The Optimization Problem of Batik Cloth Production with Fuzzy Multi-Objective Linear Programming and Application of Branch and Bound Method

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Abstract

This study discussed fuzzy multi-objective linear programming (FMOLP) and its application. This research was conducted in Rumah Batik Mentari Jambi, which produces five batik motifs typical of the Jambi. In this research, the tolerance for additional raw material capacity is included in the model. This research aims to find out the number of tolerances needed, the maximum number of batik needed to be produced, and the minimum production time so that the producer can earn the maximum profit. The decision variables in FMOLP are the number of pieces of batik measuring in $2m^2$, which means the decision variables must be an integer. Therefore, after obtaining the optimal solution from FMOLP, then proceed with the branch and bound method to obtain the integer solution. The result of this research is that the addition of raw materials needed to earn optimal solutions is as much as 50% of the tolerance assumed in the model. Thus, owner can earn the optimal profit of Rp. 5,675,800.00/week by producing as many as 67 pieces of batik with the design of angso duo, 18 pieces with the design of gentala, and 50 pieces with the design of batang hari, and the minimum production time is 270 hours/week.

Keywords: Branch and bound, fuzzy linear programming, linear programming, multi-objective linear programming, optimization

Introduction

Along with the development of the industrial world and technological advances, operations research is increasingly being applied, especially in the production field. It can solve problems related to the production of a product. Linear Programming (LP) is a model in operations research that deals with allocating possible limited resources to obtain optimal results. In LP, the objective function is linear, and the constraints are also in the form of linear equation or inequation. One of the methods to find the optimal solution in LP is the simplex method.

A good linear programming requires precisely defined data, but the certainty, reliability, and data accuracy are hard to obtain, so the data becomes uncertain. Those uncertainties can be solved by developing a linear programming model in a fuzzy environment and becoming an alternative as a decision-making model, where the resulting value is reasonable to choose. If fuzzy environment is applied in optimization model, then the mathematical model will be Fuzzy Linear Programming (FLP) [1]. In fuzzy linear programming (FLP), the constraints, which have the form of equations or inequations, can involve fuzzy numbers for the coefficients of decision variables in the left-hand side and for constant in the right-hand side. In the FLP model, the fuzzy numbers reflect the tolerance value which is allowed and subjectively estimated for additional of raw material capacity. Usually, when additional of raw material capacity is applied, it will imply the additional time for production. Furthermore, if there is more than one objective in the FLP model, the model formed is called as Fuzzy Multi-Objective Linear Programming (FMOLP).

The related paper about FLP was written by S.H. Nasseri [2] in which discussed about a method to solve FLP problem with fuzzy coefficeint in the constraints and objective functions based on multi objetive model. El-Saeed Ammar [3] wrote paper about mathematical model for solving fuzzy integer linear programming. Then Weijie Wang et all [4] modeling the problem of global supply chain network in fuzzy multi-objective mixed integer programming which aimed to maximize the delivery quality and manufacturer's profit. Badhotiya [5] modeling the integrated production and distribution planning problem in the form of FMOLP. Tai-Sheng Su [6] used FMOLP model to solve remanufacturing planning problems with multiple products, Esra Cakir et al [7] used FMOLP approach for nuclear power plant installation. Legiani, et al [8] used FMOLP to maximize profits and minimize waste costs by adding the tolerance to constraints, and applied the integer value for decision variables. Erfianti and Muhaijir [9] used FMOLP to maximize profit, and minimize the processing time, and also applied the integer-valued optimal solution. Then, Chunquan Li [10] also wrote a paper about FMOLP in which the coefficients of decision variables in objective function and in the constraints are triangular fuzzy number. In this research, we built a mathematical model in the form of FMOLP with tow objectives, i.e. to maximize the profits and minimize the production time, in which the constant in the right-hand side in the constraint applied fuzzy number. The optimal solution of the FMOLP model is built with the simplex method. However, if the optimal solution is not integer-valued, the branch and bound method are used to obtain an integer-valued optimal solution.

