# Quadratic Spline and Heligman-Pollard Methods in The Preparation of Life Tables in Gegelang Village West Lombok

Andri Azmul Fauzi[1, a)], Dara Puspita Anggraeni[1, b)], Lalu Muhammad Ridwan[1, c)]

*[1]Department of Mathematics, Universitas Nahdlatul Wathan Mataram, Indonesia*

*[a)]email: andriazmulfauzi@unwmataram.ac.id*
*[b)] email: darapuspita.anggraeni@unwmataram.ac.id*
*[c)] email: lmuhridwan@unwmataram.ac.id*

**Abstract**

In this paper, we conduct research by compiling an abridged life table using the graduation method based on real data obtained at Gegelang Village, West Lombok. The graduation methods used in this study included quadratic spline and Heligman-Pollard. Based on the results, a special case of A was chosen as a method used to make a rough life table before graduation. After the graduation process, it was found that the Heligman-Pollard method was the most suitable. This is based on the chart produced using this method, which is monotonous in nature and does not fluctuate like other graduation methods. In addition, the life expectancy obtained at 65.72 years is close to West Lombok's life expectancy based on 2015.

*Keywords: graduation process,  life expectancy, life table.*

**Introduction**

The population is the real asset of a nation. Many factors can affect population development, both in quality and quantity. The quality of life that belongs to a country or region explains the people's welfare and the success of the many programs that have been designed by the government to improve the standard of human life. Quality of life can be seen from life expectancy. Life expectancy at a certain age describes the average number of years of life that will be lived by someone who has managed to reach that age in a death situation prevailing in the community [1].

The facts show that life expectancy in developed countries is different from developing countries. We can see this from the socio-economic conditions of a country which greatly influence the size of life expectancy. To see the life expectancy of a country, we can look at the life table that is owned by that country. Life table is a table that describes the chances of survival to a certain age from the year of birth [2].

Until now, Indonesia still does not have a life table obtained from death data by age that can be used primarily to calculate life expectancy. The life tables that are still used today are in the form of an approach adapted to the Western Coale-Demeny Model Life Table. Riyana (2018) has conducted research on determining the best method for estimating the life table of the elderly population in Indonesia. To estimate the complete life table, several graduation methods were used, including the Kostaki, Heligman-Pollard, Elandt-Johnson method, and several methods for estimating the complete life table of the elderly using the Heligman-Pollard method which was modified using the death rate from the Gompertz and Makeham distributions. From this study, it was found that the Heligman-Pollard method was the best graduation method for estimating the complete life table of the several methods used. However, the results obtained are only for parents, not for all ages. There are five objectives obtained from the graduation process, including smoothing data so that the data becomes regular and consistent, making precise and appropriate results in smoothing curves based on data, helping to conclude imperfect data, making it easy to compare mortality rates, and helping forecasting [4].

West Lombok Regency is one of 10 regencies in NTB Province. Administratively, West Lombok Regency consists of 10 districts, with 122 villages and 841 hamlets. The districts include Gunungsari, Batu Layar, Lingsar, Narmada, Labuapi, Kediri, Kuripan, Gerung and Lembar districts . From year to year it is known that the socio-economic conditions and the level of health of the people of West Lombok have continued to increase. This is evidenced by the continued increase in life expectancy in this district. In 2020 the life expectancy of the people of West Lombok was 66.94 years. Then it continued to increase until 2021 respectively 67.19 years. In 2022, the combined life expectancy for West Lombok Regency also increased from 2021, namely 67.63 years .

Lingsar District consists of 15 villages with 95 hamlets. One of the villages in Lingsar District is Gegelang Village. The economic condition of Gegelang Village increases significantly every year and continues to grow. This cannot be separated from the support of various parties, especially the agriculture, industry and trade services and related agencies which provide assistance in the Gegelang Village area which is very large for improving the community's economy such as providing assistance with superior rice seeds so as to improve the people's standard of living . In addition, Gegelang Village is one of the villages that has complete data to make life tables when compared to other villages, such as population data, birth data and death data. However, until now Gegelang Village does not yet have a life table.

One method that can be used to calculate data in the life table is interpolation. Interpolation is a method used to estimate missing or missing data values among existing data values. Polynomial interpolation problems are problems that are used to find polynomials that can connect a set of known points. One form of solving polynomial interpolation is using the quadratic spline method [3][5].

