti et al., 2019; T.R., 2017). The persistence of these difficulties suggests that existing interventions have not yet bridged the gap between abstract geometric concepts and students' real-world experiences. Therefore, this study aims to address this gap by implementing an approach that explicitly connects conceptual understanding with contextual and hands-on learning to foster higher levels of geometric thinking.

Teachers play a crucial role in achieving learning objectives, particularly in geometry instruction (Chang et al., 2016; Gargrish et al., 2021; Nursyahidah et al., 2021). To improve learning outcomes, teachers must design more effective instruction by employing appropriate contexts and pedagogical approaches. In response to the persistent challenges in students' understanding of geometric concepts, such as difficulties in visualizing spatial relationships and connecting abstract ideas to real-world situations, the Realistic Mathematics Education (RME) approach has shown promising potential to address these issues (Bustang et al., 2013; Juwita et al., 2015; Nursyahidah et al., 2020; Risdiyanti & Prahmana, 2018). RME views mathematics not as static knowledge to be transferred, but as a human activity developed through meaningful interaction with real-world contexts (Gravemeijer & Terwel, 2000). This approach leverages students' surrounding environments to stimulate thinking and provide opportunities for them to construct mathematical understanding (Palinussa et al., 2021). However, despite extensive studies on RME in arithmetic and algebra, its contextual application in geometry remains underexplored, particularly in the design of tasks that connect students' spatial reasoning with their cultural and environmental surroundings. Addressing this gap, integrating locally relevant geometric contexts can serve as an innovative way to enhance students' conceptual understanding and engagement. Through such contextual geometry learning, RME can facilitate a smoother transition from everyday experiences to formal geometric reasoning (Putri & Zulkardi, 2017; Yanti et al., 2016), enabling students to explore, construct, and represent abstract mathematical ideas more effectively (Haris & Ilma, 2011; Nursyahidah et al., 2013, 2014, 2018; Prahmana et al., 2012).

Mathematics education in Indonesia, particularly in madrasas, is closely connected to the nation's socio-cultural context, which is predominantly Muslim. This context encourages the integration of Islamic values such as aqidah, sharia, and akhlaq into the teaching and learning process (Mariana, 2017). Beyond serving as moral or cultural enrichment, integrating Islamic values within mathematics education provides a theoretical foundation for holistic learning, uniting cognitive, affective, and spiritual domains (Choirudin et al., 2021). From an educational psychology perspective, this integration aligns with character education principles that foster meaningful learning experiences by connecting abstract mathematical concepts with students' belief systems and ethical reasoning (Winarso & Wahid, 2020). Consequently, mathematics learning in madrasas is not only aimed at developing computational proficiency but also at cultivating reflective, responsible, and ethically guided thinkers. Such integration strengthens students' moral reasoning and supports the formation of religious character that can help prevent moral decline among adolescents (Rahmawati & Swaditya, 2017; Achadah et al., 2022).

Integrating value-based perspectives derived from Islamic teachings into mathematics learning can foster students' social awareness, ethical reasoning, and sense of responsibility. Core values such as honesty, justice, and patience can be cultivated through fair, transparent, and reflective mathematical problem-solving processes (Hasanah et al., 2019). Beyond cognitive development, incorporating moral and affective dimensions in mathematics education contributes to shaping students' positive character and attitudes toward learning (Rahmatika et al., 2024). Learning approaches informed by Islamic ethical principles encourage students to appreciate mathematics as a discipline that promotes integrity, balance, and critical reflection in understanding real-world phenomena (Ni et al., 2017; Purwati et al., 2018).

The emphasis on moral and ethical values in Islamic-based mathematics learning can foster students' moral awareness and enhance learning outcomes by meaningfully integrating spiritual principles with cognitive development (Aldhaen et al., 2025). This approach aligns with the goals of Islamic education, which emphasize both scientific mastery and character formation grounded in Islamic teachings (Alhamuddin et al., 2022). Mathematics learning integrated with Islamic values encourages students to connect abstract mathematical concepts with real-life contexts and moral reflections, leading to deeper conceptual understanding and stronger motivation to learn (Kurniati et al., 2015).

Specific strategies that can operationalize this integration include: (1) beginning lessons with Basmallah to create spiritual mindfulness, which enhances focus and self-regulation; (2) using Islamic terms and visual illustrations to strengthen cultural relevance and engagement; (3) incorporating examples or problems related to Islamic contexts, which make learning more authentic and meaningful; (4) inserting relevant Qur'anic verses or Hadith to foster reflective thinking and ethical reasoning; and (5) relating mathematical topics to Islamic history and Kauniyah verses (signs of the universe), encouraging inquiry and curiosity about God's creation (Sa'bana, 2024). Through these mechanisms, Islamic-based strategies can measurably improve learning outcomes such as higher conceptual understanding, increased learning motivation, and positive attitudes toward mathematics while nurturing students' moral and spiritual growth. This aligns with national education goals that call for a balanced integration of science, technology, and spirituality in the learning process.

