https://journal.uinsgd.ac.id/index.php/kubik/index
Vol. 9, No. 2, 2024, pp. 278-294
e-ISSN: 2686-0341 p-ISSN: 2338-0896

A Semi Analytical Approach to the Solution of Linear and Nonlinear Telegraph Equations

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Abstract

In this paper, we apply the semi analytic iterative method to find approximate solutions of linear and nonlinear telegraph equations. To determine the accuracy and effectiveness of the method, five numerical examples are given and the results obtained are presented using tables and figures. These results are in good agreement with the exact solutions to the problems. Furthermore, the absolute errors produced by our method are found to be closer to zero than those from the other methods used in the literature. The computational simplicity, efficiency, precision and reliability of the semi analytic iterative method have been verified. The proposed method of solution is advantageous in that it does not require any non-physical restrictive assumptions underlying such methods as the Adomian decomposition method, the variational iteration method and the differential transform method for nonlinear terms, that is, the use of Adomian polynomials and Lagrange multipliers. Furthermore, the present method does not require addition of a perturbation term to the solution, as in the homotopy perturbation method and its variants. Due to its simple, straightforward and accurate implementation, as well as its rapid rate of convergence to the exact solution, the proposed method can be extended to solution of a broad variety of other linear and nonlinear partial differential equations.

Keywords: Telegraph equations, Semi-analytic iterative method, Differential transform method, Variational iteration method, Homotopy perturbation method
MSC2020: 35A01, 35A16

Abstrak

Makalah ini mengusulkan metode iteratif semi-analitik untuk memperoleh solusi pendekatan dari persamaan telegraf linier dan nonlinier. Lima contoh numerik disajikan guna mengukur keakuratan dan efektivitas metode, dengan hasil yang ditampilkan dalam bentuk tabel dan gambar. Hasilnya menunjukkan kesesuaian yang baik dengan solusi eksak, serta menghasilkan galat absolut yang lebih kecil dibandingkan metode lain yang telah ada. Keunggulan metode ini terletak pada kesederhanaan, efisiensi, dan tidak bergantung pada asumsi non-fisik seperti pada metode Adomian, iterasi variasional, maupun homotopi perturbasi. Karena implementasinya yang sederhana dan konvergensi

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Received: 17-07-2024, Accepted: 15-11-2024, Published: 30-11-2024

cepat, metode ini berpotensi diterapkan secara luas untuk berbagai jenis persamaan diferensial parsial linier maupun nonlinier.

Kata kunci: Persamaan telegram, metode iteratif semi-analitik, metode transformasi diferensial, metode iterasi variasional, metode perturbasi homotopi
MSC2020: 35A01, 35A16

Citation: C. Kasumo, "A Semi Analytical Approach to the Solution of Linear and Nonlinear Telegraph Equations", *KUBIK J. Publ. Ilm. Mat.*, Vol. 9, No. 2, pp. 278-294, 2024.

Introduction

Hyperbolic partial differential equations (PDEs) are commonly used for modelling the vibration of structures and form the basis for the fundamental equations of atomic physics. Additionally, these equations are used in signal analysis for transmission and propagation of electrical signals. A case in point is the one-dimensional telegraph equation which is mostly used in wave propagation of electrical signals in a cable transmission line [1]. The telegraph equation models a mixture between diffusion and wave propagation by inserting a term that accounts for effects of finite velocity to the standard heat or mass transport equation [2]. Also known as a *damped* wave equation, the telegraph equation has many applications which include propagation of pressure waves occurring in pulsating blood flow in arteries, random motion of a bug along a hedge, digital image processing and propagation of electrical signals along a telegraph or cable transmission line [3].