Based on the above problems, this research will reconstruct a life table based on real data from Gegelang Village, West Lombok, after previously smoothing the model using the graduation method. In particular, the purpose of this study was to estimate the probability of death, determine the rough life table, carry out the graduation, and determine the appropriate life table model based on several methods used. So that the life expectancy of Gegelang Village can be obtained based on the calculation of the life table.

**Method**

In this study, the data used is secondary data obtained from Gegelang Village, West Lombok. The data taken for this study were population data for 2016, birth data for the 2014–2018 period, and death data for the 2014–2018 period. After data collection, the next step is to calculate the parameter q (probability of death) using the moment estimator method to obtain a rough life table model. Next, perform a graduation on the rough-life table using the graduation methods. The graduation methods used are the Heligman-Pollard and quadratic spline methods. After graduation, the next step is to choose a life table model based on the two models obtained.

1. Calculating the Probability of Death for an Incomplete Data Sample Using the Moment Method

The moment method is a method used to obtain an estimate of the parameter $\beta$ by equating the moments of the population with the moments of the sample [6] . The moment method is one of the methods used to assess the probability of death in both complete and incomplete data. In this study, for incomplete data using the moment method, several methods were used to assess the probability of death, including the general case method, special case A, special case B, special case B, special case C, the Hoem approach and the actuarial approach.

An incomplete data sample is a case where, at the end of the observation period, there are still individuals who survive [7]. To calculate the probability of death for an incomplete data sample when observations are made, it is necessary to know the age vector and the duration vector. The age vector

($\boldsymbol{v}_i'$) is defined as $\boldsymbol{v}_i'$=$[y_i, z_i, \theta_i]$, where $y_i$ is the exact age at the start of observation of individuals, $z_i$ is the exact age at the end of the observation period, dan $\theta_i$ is the exact age at death during the observation process. While the duration vector ($\boldsymbol{u}_{i,x}'$) is defined as $\boldsymbol{u}_{i,x}'$=$[r_i, s_i, \iota_i]$, where $x$ is the age of the ith individual observed, $r_i$ is the initial observed value, $s_i$ is the final value of the observation, dan $\iota_i$ s the value when death occurs during the observation [7]. The duration vector ($\boldsymbol{u}_{i,x}'$) is calculated to determine the initial and final values of the ith individual observations at each age interval $(x, x+5]$.

The expected value of death at the age interval $(x, x+5]$ if there are $N_x$ individuals observed is

$$E(D_x) = \sum_{i=1}^{N_x} {}_{s_i - r_i}q_{x+r_i}. \tag{1}$$

For the general case method, it is assumed that $0 \leq r_i < 5$, $0 < s_i \leq 5$, $0 \leq \iota_i < 5$ where $\iota_i < s_i$, and $t_i$ = $s_i$ for $\delta_i$ = 0 and $t_i$ = $\iota_i$ for $\delta_i$ = 5, by equating the number of deaths to the expected value, the moment equation is obtained, that is

$$E(D_x) = \sum_{i=1}^{N_x} {}_{t_i - r_i}q_{x+r_i}^G = D_x. \tag{2}$$

Furthermore, if it is assumed that ${}_{t_i - r_i}q_{x+r_i}^G \approx (t_i - r_i) \cdot q_x^G$, then equation (2) can be formulated also in the form

$$E(D_x) = q_x^G \cdot \sum_{i=1}^{N_x} (t_i - r_i) = D_x, \tag{3}$$

so that the estimator of the parameter q is obtained for each age interval $(x, x+5]$ i.e.

$$\hat{q}_x^G = \frac{D_x}{\sum_{i=1}^{N_x}(t_i - r_i)}. \tag{4}$$

For special case A, by equating the number of deaths to the expected value, if there are $N_x$ individuals observed at the age interval $(x, x+5]$, assuming $r_i = 0$ and $s_i = 5$ the moment equation is obtained, namely

$$E(D_x) = N_x \cdot q_x^A = D_x, \tag{5}$$

so that the estimator of the parameter q for each age interval $(x, x+5]$ is

$$\hat{q}_x^A = \frac{D_x}{N_x}. \tag{6}$$

In the special case B method, by equating the number of deaths to the expected value, it is assumed that $0 \leq r_i < 5$ and $s_i = 5$, and there are $N_x$ individuals observed at the age interval $(x, x+5]$, so that the moment equation is obtained, namely