Developing a learning trajectory is essential to support and facilitate students' learning processes. In its early stages, the learning trajectory is formulated as a hypothesis about how students will progress when guided by a designed Hypothetical Learning Trajectory (HLT). Once tested and refined, HLT can evolve into a Local Instructional Theory (LIT) that informs broader educational practices (Doorman, 2019; Gravemeijer & Cobb, 2013). HLT is considered a vital tool for implementing innovation in learning (Bahamonde et al., 2017; Muhtadi et al., 2025), as it serves to bridge research and classroom practice, thereby helping to create effective learning environments that support students' understanding of specific mathematical concepts (Andrews-Larson et al., 2017; Fauzan et al., 2018). Numerous studies highlight the crucial role of HLTs in improving educational practices by offering structured, theory-based instructional approaches (Akbar & Hasby, 2019; Baroody et al., 2022). However, most existing HLT studies have focused primarily on cognitive and procedural aspects of learning mathematics, with limited attention to how faith-based or value-oriented contexts can be systematically integrated into the design and validation of HLTs. This study addresses the theoretical and practical gap by developing an HLT that not only supports students' conceptual understanding but also aligns with faith-integrated learning principles, thereby offering a novel perspective on the growing body of research on contextually grounded instructional design.

Despite these advancements, the integration of Islamic values into HLT-based mathematics learning remains limited. Previous studies have discussed the inclusion of Islamic perspectives in mathematics education, emphasizing their potential to connect mathematical concepts with spiritual meaning and promote the unity of knowledge (Elida, 2023; Fitrah & Kusnadi, 2022; Nasaruddin, 2014). However, such integration has not been systematically operationalized, particularly in geometry learning within Islamic educational settings. This study seeks to fill that gap by developing a realistic mathematics learning trajectory (HLT) that embeds Islamic values into junior high school and Islamic junior high school mathematics instruction. Theoretically, this research contributes to expanding the framework of Realistic Mathematics Education (RME) by introducing a culturally and spiritually responsive dimension that connects mathematical reasoning with ethical and moral development. Globally, it highlights how localized cultural and religious values can enrich the discourse on mathematics education, offering insights for other multicultural or faith-based educational contexts seeking to balance cognitive, moral, and spiritual growth.

METHOD

This research is a design research study (design research) that focuses on the development of learning theory and activities (Gravemeijer & Cobb, 2013). This research was conducted based on the design research framework proposed by Gravemeijer & Cobb (2013), which includes three iterative phases, namely: (1) preparing for the experiment, (2) conducting the experimental design, and (3) conducting retrospective analysis. As shown in Figure 1, this research process is cyclical, allowing for continuous refinement of theory and practice.

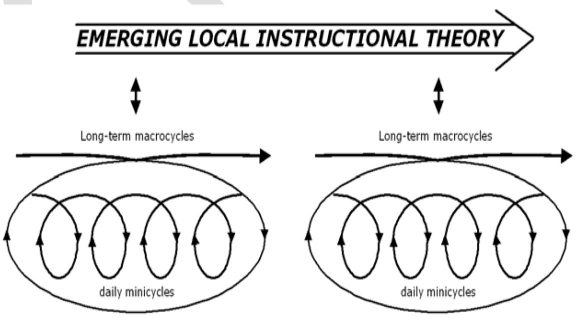

Figure 1. Research Design as a Cyclical Process

Source Gravemeijer & Cobb (2013)

This framework is considered most aligned with the research objectives and context, namely the design of a Hypothetical Learning Trajectory (HLT) on the topic of cuboidal geometry, using objects relevant to students' daily lives and integrated with Islamic values. The cyclical and theory-based nature of design research supports the development of local instructional theories that evolve through iterative design, classroom implementation, and reflection on learning outcomes. Thus, this approach allows researchers to optimize contextually and culturally meaningful learning activities while making theoretical contributions to understanding the development of students' geometric reasoning through meaningful, value-based learning experiences.

The sampling strategy used purposive sampling, given that all participating schools are state madrasas (Islamic schools) with an Islamic-based educational vision, making them relevant to the research focus on integrating the Realistic Mathematics Education (RME) approach with Islamic values in mathematics learning. Furthermore, all three madrasas have mathematics teachers who are active and experienced in developing learning innovations, thus providing substantial contributions to the development and implementation of HLT. Based on these considerations, the subjects of this study were grade VIII students from three madrasas, namely MTsN 5 Padang, MTsN 1 Bandung, and MTsN 1 Malang.