Methods

1. Linear Programming (LP)

Linear programming is the most basic mechanism in OR for formulating various problems with simple efforts characterized by linear objective functions and constraints. Problems in LP can be solved with the simplex method [11][12][13][14]. Linear programming is the most basic mechanism for formulating various problems with simple efforts characterized by linear objective functions and constraints [15]. LP problems can be found in various fields and used as an appropriate alternative in decision-making so that the best solution is obtained [16].

The model formulation is the most decisive step in LP which includes the identification of matters relating to the objectives and constraints of the problem. Some basic elements in formulating the LP model are: [14][17]

1) Decision variables $\left(X_j\right)$: variables that we seek to identify its value;
2) The objective function: a function that describes the goals or objectives in LP problems related to the optimal utilization of resources to obtain maximum profit or use minimum cost.
3) The constraint function: a formulation of the availability of resources in achieving the goal.

In general, the mathematical model of the LP problem is

$$\begin{aligned} &\text{Maximize} && f(\mathbf{x}) = \mathbf{c}^T\mathbf{x} \\ &\text{with constraints} && \\ & && A\mathbf{x} \le \mathbf{b} \\ & && \mathbf{x} \ge \mathbf{0}, \end{aligned} \tag{1}$$

with $\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_n \end{bmatrix} \in \boldsymbol{R^n}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} \in \boldsymbol{R^n}, \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_m \end{bmatrix} \in \boldsymbol{R^m}, A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \in \boldsymbol{R^{m\times n}}, \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \end{bmatrix}$ is a zeros vector in $\boldsymbol{R^n}$ [18] .

2. Simplex Method

The steps in the simplex method are as follows [19], [20], [21]:

1) Identify decision variables and formulate them in mathematical symbols;
2) Identify the objective function and constraints;
3) Formulate the objective function and constraints in a mathematical model;
4) Convert the inequality in the constraint into an equation;
5) Input the coefficients on the objective and constraint functions into the simplex table. The coefficients of the objective function are written in the top row;
6) Identify the pivot column, i.e. the most negative entry in the top row;
7) Calculate the quotients which are computed by dividing the value on the right column (the value of $\mathbf{b}$), with the value in pivot column. If denominator is zero or negative, then quotient is ignored. The smallest quotients will be the pivot row;
8) State the pivot element, i.e. the number in intersection of pivot column and pivot row;
9) Make all other entries in pivot column to be zero and perform pivoting. This is just like the method of Gauss-Jordan;
10) The simplex method is finished or stopped if there are no more negative values in the top row. Otherwise, repeat the algorithm from step 6.
11) When the simplex method is considered finished, then state or identify the basic solution for optimal condition, i.e. look at the columns which have 1 and all other elements zeros. The maximum value appears in the top right hand corner.

3. Fuzzy Linear Programming (FLP)

The definitions of some of the terms used in Fuzzy Linear Programming (FLP) are as follows [1]:

Definition 1: Assume X is a set of objects, then the set containing ordered pairs

$$\bar{A} = \{(x, \mu_{\bar{A}}(x)) : x \in X\}$$

where $\mu_{\bar{A}}: X \to [0,1]$, is called the fuzzy set in X and $\mu_{\overline{A}}(x)$ is called the membership function.

Definition 2: Assume $\bar{A}$ is a fuzzy set in X and $\lambda \in [0,1]$ is a real number. Then, a set

$$\overline{A}^{\lambda} = \{x \in X : \mu_{\overline{A}}(x) \ge \lambda\}$$

is called $\lambda -$ cut of $\overline{A}$.

There are various types of FLP depend on the models and it's solutions that have been summarized by Reza Ghanbari et all in [22]. The formation of the FLP model is derived from the classical LP model in optimization model (1). Let say the value of the objective function $f(\mathbf{x})$ in model (1) is denoted by z, and each constraint is modeled as a fuzzy set, then the FLP model for such LP problem (for maximization objective) is generally written as follows