$$E(D_x) = \sum_{i=1}^{N_x} {}_{1 - r_i}q_{x+r_i}^B = D_x. \tag{7}$$

Furthermore, using Balducci's assumption that is ${}_{b-a}q_{x+a} = \frac{(b-a)q_x}{1-(1-b)q_x}$, then equation (7) can also be formulated in the form

$$E(D_x) = q_x^B \cdot \sum_{i=1}^{N_x} (1 - r_i) = D_x, \tag{8}$$

so that the estimator of the parameter q is obtained for each age interval $(x, x+5]$ i.e.

$$\hat{q}_x^B = \frac{D_x}{\sum_{i=1}^{N_x} (1 - r_i)}. \tag{9}$$

For special case C, by equating the number of deaths to the expected value, it is assumed that $r_i = 0$ and $0 < s_i \leq 5$, and there are $N_x$ ndividuals observed at the age interval $(x, x+5]$, then the moment equation is obtained, namely

$$E(D_x) = \sum_{i=1}^{N_x} {}_{s_i}q_x^C = D_x. \tag{10}$$

Furthermore, using the assumption of a uniform distribution of deaths, ${}_{b-a}q_{x+a} = \frac{(1-a)q_x}{1-a\cdot q_x}$, then equation (10) can also be formulated in the form

$$E(D_x) = q_x^C \cdot \sum_{i=1}^{N_x} s_i = D_x, \tag{11}$$

so that the estimator for the parameter q for each age interval $(x, x+5]$ is

$$\hat{q}_x^C = \frac{D_x}{\sum_{i=1}^{N_x} s_i}. \tag{12}$$

In Hoem's approach, assuming the length of time the i-th individual participates in the age interval $(x, x+5]$ from the start of joining $(r_i)$ to the end of joining $(s_i)$ even though there is death $(\iota_i)$ which means $\iota_i = s_i$, then by equating the number of deaths to the expected value, the moment equation is

$$E(D_x) = \sum_{i=1}^{N_x} {}_{s_i - r_i}q_{x+r_i}^H = D_x. \tag{13}$$

Furthermore, if it is assumed ${}_{s_i - r_i}q_{x+r_i}^H \approx (s_i - r_i) \cdot q_x^H$, then equation (13) can also be formulated in the form

$$E(D_x) = q_x^H \cdot \sum_{i=1}^{N_x} (s_i - r_i) = D_x, \tag{14}$$

so that the estimator of the parameter q is obtained for each age interval $(x, x+5]$ is

$$\hat{q}_x^H = \frac{D_x}{\sum_{i=1}^{N_x} (s_i - r_i)}. \tag{15}$$

In the actuarial approach, it is assumed that every i-th individual who dies at the age interval $(x, x+5]$, has reached the end of the age interval $(x, x+5]$ which means $s_i$ = 5. By equating the number of deaths to the expected value, the moment equation is

$$E(D_x) = \sum_{i=1}^{N_x} \left( {}_{s_i - r_i}q_{x+r_i}^L + {}_{1-s_i}q_{x+s_i}^L \delta_i \right) = D_x. \tag{16}$$

Furthermore, if it is assumed $\sum_{i=1}^{N_x} {}_{s_i-r_i}q_{x+r_i}^L + {}_{1-s_i}q_{x+s_i}^L \delta_i \approx (s_i - r_i) \cdot q_x^L + (5 - s_i) \cdot q_x^L \cdot \delta_i$, then equation (16) can also be formulated in the form

$$E(D_x) = q_x^L \cdot \sum_{i=1}^{N_x} (s_i - r_i) + q_x^L \cdot \sum_{i=1}^{N_x} (1 - s_i)\delta_i = D_x, \tag{17}$$

so we get an estimator of the parameter q for each age interval $(x, x+5]$ namely

$$\hat{q}_x^L = \frac{D_x}{\sum_{i=1}^{N_x}(s_i - r_i) + \sum_{i=1}^{N_x}(1 - s_i)\delta_i}. \tag{18}$$

2. Interpolation

Interpolation is a calculation process to find the value of a function whose graph passes through a certain set of points. These points are the result of the known function [8]. The main goal of obtaining an approximation polynomial is to replace functions with complex forms into functions that are simpler and easier to manipulate .