In the prepare for experiment phase, the activities conducted include: 1) needs analysis to determine the learning design needed to teach spatial geometry, 2) conceptual analysis to analyze the essential concepts that will be taught to students and analyze Islamic values that can be used for spatial geometry, 3) literature analysis to determine the learning trajectory design process seen from the results of previous research. Furthermore, the preparation results for the experiment phase are used in the design phase. In this phase, the HLT design is carried out. The HLT design begins with the Iceberg HLT, which describes how students develop their knowledge, from the concrete to the abstract, in spatial structures. It then proceeds to design a complete HLT, including objectives, activities, predictions, and learning anticipation.

Moreover, the HLT activities were developed based on the core principles of Realistic Mathematics Education (RME), integrating realistic contexts that also embed Islamic values. The suitability and quality of the designed HLT were validated by RME experts, experts in integrating mathematics with Islamic perspectives, and learning technology experts to ensure content and construct validity, with inter-rater reliability established through consistency checks among reviewers. The retrospective analysis involved implementing the HLT with grade VIII students at MTsN 5 Padang, MTsN 1 Malang, and MTsN 1 Bandung, where systematic observations and video recordings were conducted to document learning interactions. After each implementation, reflective discussions among the researcher, teacher, and observer ensured data triangulation from multiple perspectives. The data sources included needs analysis questionnaires, students' written work, video transcripts, and observation sheets, which were analyzed using a thematic coding framework aligned with the RME stages. Descriptive analysis was applied to interpret how students constructed their understanding of spatial shapes, ensuring credibility, dependability, and methodological rigor of the findings.

RESULT AND DISCUSSION

Results of Preparing for the Experiment

The preliminary analysis indicates that the initial learning design implemented by teachers has not effectively facilitated students' independent construction of concepts. Observational data indicate that the learning trajectory remains predominantly textbook-centred, limiting students' opportunities to engage in exploratory or problem-based learning. This finding aligns with constructivist theory, which emphasizes active student participation in meaning-making processes (Vygotsky, 1978). Furthermore, the analysis shows that teachers' instructional plans do not sufficiently account for students' cognitive development stages or diverse learning characteristics. The absence of contextual and value-based integration, particularly the incorporation of Islamic values relevant to the students' environment, further reduces the relevance and engagement of the lessons. Consequently, students' mathematical reasoning and conceptual understanding have not developed optimally, as supported by the limited indicators of higher-order thinking observed during classroom activities.

In addition, textbooks used in learning have not integrated Islamic values into the learning process, so efforts are needed to internalize these values in students' daily lives. The Independent Curriculum itself emphasizes the importance of understanding students' needs, interests, and potential holistically. Thus, education is directed to support the holistic development of individual students, including cognitive, affective, and spiritual aspects. The Realistic Mathematics Education (RME) approach plays a crucial role in helping students understand mathematical concepts through real-life contexts. Through RME, students can see the connection between mathematical concepts and their application in everyday life, for example, by using blocks or other geometric shapes to solve real-life problems, thereby making mathematics learning more meaningful and contextual. This aligns with research by Suryani et al. (2023), which found that implementing the RME approach can improve students' understanding of contextual problems. A similar study by Desvita and Turdjai (2020) also concluded that implementing the RME learning model effectively improves students' mathematics achievement and learning outcomes.

Result of Design for Experiment

Based on the preliminary analysis results, HLT was designed around the topic of cuboids, integrated with RME and Islamic values. HLT consists of learning objectives, activities, and student learning hypotheses (Simon, 1995). The following are the results of the design of the learning trajectory for the surface area and volume of cuboids:

Table 1. Hypothetical Learning Trajectory

Aim	**Learning Activities**	**Hypothetical Learning Trajectory**	**Islamic values**
Finding the concept of the Surface Area of a Cuboid	**Kaaba Covering Cloth:**  Prophet Ismail and Prophet Ibrahim are inseparable from the history of the Kaaba. The *Kiswah* (covering cloth) of the Kaaba is replaced annually. The *Kiswah* was made by Majma' Malik Abdul Aziz li Kiswatil Kaaba Al-Musyarrafah. It includes around 220 Saudi Arabian artists who work on making the Kiswah. The kiswah cloth is made using the most expensive materials in the world. Each year, it costs around 25 million Saudi riyals. "The material consists of 760 kg of Italian silk, 120 kg of gold, and 100 kg of silver from Germany.	Students draw the net of a cuboid by drawing five rectangular sides. Students determine the area of the cloth covering the Kaaba by adding the areas of the individual flat shapes. Students determine the area of the flat shapes that form a cuboid. Students with non-formal abilities will add the area of shape 1 to the area of shape 5. While students who have acquired non-formal knowledge will not add up all the flat shapes that form a cuboid, they will not measure the parallel sides using their knowledge. By providing an assumption, the teacher encourages students to assume that all the surfaces are covered, including the	1.Ittiba' 2.Persistence 3.Sincerity 4.Faith in Allah 5.Patience 6.Ibadah