Find $\mathbf{x}$ such that

$$\begin{aligned} \mathbf{c}^T\mathbf{x} &\gtrsim z \\ A\mathbf{x} &\lesssim \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0}, \end{aligned} \tag{2}$$

with $\mathbf{c} \in \boldsymbol{R^n}, \mathbf{x} \in \boldsymbol{R^n}, \mathbf{b} \in \boldsymbol{R^m}, A \in \boldsymbol{R^{m\times n}}$ [23].
However, if model (1) has a minimization objective, then the FPL model is generally written as
Find $\mathbf{x}$ such that

$$\begin{aligned} \mathbf{c}^T\mathbf{x} &\lesssim z \\ A\mathbf{x} &\gtrsim \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0}. \end{aligned}$$

Notation $\gtrsim$ is the fuzzy version of $\geq$ and implies "essentially greater than or equal to". While $\lesssim$ is the fuzzy version of $\leq$ and means "essentially less than or equal to" [24].

Model (2) can be written into a new problem, i.e. to find $\mathbf{x}$ of the model

$$\begin{aligned} B\mathbf{x} &\lesssim \mathbf{d} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned} \tag{3}$$

with

$$\begin{pmatrix} -\mathbf{c}^T \\ A \end{pmatrix} = B \in \boldsymbol{R}^{(\boldsymbol{m}+\mathbf{1})\times \boldsymbol{n}}, \text{ and } \begin{pmatrix} -z \\ \mathbf{b} \end{pmatrix} = \mathbf{d} \in \boldsymbol{R}^{(m+1)\times 1} \text{ for maximization problem,}$$

or

$$\begin{pmatrix} \mathbf{c}^T \\ -A \end{pmatrix} = B \in \boldsymbol{R}^{(\boldsymbol{m}+\mathbf{1})\times \boldsymbol{n}}, \text{ and } \begin{pmatrix} z \\ -\mathbf{b} \end{pmatrix} = \mathbf{d} \in \boldsymbol{R}^{(m+1)\times 1} \text{ for the minimization problem.}$$

Thus model (3) contains $(m+1)$ rows [18]. Each inequality in model (3) is represented as a fuzzy set. The membership function of the fuzzy "decision" set of model (3) is

$$\boldsymbol{\mu_D}(\mathbf{x}) = \min_{\boldsymbol{i}}\{\boldsymbol{\mu_i}(\mathbf{x})\}.$$

$\boldsymbol{\mu_i}(\mathbf{x})$ is the membership function of the row i-th and can be interpreted as the degree to which $\mathbf{x}$ satisfies the inequality $B_{\boldsymbol{i}}\mathbf{x} \lesssim \mathbf{d_i}$, with $\boldsymbol{B_i}$ is row i-th of matrix B and $\boldsymbol{d_i}$ is row $\boldsymbol{i}$-th of vector $\boldsymbol{d}$ [18].

A solution that has the largest membership value is the best solution, so the true optimal solution will be

$$\max_{\boldsymbol{x}\geq\mathbf{0}} \boldsymbol{\mu_D}(\mathbf{x}) = \max_{\boldsymbol{x}\geq\mathbf{0}} \min_{\boldsymbol{i}}\{\boldsymbol{\mu_i}(\mathbf{x})\}. \tag{4}$$

If the boundaries/constraints and objective function are not satisfied, then $\mu_{\boldsymbol{i}}(\mathbf{x}) = 0$. Conversely, if the constraints and objective function are completely fulfilled, then $\mu_{\boldsymbol{i}}(\mathbf{x}) = 1$. One of the simple membership functions that can be used in this situation is

$$\mu_i(\boldsymbol{x}) = \begin{cases} 1 & ; \boldsymbol{B_i}\mathbf{x} \leq d_i \\ 1 - \dfrac{\boldsymbol{B_i}\mathbf{x} - d_i}{p_i} & ; d_i < \boldsymbol{B_i}\mathbf{x} \leq d_i + p_i \\ 0 & \boldsymbol{B_i}\mathbf{x} > d_i + p_i \end{cases} \tag{5}$$

for $i = 1,2,\ldots,m+1$, with p_i is a constant that can be chosen subjectively from tolerable violations of the constraints and objective function [18], see Figure 1.