Interpolation polynomials depend on the number of values given [9]. One example is the short-term mortality probability curve in humans. This probability does not increase significantly at the youngest age due to neonatal death, relatively flat until early adolescence, increases slowly in adolescence, fluctuates in the age range of 18-25 years (due to accidents), then increases slowly but increases for higher ages [10].

3. Graduation Process

The first graduation method used is the quadratic spline. The quadratic spline function (Q) is the set of quadratic polynomial pieces on the interval $[a, b]$ that have the form

$$Q(x) = \begin{cases} Q_0(x) & ; \quad x \in [t_0, t_1] \\ Q_1(x) & ; \quad x \in [t_1, t_2] \\ \vdots & \\ Q_{n-1}(x) & ; \quad x \in [t_{n-1}, t_n]. \end{cases}$$

For each interval $[t_i, t_{i+1}]$, $Q_i(x)$ is defined as

$$Q_i(x) = \left( \frac{z_{i+1} - z_i}{2(t_{i+1} - t_i)} \right) (x - t_i)^2 + z_i (x - t_i) + y_i \tag{19}$$

The coefficients $z_0, z_1, \ldots, z_n$ are generated by selecting the value $z_0$. Then for $z_1, z_2, \ldots, z_n$ find by using the equation $z_{i+1} = -z_i + 2\left(\frac{y_{i+1}-y_i}{t_{i+1}-t_i}\right)$, where $(0 \le i \le n-1)$.

The second graduation method used is the Heligman-Pollard method, which is an interpolation method that can represent death over the span of an entire life. Suppose a function is defined by $F(x;c)$, namely a function with the age variables $x$ and $c$, which are parameter vectors in the equation, then the Heligman-Pollard equation will become:

$$\frac{q_x}{p_x} = F(x;c) \tag{20}$$

with $F(x;c) = GH^x$ [11].

The relationship between ${}_nq_x$ and $q_x$ n the Heligman-Pollard method is

$${}_nq_x = 1 - \prod_{i=0}^{n-1}(1 - q_{x+i}). \tag{21}$$

Equation (21) in the Heligman-Pollard method can be proven as follows:

$$\begin{aligned} {}_np_x &= 1 - {}_nq_x \\ &= 1 - \frac{{}_nd_x}{l_x} \\ &= \frac{l_{x+n}}{l_x} \\ &= \frac{l_{x+1}}{l_x}\frac{l_{x+2}}{l_{x+1}}\cdots\frac{l_{x+n}}{l_{x+n-1}} \\ &= p_x p_{x+1} \cdots p_{x+n-1} \\ &= \prod_{i=0}^{n-1} p_{x+i} \\ {}_nq_x &= 1 - \prod_{i=0}^{n-1} p_{x+i} \\ {}_nq_x &= 1 - \prod_{i=0}^{n-1}(1 - q_{x+i}). \end{aligned}$$

4. Life Table

There are three demographic components that influence population structure, namely mortality, fertility, and population mobility. Mortality is an event of permanent loss of all signs of individual life that can occur at any time[12][13]. Mortality indicator is a number used to determine the high or low mortality rate of a population. Numbers cannot represent the state of death of a population [14]. Basically the mortality table is a hypothetical table that combines various mortality rates at various ages into one statistical model [15].

Life table is a table that describes a person's chances of surviving to a certain age from the year of birth [2]. Life table is one way to analyze mortality at a certain age, calculate the probability of survival and the average life expectancy of the population. In compiling a life table, several notations and functions are used. Notations and their functions include:

1. The age of the population observed in the life table is denoted by $x$
2. The number of survivors at the exact age $x$ is denoted by $l_x$
3. The number of deaths in the population in the preparation of the life table in the age range $x$ to $x+n$ is denoted by ${}_nd_x$ , with the formula

$$ {}_nd_x = l_x - l_{x+n}. \tag{22} $$

4. Furthermore, the case of the number of deaths in the population between the ages of $x$ to $x+1$ is denoted by $d_x$ with the formula

$$ d_x = l_x - l_{x+1}. \tag{23} $$

5. Individuals aged $x$ right who have a chance of surviving to age $x+1$ are denoted by ${}_np_x$, where

$$ {}_np_x = \frac{l_{x+n}}{l_x} \tag{24} $$

6. For the case where the probability of survival of a population aged $x$ will reach exactly the age $x+1$ is denoted by $p_x$and defined as