Aim	Learning Activities	Hypothetical Learning Trajectory	Islamic values
	If the size of the kiswah cloth on each side is 6.3 m x 3.3 m, how much cloth is needed to cover the Kaaba? If it is assumed that the bottom cover cloth is also provided, estimate the amount of cloth required. If a building has all the exact rib sizes, how much cloth is needed?	bottom. Students will find that the surface area of the cuboid is $L_p = 2\{(p \times l) + (p \times t) + (l \times t)\}$. If there are no students who have achieved formal understanding, the teacher will give a trigger question: "Does a cuboid have parallel sides? If so, do we have to count all the plane shapes on the cuboid net? Can we add parallel sides?" Meanwhile, students who have acquired non-formal knowledge will not add up the areas of all the flat shapes that form a cuboid; they will measure the area of 1 flat shape, and then the areas of the other flat shapes will be the same. So, students will find the surface area of the cube as $\sqrt{6s^2}$.	
Finding the concept of cuboid volume	**Concrete cube:**  Al-Hakim Mosque is one of the mosques located on the beachside. To avoid abrasion, the concrete safety wall will be installed at the tip of the beach if the government plans to install a safety wall measuring *20m x 5m x 3m*. How many concrete cubes are needed?	Students will imagine how many concrete cubes are needed if • 4 x 3 x 2 = 24 • 5 x 4 x 2 = 40 • and so on. If the volume formula for a block-shaped safety wall is length x width x height = Number of concrete cubes required, then: *Cuboid* $V = l \times w \times h =$ *cube* $V = s \times s \times s = s^3$ Students are asked, "*Does the bath at home have the shape of a cube, cuboid, or*	7.Ittiba' 8.Persistence 9.Sincerity 10. Faith in Allah 11. Patience Ibadah

Aim	Learning Activities	Hypothetical Learning Trajectory	Islamic values
		other shapes? If so, have you ever asked or found out whether the volume of water in a bath meets the requirements for a valid wudhu volume? Remember that, according to the Prophet's hadith, the volume of water required for a valid wudu is two qullah (±216 Litres)? By studying the bath's volume, you will be able to answer this problem!	

Based on the HLT design table above, the contextual problem related to the surface area of a cuboid involves providing students with re-stimulation through the shape of the Kaaba and its covering cloth (Kiswah). This context is intended to help students construct their mathematical understanding while simultaneously strengthening their Islamic values. For the volume of a cuboid problem, students are stimulated with a contextual question related to ablution (wudhu):

"Is there a bathtub in your house that is shaped like a cube, cuboid, or another geometric shape? If so, have you ever asked or found out whether the volume of water in the bathtub meets the requirements for valid ablution according to the hadith, which is two *qullah*?"

The HLT design for the topic of surface area and volume of cuboids, based on Realistic Mathematics Education (RME) and the integration of Islamic values, consists of the following stages:

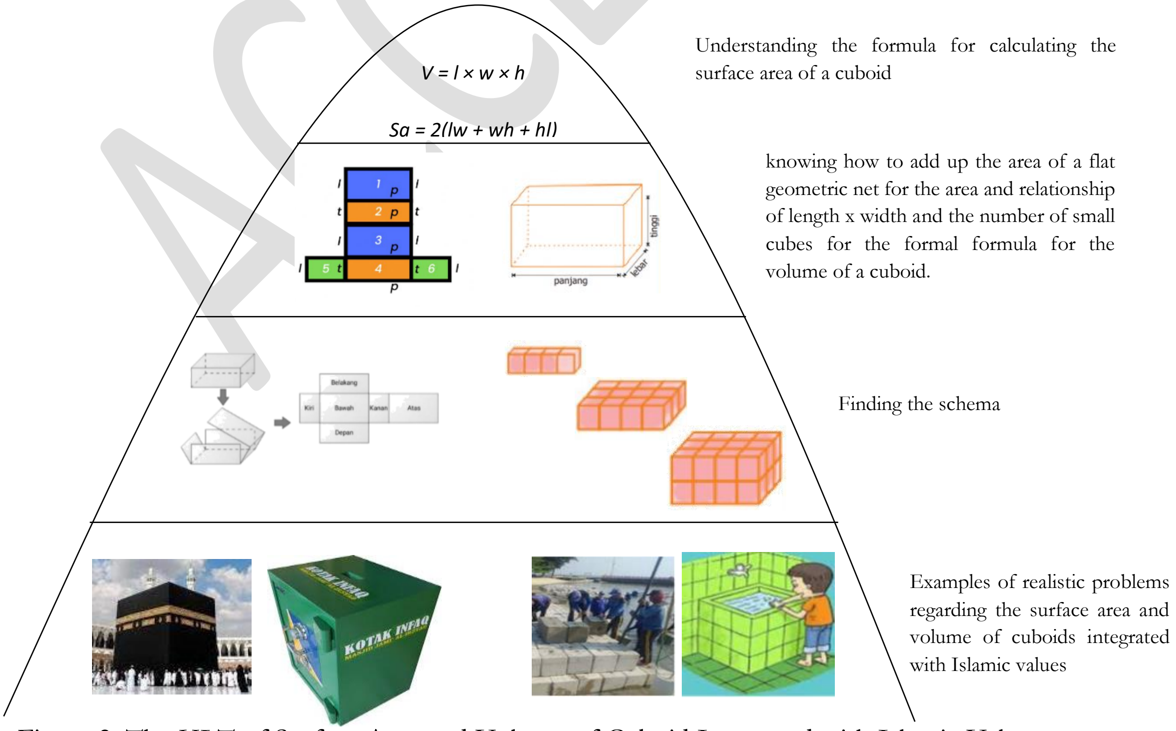

Figure 2. The HLT of Surface Area and Volume of Cuboid Integrated with Islamic Values

Figure 2 above depicts a hypothetical learning trajectory (HLT) on the surface area and volume of cuboids integrated with Islamic values. This trajectory demonstrates an innovative yet procedural learning design, as each stage is systematically structured from a concrete context to conceptual and formal understanding. In the initial stage, learning begins with a realistic Islamic-themed context, such as the shape of the Kaaba, a donation box, or building a mosque using concrete as a concrete object. This stage is based on the principle of social constructivism, where contextual experiences are used to construct meaning and connect mathematical concepts to everyday life. Islamic values such as caring, honesty, and cooperation are part of the learning context that not only strengthen moral aspects but also trigger students' cognitive processes in meaningfully recognizing the concepts of volume and surface area.

In the next stage, students are guided to discover conceptual schemes through exploration of the concrete form of cuboids and their nets. This activity fosters the understanding that cuboids are composed of multiple small cubes and have rectangular surfaces. The next stage leads students to the formulation of mathematical concepts, as they discover the relationships among the dimensions of length, width, and height, and develop formulas for the surface area and volume of cuboids. The final stage is conceptual formalization and reflection, where students understand and can use the formulas $V=l\times w\times h$ and $SA=2(l1+wh+lh)$ with conceptual understanding, not just procedural memorization.

Thus, this learning trajectory highlights pedagogical reasoning and theoretical contributions at every stage of learning. The integration of the Islamic context goes beyond strengthening character values and serves as a conceptual bridge that helps students construct deep, contextual meanings in mathematics. This HLT thus represents a learning process oriented toward integrated conceptual, moral, and spiritual understanding.

Results of Retrospective Analysis

After the HLT was designed with all its predictions and anticipations, the learning trajectory was tested in one-to-one, small-group, and field-test settings. The one-to-one test was conducted on three students with high, medium, and low abilities. The aim was to directly observe the designed learning trajectory, focusing on instructions that were difficult to understand. The results of the one-to-one test showed that the three students could understand the contextual problems. Still, the students had difficulty understanding the activities because they had never seen the Kiswah directly and did not know its size. The solution was provided in the context problem, which gave information about the actual size of the Kaaba by asking students to search directly. Students also had doubts about the volume/amount of water required for wudhu (2 *qullah*); the solution was for students to ask the religious teacher.

After the one-to-one evaluation stage and improvements to the HLT design, the next activity was a small-group test with nine students of varying abilities. Overall, no significant obstacles were encountered when students understood the concepts of surface area and volume of the cuboid designed, and the predictions were found correct. Furthermore, after being revised, it was continued with a field trial. The following are the results of students' answers to the problems given:

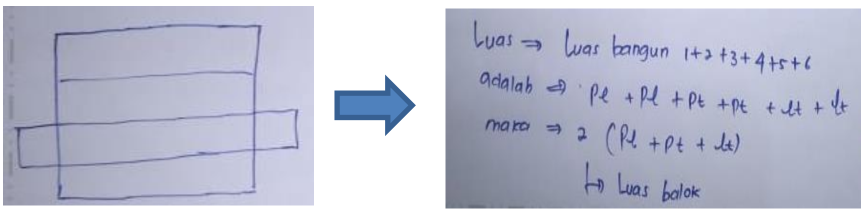

Figure 3. Student answers related to the surface area of a cuboid

Based on Figure 3 above, the realistic problems provided can help students construct the concept of the surface area of a cuboid. The students' written responses indicate that they are able to identify and combine the surface areas of the six faces of a cuboid, leading to the generalization of the formula $2(pl + pt + lt)$. This finding shows that students engaged in cognitive processes of conceptual understanding and reasoning, as they connected contextual representations with abstract mathematical relationships. The discussions among group members also promoted social constructivist learning, where knowledge was co-constructed through interaction. Theoretically, this aligns with the principles of Realistic Mathematics Education (RME), in which contextual problems serve as a bridge between informal strategies and formal mathematical concepts. Thus, the context provided effectively guided students in formulating the surface area formula for a cuboid through meaningful learning. Furthermore, the problem of the volume of a cuboid can be seen in the following figure:

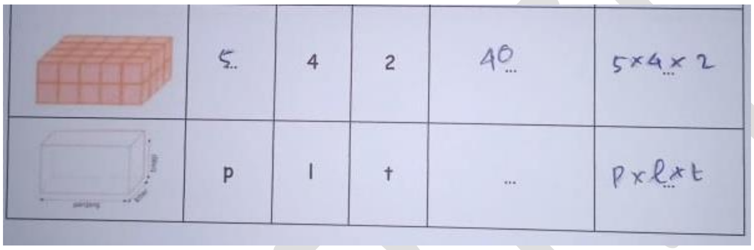

Figure 4. Student answers regarding the surface volume of a cuboid

In Figure 4, the students' answers are visible, and it can be seen that they have developed an understanding of the concept of cuboid volume. Students represented the structure of the cuboid as repeated units and expressed the relationship among length, width, and height symbolically as $V = p \times l \times t$. This indicates that students engaged in higher-order cognitive processes of abstraction and generalization, as defined by Bloom's taxonomy. Collaborative discussion enabled them to connect spatial visualization with symbolic reasoning, supporting the development of relational understanding rather than rote memorization. From a theoretical perspective, this finding supports the constructivist view that knowledge is built through active engagement and contextual exploration. Therefore, it can be concluded that the given context not only guided students in finding the cuboid volume formula but also fostered deeper conceptual understanding through meaningful cognitive engagement.

Based on the stages of the developed learning trajectory, the design effectively facilitates students in constructing mathematical concepts through active engagement and contextual understanding. From a constructivist perspective, this process aligns with the idea that knowledge is built through learners' interaction with meaningful experiences. The incorporation of realistic problems designed by educators situates learning within the framework of realism, emphasizing that mathematical knowledge is not isolated from the real world but emerges from observable, authentic phenomena. According to Akgul and Yilmaz (2023), the construction of mathematical concepts is a multifaceted process that significantly contributes to the development of mathematical realism. By forming complex thought structures, shifting focus to symbolic manipulation, and integrating personal, social, and cultural influences, this process helps in understanding and validating the existence of mathematical entities. Educational practices that emphasize active learning and real-world applications further reinforce the realist perspective, making mathematical concepts more tangible and relevant. In addition, according to Sumirattana et al. (2017), the construction of mathematical concepts often involves interaction with real-world experiences and artefacts. This interaction is crucial for understanding and learning mathematics, as it helps bridge the gap between abstract mathematical ideas and tangible experiences.

Furthermore, integrating Islamic contexts, particularly those related to daily worship, serves as a bridge between abstract mathematical ideas and students' lived experiences, fostering a holistic worldview. This integration reflects the Islamic epistemological view that knowledge and faith are interconnected, and that understanding reality through mathematics is part of understanding divine order. Hence, this study not only supports students' conceptual construction but also strengthens the connection between realism and Islamic values in mathematics learning, particularly in introducing the concept of cuboids through culturally and religiously meaningful contexts.

Integrating Islamic values into mathematics education has the potential to foster deeper discussions on cultural diversity and broaden students' global perspectives on mathematics. This culturally contextualized approach aligns with global research emphasizing the importance of connecting mathematical learning to students' socio-cultural backgrounds to enhance engagement and conceptual understanding (Nasir et al., 2008). By linking local Islamic values with mathematical ideas, students can develop a richer appreciation for the mathematical contributions of various civilizations, fostering both cultural awareness and global-mindedness (Ardi et al., 2024). However, in the context of Indonesia, such integration must be implemented thoughtfully and in harmony with the national curriculum framework. Effective implementation also requires teachers to possess a deep understanding of both mathematical content and the underlying Islamic principles, ensuring that cultural integration supports rather than overshadows mathematical rigor.