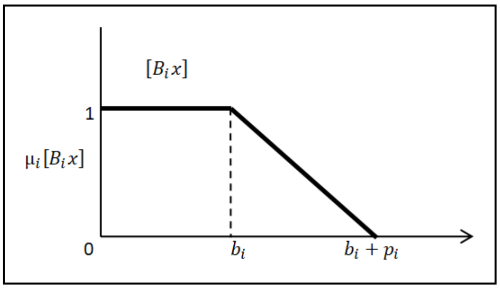

Figure 1. Illustration of Membership Function [23]

If formula (5) is substituted into (4) and with some additional assumptions, we get

$$\max_{\mathbf{x}\geq \mathbf{0}} \boldsymbol{\mu_D}(\mathbf{x}) = \max_{\mathbf{x}\geq \mathbf{0}} \min_{\boldsymbol{i}} \left\{ \mathbf{1} - \frac{\boldsymbol{B_i}\mathbf{x} - \boldsymbol{d_i}}{\boldsymbol{p_i}} \right\}. \tag{6}$$

By introducing a new variable, named λ, which is basically related to equations (4), (5) and (6), i.e. by defining

$$\lambda = 1 - \frac{\boldsymbol{B_i}\mathbf{x} - d_i}{p_i} \Longrightarrow \lambda p_i = p_i - (\boldsymbol{B_i}\mathbf{x} - d_i) \Longrightarrow \lambda p_i + \boldsymbol{B_i}\mathbf{x} = d_i + p_i,$$

so that the model becomes [25]

$$\begin{aligned} &\text{maximize} && \lambda \\ &\text{with constraint} && \\ & && \boldsymbol{\lambda p_i} + \boldsymbol{B_i}\mathbf{x} \leq \boldsymbol{d_i} + \boldsymbol{p_i} \\ & && \mathbf{x} \geq \mathbf{0}, \end{aligned} \tag{7}$$

By solving the optimization problem in model (7) and assuming that the optimal solution of model (7) is $(\lambda, \mathbf{x}^*)$, then we can say that $\mathbf{x}^*$ is the optimal solution for the model (2) based on the assumption of membership function (5). Furthermore, $\lambda - cut$ can be found from equation $\lambda = 1 - t$, with $d_i + tp_i$ is the constant in right-hand side of the i^{th} constraint [18].

4. Fuzzy Multi-objective Linear Programming (FMOLP)

There are many fields of science where optimal decisions depend on two or more objectives, so that they apply multiobjective optimization. An optimization problem which involves more than one objective functions are called a multiobjective optimization problem [26]. Fuzzy multi-objective linear programming (FMOLP) is an optimization method that has more than one objective function subject to multiple constraints. The solution of an FMOLP problem can be obtained using the same method as an LP problem that has one objective function [23]. Other methods which also can be used to solve FMOLP problem are epsilon-constraint method [27], by using fuzzy dominant degrees [28], by using game theory approach [29], and the two-stage method which in the first stage, the FMOLP is transformed into the interval FMOLP, and then transformed again into the crisp multi-objective LP. Then in the second stages, the crisp multi-objective LP is converted into mono-objective program [30], and other methods as explained in [31] and [32] .

5. Branch and Bound Method

The Branch and Bound method is a well-known technique applied to solve Integer Programming problems, which is a problem that requires the decision variable to be an integer. The basic concept of this method is the branching stage and the stage of determining the boundaries for the existence of feasible solutions (bounding) [13]. Here is the algorithm of Branch and Bound method for maximization problem [14].

1) Set the initial lower bound, i.e. $z = -\infty$ and $i = 0$.
2) Fathomed/bounding stage.

   Choose a LP_i that is a sub-problem to be tested and found a solution. Next, try to achieve a fathomed condition through one of the following conditions:
   a) The value of z, which is the optimal value of LP_i, cannot produce an objective value that is better than the current lower bound value,
   b) LP_i produces an integer solution that is feasible and better than the current lower bound,
   c) LP_i has no feasible solution.

   As a result, there will be two possibilities:
   a) If LP_i fathomed and a better feasible solution is found, then the lower bound should be updated. z must be updated. If all sub-problems have reached the fathomed state, then stop branching, the lower bound is said to have provided the optimal solution. If this is not the case, then set $i = i + 1$. And repeat step 2).
   b) If L Pi is not fathomed, then go to step 3) for branching.