$$ p_x = \frac{l_{x+1}}{l_x}. \tag{25} $$

7. The value denoting the probability that an individual aged $x$ will die before reaching age $x+n$ is denoted by${}_nq_x$, where

$$ {}_nq_x = 1 - {}_np_x = \frac{{}_nd_x}{l_x}. \tag{26} $$

8. For the case where the probability of death of a population aged $x$ will die before reaching the exact age $x+1$ is denoted by $q_x$ and defined as

$$ q_x = 1 - p_x = \frac{d_x}{l_x}. \tag{27} $$

9. The symbol that shows the age of a number of$l_x$ residents in the time interval from age $x$ to age $x+n$ is ${}_nL_x$ , where

$$ {}_nL_x = \int_0^n l_{x+s}\, ds = n \cdot l_x - \frac{n}{2}\, {}_nd_x = \frac{n}{2}(l_x + l_{x+n}). \tag{28} $$

10. While the symbol that shows longevity with the number $l_x$ in the time interval from age $x$ to age $x+1$ is $L_x$, where

$$ L_x = \int_0^1 l_{x+s}\, ds. \tag{29} $$

11. The symbol that shows the remaining time that will be lived by an individual with the right age $x$ is denoted by $T_x$, and is defined by

$$ T_x = \sum_{a=0}^{\omega} {}_nL_{x+an} \tag{30} $$

where ω is the maximum age of a person in the life table.

12. The life expectancy of a population aged $x$ in the life table is denoted by $\dot{e}_x$, by the formula

$$\dot{e}_x = \frac{T_x}{l_x}. \tag{31}$$

**Results and Discussions**

1. The Calculation of Crude Life Table

Based on Gegelang Village data, the calculated crude life table is obtained by using the moment method which the life expectancy calculation results provided on Table 1 below.

**Table 1.** The calculation results of life expectancy crude life table

| No | Method | Life Expectancy |
|---|---|---|
| 1 | *General case* | 72.79 |
| 2 | *Special case A* | 62.21 |
| 3 | *Special case B* | 76.49 |
| 4 | *Special case C* | 75.60 |
| 5 | Hoem Approach | 72.92 |
| 6 | Actuarial Approach | 72.93 |

Based on table 1, different life expectancy is obtained. The highest value is obtained using the special case B method, while the lowest value is obtained using the special case A method. This happened because of using the different method and data condition. For example, the special case A method used the group data, while the general case method used the complete data.

2. Graduation Model

To do graduation, one method that which is close to the life expectancy of West Lombok Regency will be chosen. In 2015, it is known that the life expectancy of West Lombok Regency is 65,10 years. Based on Table 1, special case A is chosen because its life expectancy result is closest to West Lombok Regency's in 2015. After the graduation is done to the crude life table, then a chart as shown in Figure 1 and 2 will be obtained.