A growing body of research highlights the potential of integrating cultural and spiritual contexts, including Islamic symbols, into mathematics learning as a means to support students' character development. This approach is not aimed at altering the epistemological nature of mathematics, but at enhancing students' motivation, sense of relevance, and socio-emotional engagement in learning (Rahmat & Yahya, 2022). From an educational psychology standpoint, contextual learning that incorporates familiar cultural or value-based elements can strengthen students' intrinsic motivation and identity formation through meaningful learning experiences. Empirical studies suggest that such contextualization promotes not only cognitive understanding but also affective and moral growth, contributing to well-rounded educational outcomes (Al Migdadi, 2011).

By understanding the concepts of surface area and volume of cuboids, students can appreciate the complexity and beauty of the universe's design. According to Shaw (2022), geometry serves as a profound medium for spiritual reflection across various cultures and disciplines. Whether through the intricate patterns of Islamic art, the mathematical metaphors in theology, or the sacred designs in architecture, geometry provides a unique language for exploring and expressing spiritual ideas. This intersection of geometry and spirituality fosters a deeper understanding of the divine and the metaphysical, enriching both artistic and religious experiences. This can foster a sense of gratitude for the infinite creation of Allah SWT and an understanding of how beautiful the world He has created is. Learning about the surface area and volume of cuboids also involves mathematical concepts such as comparison, equality, and justice. This can arouse an understanding of the importance of balance in the universe created by Allah SWT. Students can understand the importance of sharing and supporting those less fortunate by understanding the concept of volume and using cuboids in everyday life. This reflects the social values and caring taught in Islam.

The integration of Islamic principles into Realistic Mathematics Education (RME) represents not merely a pedagogical adaptation but an epistemological and axiological expansion of the RME framework. Within the Indonesian educational context, this integration aligns with the National Curriculum's emphasis on character education (*Pendidikan Karakter*), which seeks to cultivate students' spiritual, moral, and social values alongside cognitive development. Character education aims to develop students' good character through attitudes, behaviors, and qualities that differentiate individuals. Integrating character education into mathematics involves embedding character values into learning materials, which serve as guides for teachers (Anggraini & Retnawati, 2022). By embedding mathematical problems in religious and culturally familiar contexts such as the geometric proportions of the Ka'bah, the symmetry in rows during congregational prayer, or the measurement of ablution spaces, students engage in mathematics learning that is both contextually meaningful and morally grounded. This approach fosters not only conceptual understanding but also the internalization of virtues such as discipline, order, cleanliness, and unity (Khalid, 2018).

From a theoretical standpoint, this integration broadens the conception of "realistic" in Freudenthal's (1991) original RME theory. Freudenthal positioned mathematics as a *human activity* where learners reinvent mathematical ideas through engagement with real-world phenomena. However, "reality" in classical RME has predominantly been interpreted through socio-cultural and experiential lenses, emphasizing the development of mathematical reasoning within every day or community-based contexts (Gravemeijer & Cobb, 2013). In contrast, Islamic-based RME extends this notion by incorporating *transcendental* and *moral* realities that are integral to students' lived experiences in faith-based societies. Thus, the contextualization of mathematics through religious and ethical dimensions transforms RME from a purely constructivist model into a value-oriented educational paradigm.

Critically, this integration also strengthens curriculum alignment by embedding the *Profil Pelajar Pancasila* competencies, particularly faith, integrity, and mutual cooperation, into mathematics learning. While classical RME frameworks primarily focus on fostering mathematical literacy and problem-solving, the character-based adaptation introduces a dual focus: cognitive mastery and moral formation. This duality situates mathematical learning within an *axiological realism*, in which knowledge acquisition and value internalization are mutually reinforcing. Hence, integrating Islamic principles into RME not only contextualizes mathematical content but also extends the theoretical boundaries of RME, offering a culturally responsive and morally anchored model of mathematics education that reflects the holistic goals of the Indonesian curriculum (Freudenthal, 1991; Gravemeijer & Cobb, 2013; Khalid, 2018).