3) Branching stage

   Choose a variable x_j (whose constraint is integer) whose optimal value is x_j^* in the solution of the LP_i is *non integer*, such that

$$x_j \leq [\![x_j^*]\!] \text{ and } x_j \geq [\![x_j^*]\!] + 1.$$

   Then assign $i = i + 1$ and repeat step 2).

Results and Discussion

In this research, there are five decision variables for modelling the optimization problem. They express the number of batik cloths to be produced with 5 different motifs, namely

X_1 = production number of batik motif tampuk manggis per week
X_2 = production number of batik motif duren pecah per week
X_3 = production number of batik motif angso duo per week
X_4 = production number of batik motif gentala per week
X_5 = production number of batik motif per week
The length of each piece of batik cloth is 2m^2.

The data used in this research are primary data from Rumah Batik Mentari, which produces five batik motifs. The main raw materials to produce batiks are fabric, wax and fabric dye. The use or consumption of raw materials for each batik motifs is presented in Table 1.

Table 1. Data on consumption of raw materials, production time and profit

Raw Materials, profit and production time	Tampuk Manggis	Duren Pecah	Angso Duo	Gentala	Batanghari	Capacity	Tolerance
Fabric (meter)	2	2	2	2	2	240	60
Wax (ounce)	3	3.3	2.2	1.8	2.8	300	40
Fabric dye (gram)	40	55	40	45	50	5800	400
Profit per piece (IDR)	43,000	44,300	41,200	40,300	43,800		
Production time per piece (hour)	2.5	2.5	2	2	2		

Based on the data in Table 1, we want to analyze the optimal solutions if we want to maximize the profit and minimize the production time to avoid overtime. To achieve this goals, then we build a mathematical model in the form of multi-objective linear programming where the first objective is maximizing profit (Z_1), while the second objective is minimizing production time (Z_2). For this research, the owner of Rumah Batik Mentari did not state the minimum production for each motives. Therefore, the multi-objective linear programming is as follow:

$$\begin{array}{ll} \text{Maximize:} & Z_1 = 43{,}000X_1 + 44{,}300X_2 + 41{,}200X_3 + 40{,}300X_4 + 43{,}800X_5 \\ \text{Minimize:} & Z_2 = 2.5X_1 + 2.5X_2 + 2X_3 + 2X_4 + 2X_5 \\ \text{with constraints:} & \\ \text{(a):} & 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 \leq 240 \\ \text{(b):} & 3X_1 + 3.3X_2 + 2.2X_3 + 1.8X_4 + 2.8X_5 \leq 300 \\ \text{(c):} & 40X_1 + 55X_2 + 40X_3 + 45X_4 + 50X_5 \leq 5{,}800 \\ \text{(d):} & X_1, X_2, X_3, X_4, X_5 \geq 0 \end{array} \tag{8}$$

Constraint (a) in model (8) is related to usage and capacity of fabric, constraint (b) is related to usage and capacity of wax, and constraint (c) is related to usage and capacity of fabric dye, which coefficients in the left-hand side and constant in the right-hand side are referred to the data in Table 1. Based on model (8), then inequality (3) can be formed with

$$B = \begin{pmatrix} -43{,}000 & -44{,}300 & -41{,}200 & -40{,}300 & -43{,}800 \\ 2.5 & 2.5 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3.3 & 2.2 & 1.8 & 2.8 \\ 40 & 55 & 40 & 45 & 50 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}, \mathbf{d} = \begin{pmatrix} -Z_1 \\ Z_2 \\ 240 \\ 300 \\ 5{,}800 \end{pmatrix}.$$