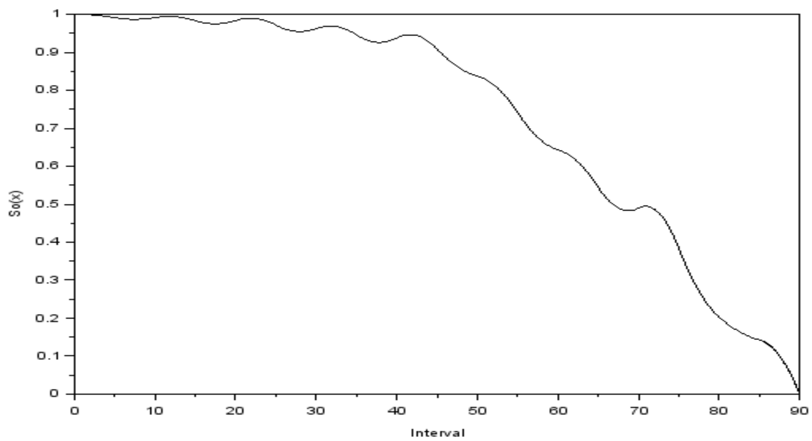


**Figure 1.** Special case A chart using quadratic spline method

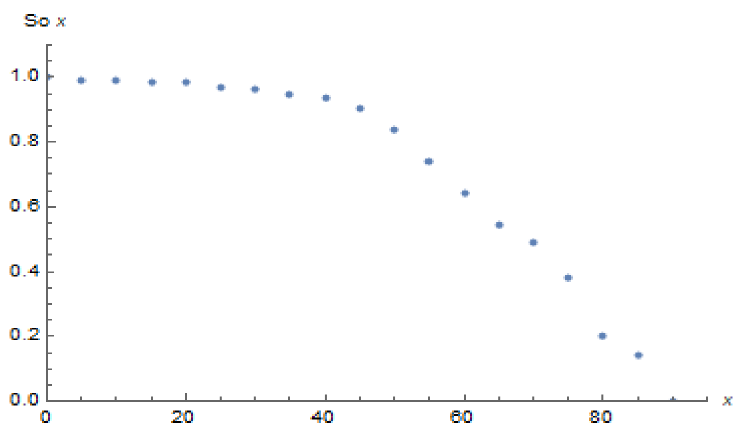


**Figure 2.** Special case A chart using Heligman-Pollard method

Based on Figure 1 and 2, it can be seen that the chart made by using quadratic spline method is fluctuative and is not monotonically descending compared to the chart made by using Heligman-Pollard which is monotonically descending. Furthermore, different life expectancies are obtained after the calculation is done. The amount of life expectancy using quadratic spline method is 65.80 years, while using the Heligman-Pollard method is 65.72 years.

3. The Selection of Life Table Models

The selection of Life Table model is based on the chart model and the life expectancy which is the closest to life expectancy of West Lombok Regency in 2015. It should be noted that a life table is said to be good if the line is monotonically descending. Based on the chart images above, from both of the chart resulted, the chart model of Heligman-Pollard is said to be appropriate. This is because it produced the monotonically descending line compared to the quadratic spline model. On quadratic spline model the resulted chart is fluctuative and is not monotonically descending as previously explained. Furthermore, the life expectancy that is closest to the life expectancy of West Lombok Regency in 2015 is the result of Heligman-Pollard method.

Based on the discussion above, the approppriate life table model is the life table result of the Heligman-Pollard method. The complete calculation result of the life table using Heligman-Pollar method is presented on Table 3.

**Table 3**. *Life Table* using Heligman-Pollard method

| $x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $T_x$ | $\dot{e}_x$ |
|---|---|---|---|---|---|
| 0 | 100000 | 193 | 99904 | 6571826 | 65.72 |
| 1 | 99807 | 519 | 398191 | 6471923 | 64.84 |
| 5 | 99288 | 527 | 495125 | 6073732 | 61.17 |
| 10 | 98762 | 523 | 492502 | 5578606 | 56.49 |
| 15 | 98239 | 478 | 490001 | 5086105 | 51.77 |
| 20 | 97761 | 568 | 487387 | 4596104 | 47.01 |
| 25 | 97194 | 523 | 484661 | 4108717 | 42.27 |
| 30 | 96671 | 1045 | 480741 | 3624057 | 37.49 |
| 35 | 95625 | 1568 | 474207 | 3143316 | 32.87 |
| 40 | 94057 | 3659 | 461139 | 2669109 | 28.38 |
| 45 | 90398 | 6795 | 435003 | 2207970 | 24.42 |
| 50 | 83603 | 9409 | 394493 | 1772967 | 21.21 |
| 55 | 74194 | 10454 | 344835 | 1378474 | 18.58 |
| 60 | 63740 | 9932 | 293871 | 1033639 | 16.22 |
| 65 | 53808 | 4678 | 257348 | 739768 | 13.75 |
| 70 | 49131 | 11004 | 218145 | 482420 | 9.82 |
| 75 | 38127 | 18295 | 144898 | 264275 | 6.93 |
| 80 | 19832 | 5873 | 84479 | 119378 | 6.02 |
| 85+ | 13960 | 13960 | 34899 | 34899 | 2.50 |

The life table compiled using the Heligman-Pollard method is expected to be a base by Gegelang Village officials in West Lombok to make policies that can improve people's welfare in the future.

## Conclusion

The best method to make crude life table is the special case A method. After the graduation is done to the crude life table, the most appropriate graduation method between the two models used is obtained, which is the Heligman-Pollard. Furthermore, based on Gegelang Village data from 2014 until 2018 using the moment method for special case A and graduation method Heligman-Pollard, the life expectancy in the amount of 65.72 years is obtained.

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