The development of higher-order thinking skills (HOTS) is also facilitated by this pedagogy. Students engage in contextual, nuanced analysis, modelling, and problem-solving when taught geometric concepts such as the volume and length of a surface through religious or social activities (Widiawati et al., 2018). For example, calculating the volume of air pressure or estimating the size of a room based on the number of worshipers encourages students to be critical and reflective. This aligns with the higher cognitive levels of Bloom's taxonomy, particularly analyzing, evaluating, and creating, as students are required not only to apply formulas but also to interpret, justify, and construct solutions relevant to real-life contexts. By embedding mathematical learning within meaningful social and spiritual experiences, students move beyond lower-order skills (remembering and understanding) toward HOTS-oriented learning that integrates reasoning, critical thinking, and creative problem-solving. This approach illustrates that mathematics can be seen as a science inseparable from social and spiritual life, contributing to students' understanding of justice, balance, and the creation of Allah SWT. Darwis et al. (2024) stated that Higher-order thinking and contextual learning are interrelated concepts that, when effectively integrated, can significantly enhance students' cognitive abilities and engagement. By employing active learning strategies, technology integration, project-based learning, and experiential learning, educators can create dynamic and effective learning environments that foster higher-order thinking skills. These approaches prepare students for real-world challenges and contribute to their overall academic and professional success.

The Hypothetical Learning Trajectory (HLT) design provides a systematic and flexible framework for influencing students' learning processes (Irma & Nada, 2024). This aligns with the findings of Yulia et al. (2020), who state that HLT can be used as a guide for educators in implementing learning and developing other learning pathways. Through a sequence of organized, progressive activities, HLT enables teachers to guide students from contextual exploration to mathematical abstraction while simultaneously promoting religious and cultural understanding. Besides that, it could also help students reinvent the concepts and build more confidence in using their own strategies to solve contextual problems (Yulia et al., 2019). The novelty of this HLT lies in its integration of Islamic values and Realistic Mathematics Education (RME) principles within the local cultural context, an approach that has rarely been theorized in previous HLT studies. This integration not only situates mathematical concepts in meaningful, culturally relevant experiences but also extends the theoretical scope of HLT by embedding moral and spiritual dimensions into mathematical learning design. Consequently, this study contributes to the theoretical advancement of mathematics education by demonstrating how HLT can serve as a model for culturally responsive, faith-integrated learning trajectories that bridge the gap between abstract mathematical reasoning and students' lived experiences.

Several previous studies have incorporated students' daily activities into mathematics learning, such as Indonesian handicrafts in measurement and geometry (Prahmana & D'Ambrosio, 2020), traditional games in multiplication, addition, and social arithmetic (Prahmana et al., 2012), wayang stories and *uno stucko* in number patterns (McGeough, 2016), and historical buildings in geometric transformations (Bedewy et al., 2022). Other cultural contexts have also been explored, including *Tepuk Bergambar* in number operations (Mayasari et al., 1983), *Gasing* in learning to measure time (Nubatonic et al., 2023), the integration of Sundanese culture in fractions (Abdullah, 2017), the use of everyday representations in understanding fractions (Gabriel et al., 2013), and contextual approaches in arithmetic averages and statistics (Crowley, 2023). Collectively, these studies highlight the potential of local and cultural contexts to enhance mathematical understanding through meaningful and relatable experiences.

However, most of these studies emphasize the use of cultural artefacts or traditional practices as mere pedagogical tools to make mathematical concepts more accessible. In contrast, the present study synthesizes *Realistic Mathematics Education (RME)* principles with Islamic values within a *Hypothetical Learning Trajectory (HLT)* framework. This integration goes beyond cultural contextualization by embedding moral, spiritual, and social dimensions into the learning process. Consequently, this research contributes to contextual mathematics education by providing culturally relevant learning scenarios and by theorizing how contextual and faith-based elements can systematically shape students' cognitive and affective development in mathematics.

CONCLUSION

This study concludes that the mathematics learning trajectory developed through a realistic approach integrated with Islamic values is effective in supporting students' conceptual construction of the surface area and volume of cuboids. Quantitative indicators, such as improvements in students' problem-solving accuracy and the quality of conceptual explanations during classroom discussions, demonstrate a positive impact on students' mathematical thinking skills. In addition, the integration of Islamic values, such as discipline, honesty, and gratitude, was observed to strengthen students' religious character and foster meaningful connections between mathematics and everyday experiences.

However, the study is limited by its small sample size and focus on a single mathematical topic, which may constrain the generalizability of the findings. Future studies should involve a broader range of participants and explore additional topics to validate the approach's consistency further.

Theoretically, this research contributes to the literature on Realistic Mathematics Education (RME) by demonstrating how contextual learning can be meaningfully combined with values-based education to enhance both cognitive and affective learning outcomes. Practically, it provides a structured learning trajectory that can guide mathematics teachers, particularly in Madrasah or value-oriented educational settings, in designing lessons that integrate real-life contexts with religious values. Future research is recommended to investigate the longitudinal effects of this approach and to develop technology-assisted learning tools that further support the integration of contextual and spiritual dimensions in mathematics education.

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