Let B_i means the entry in i^{th}-rows in matrix B. B_3 comes from the coefficient of the decision variables in constraint (a) in model (8), B_4 is the coefficient in constraint (b) in model (8) and B_5 is the coefficient in constraint (c) in model (8). Furthermore, if a tolerance is added to each constraint, namely an additional maximum of 60 meters for fabric capacity, an additional maximum of 40 ounces for wax capacity, and an additional 400 grams for dye capacity, then model (8) is written as follows

$$\begin{array}{ll} \text{Maximize:} & Z_1 = 43{,}000X_1 + 44{,}300X_2 + 41{,}200X_3 + 40{,}300X_4 + 43{,}800X_5 \\ \text{Minimize:} & Z_2 = 2.5X_1 + 2.5X_2 + 2X_3 + 2X_4 + 2X_5 \\ \text{with constraints:} & \\ \text{(a):} & 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 \leq 240 + 60t \\ \text{(b):} & 3X_1 + 3.3X_2 + 2.2X_3 + 1.8X_4 + 2.8X_5 \leq 300 + 40t \\ \text{(c):} & 40X_1 + 55X_2 + 40X_3 + 45X_4 + 50X_5 \leq 5{,}800 + 400t \\ \text{(d):} & X_1, X_2, X_3, X_4, X_5 \geq 0 \\ \text{(e):} & 0 \leq t \leq 1 \end{array} \tag{9}$$

From model (9), for the sake of consistency of the index for constraints, it can be said that $p_3 = 60$, $p_4 = 40$ and $p_5 = 400$. The state $t = 0$ indicates no addition of raw materials and $t = 1$ indicates the addition of raw materials. The optimal solution of the model (9) in the state $t = 0$ and $t = 1$ can be found by the simplex method, which is shown in Table 2.

Table 2. Solution of Model (9) when $t = 0$ and $t = 1$

Decision Variables	$t = 0$	$t = 1$
X_1	0	0
X_2	0	0
X_3	2.86	128.57
X_4	34.29	2.86
X_5	82.86	18.57
Z_1	5,128,571.43	6,225,714.29
Z_2	240	300

Based on the value of Z_1 on the state $t = 0$ and Z_1 on the state $t = 1$, then we have

$$p_0 = 6{,}225{,}714.29 - 5{,}128{,}571.43 = 1{,}097{,}142.86.$$

Afterwards, the FMOLP model can be built which will maximize λ with both objective functions in model (8) being constraints on FMOLP. The first objective function related to maximize profit can be written as

$$\text{Minimize: } -Z_1 = -43{,}000X_1 - 44{,}300X_2 - 41{,}200X_3 - 40{,}300X_4 - 43{,}800X_5.$$

Then the FMOLP model formed is as shown in model (10).

$$\text{Maximize: } \lambda$$

with constraints:

$$\begin{array}{lcr}
\text{(a):} & 43{,}000\,X_1 + 44{,}300X_2 + 41{,}200X_3 + 40{,}300X_4 + 43{,}800X_5 - 1{,}097{,}142.86\lambda & \geq 5{,}128{,}571.43 \\
\text{(b):} & 2.5X_1 + 2.5X_2 + 2X_3 + 2X_4 + 2X_5 + 60\lambda & \leq 300 \\
\text{(c):} & 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 + 60\lambda & \leq 300 \\
\text{(d):} & 3X_1 + 3.3X_2 + 2.2X_3 + 1.8X_4 + 2.8X_5 + 40\lambda & \leq 340 \\
\text{(e):} & 40X_1 + 55X_2 + 40X_3 + 45X_4 + 50X_5 + 400\lambda & \leq 6{,}200 \\
 & \lambda, X_1, X_2, X_3, X_4, X_5 & \geq 0
\end{array} \quad (10)$$

The constraint (a) in model (10) is taken from the maximization objective in model (9) and the constrain (b) is taken from the minimization objective in model (9). While the constraints (c), (d) and (e) in model (10) are respectively derived from the constraints (a), (b) and (c) in model (9), respectively. The optimal solution of model (10) is found using the two-phase technique because there is an inequality in the form of greater than equal (≥) in the constraints. The solution of model (10) is shown in Table 3

Table 3. Solution of model (10)

Solution	
X_1	0
X_2	0
X_3	65.71
X_4	18.57
X_5	50.71
λ	0.5

Since $\lambda = 0.5$, then we obtain $t = 1 - 0.5 = 0.5$. By substituting $t = 0.5$ into model (9), then the total of inventory capacity required is

$$\text{Fabric capacity: } B_3\mathbf{x} = 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 = 270.$$

Wax capacity: $B_4\mathbf{x} = 3X_1 + 3.3X_2 + 2.2X_3 + 1.8X_4 + 2.8X_5 = 320.$

Fabric dye capacity: $B_5\mathbf{x} = 40X_1 + 55X_2 + 40X_3 + 45X_4 + 50X_5 = 6{,}000.$

Thus, the inventory of raw materials needed to produce five batik motifs is 270 meters of fabric/ material, 320 ounces of wax and 6,000 grams of fabric dye.

The membership function on the constraints is used to determine the degree of membership value for each constraint. Based on the membership function formula in equation (5), for this case, the membership function value for each constraint of model (8) is obtained as follows

$$\mu_3[\mathbf{x}] = 1 - \frac{B_3\mathbf{x} - d_3}{p_3} = 1 - \frac{270 - 240}{60} = 0.5.$$

$$\mu_4[\mathbf{x}] = 1 - \frac{B_4\mathbf{x} - d_4}{p_4} = 1 - \frac{320 - 300}{40} = 0.5.$$

$$\mu_5[\mathbf{x}] = 1 - \frac{B_5\mathbf{x} - d_5}{p_5} = 1 - \frac{6{,}000 - 5{,}800}{400} = 0.5.$$

The membership degree value for each constraint of 0.5 indicates that, the addition of raw materials needed to obtain the optimum solution is 0.5 times the tolerance of each constraint. That is, for fabric capacity an additional $0.5 \times 60 = 30$ meters is needed. For the wax capacity, it takes an additional $0.5 \times 40 = 20$ ounces of wax. And for the dye capacity, it takes an additional $0.5 \times 400 = 200$ grams of dye.

Thus, if the constants in right-hand side in the first constraint, the second constraint and the third constraint respectively are applied 270, 320 and 6,000 to model (9), then using the simplex method, the optimal solution can be obtained, namely $X_1 = 0, X_2 = 0, X_3 = 65.71, X_4 = 18.57, X_5 = 50.71$ with optimum values $Z_1 = 5{,}677{,}142.86$ and $Z_2 = 270$.

The decision variables $X_i; i = 1,2,3,4,5$ in this optimization problem is the number of fabric in pieces (2m^2), so an integer solution is required in this problem. Unfortunately, FMOLP gives a non-integer solutions, so it is necessary to apply the Branch and Bound method to the model by adding the integer constraints X_1, X_2, X_3, X_4 and X_5. The Branch and Bound chart to find the integer solution is shown in Figure 2 with $t = 0.5$. Through the Branch and Bound method, for the purpose of maximizing Z_1 and minimizing Z_2, the optimum solution is finally obtained in sub-problem 36 because in this sub-problem, the value of Z_1 is the highest, while the value of Z_2 is the lowest, with condition that all decision variables $(X_1, X_2, X_3, X_4, X_5)$ have integer value, i.e

$$X_1 = 0, X_2 = 0, X_3 = 67, X_4 = 18, X_5 = 50, Z_1 = 5{,}675{,}800, Z_2 = 270.$$

Conclusion

The optimization problem of batik cloth production at Rumah Batik Mentari is analyzed through the Fuzzy Multi Objective Linear Programming (FMOLP) model by considering two objective functions, namely maximizing profit and minimizing production time. The optimum solution of the FMOLP is to produce batik cloth with a total of 135 pieces of batik cloth consisting of 67 pieces of angso duo motif, 18 pieces of gentala motif and 50 pieces of batang hari motif. Meanwhile, the motifs of tampuk mangosteen and durian pecah do not contribute to gain maximum profit based on current data. With this result, maximum profit can be obtained is IDR 5,675,800 / week with the required production time of 270 hours / week. Eventhough current result might be unsatisfactory for the owner because there are two motifs that do not contribute to gain maximum profit based on this research analysis, but due to some people's interest in such two motifs, the owner still produce batik with motifs of tampuk mangosteen and durian pecah in few number of production. While in the meantime, the owner needs to review the capacity comparing to materials consumptions such that the analysis will give result of optimal solution for all motifs.

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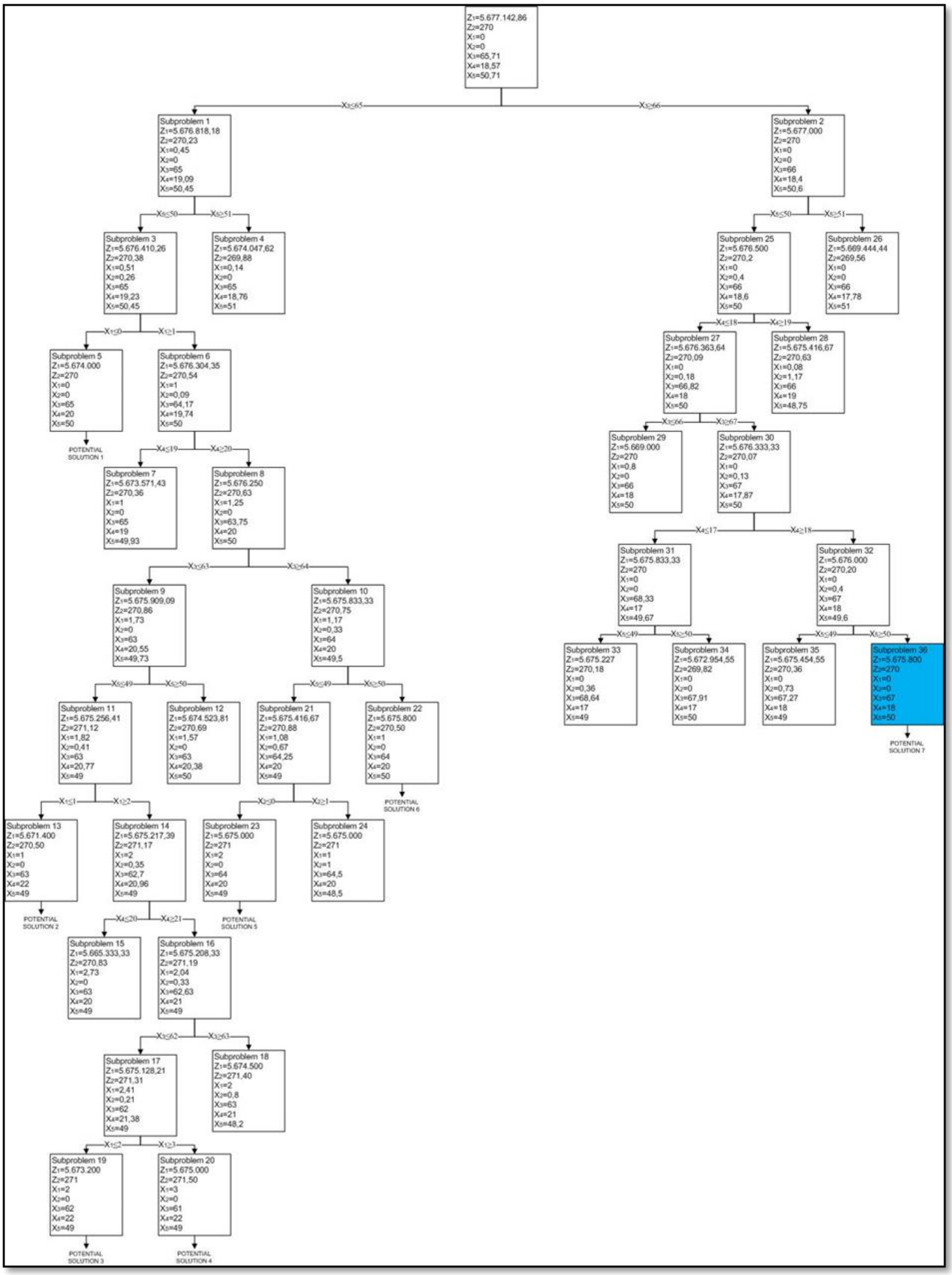

Figure 2. Chart of Branch and Bound Method

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