This paper is concerned with finding solutions to linear and nonlinear telegraphic equations using the semi analytic iterative method (SAIM) which was first proposed by Temimi and Ansari [4]. This method has been used for solving all kinds of linear and nonlinear ordinary differential equations, partial differential equations and higher-order integrodifferential equations [4],[5],[6]. More recently it has been applied to solution of Duffing equations [7], Blasius equation [8], nonlinear thin film flow problems [9], chemistry problems [10] and KdV equations [11]. To the best of the author's knowledge, this method has not been applied to the solution of one-dimensional second-order telegraph equations.

In this paper we apply the SAIM to the solution of linear and nonlinear telegraph equations which take the form

$$u_{tt} + (\alpha + \beta)u_t + \alpha\beta F(u) = c^2 u_{xx} + f(x,t) \tag{1}$$

in the region $\Omega = \{(x,t) | a < x < b, 0 < t < T\}$ subject to the initial and boundary conditions

$$\begin{gathered} u(x,0) = g_1(x), u_t(x,0) = g_2(x), a \le x \le b, \\ u(0,t) = h_1(t), u(L,t) = h_2(t), 0 \le t \le T, \end{gathered} \tag{2}$$

where L is the length of the cable and $u = u(x,t)$ is the unknown function representing the voltage flowing through the wire at any position x and any time t. Also, $F(u) = a_1u^3 + a_2u^2 + a_3u$, so that if $a_1 = a_2 = 0$ then (1) is linear, otherwise it is non-linear. $\alpha, \beta, a_1, a_2, a_3$ are known real constants and f, g_1, g_2, h_1, h_2 are known continuous real-valued functions. In equation (1), we have

$$\alpha = \frac{G}{C}, \beta = \frac{R}{L}, c^2 = \frac{1}{LC},$$

where R and G represent the resistance and conductance, respectively, of the resistor per unit length of cable, while C and L are, respectively, the capacitance and conductance of the capacitor per unit length of cable. For derivation of (1), see Srivastava et al. [12].

Equation (1), with initial and boundary conditions (2), does not have an analytical solution. Therefore, several methods and schemes have been devised in the literature for finding approximate solutions to the telegraph equation. These include the Adomian decomposition method (ADM) [13], homotopy perturbation method (HPM) [14], the variational iteration method (VIM) [15] and the differential transform method (DTM) [16], to mention but a few. The ADM decomposes a differential equation into simpler sub-problems, the VIM uses Lagrange multipliers to optimize solutions, the HPM gradually deforms the problem into a simpler one while adding a perturbation term to the solution [17] and the DTM is an iterative method based on Taylor's series. However, these methods have their drawbacks. The ADM requires the use of computation-intensive Adomian polynomials, the VIM requires the use of Lagrange multipliers and the HPM requires inclusion of a perturbation term to the solution. Though the DTM finds the solution of the telegraph equation directly without linearization, transformation, discretization or restrictive assumptions, it has the limitations of a computation-intensive implementation for nonlinear terms and a small domain of convergence, with results being valid only in intervals close to the initial conditions [18]. The foregoing weaknesses of the other methods used in the literature have motivated the use of the SAIM in this paper.

In the rest of the paper we will give a description of the proposed method of solution, present results of numerical experiments based on five test problems and compare these results with the exact solution and solutions from previous methods used in the literature and draw conclusions from the results.

Methods

This section describes the proposed method of solution of equation (1), namely, the *semi analytic iterative method* (SAIM). The SAIM was used by [6] to solve higher order integro-differential equations and by [11] to solve KdV equations. This method uses an iterative approach together with analytical computations to provide a solution of a modified reformulated linear problem. The SAIM was inspired by the homotopy analysis method (HAM) which is a general approximate analytical approach for obtaining convergent series solutions of strongly nonlinear problems [5]. The SAIM offers several advantages over existing methods such as Picard's successive approximations method (SAM) and the Adomian decomposition method (ADM) in that it is very easy to implement since it avoids the calculation of Adomian polynomials for the nonlinear term in the ADM or Lagrange multipliers in the VIM, thus demanding less computational work [19]. In this paper we propose to use the SAIM to solve linear and nonlinear telegraph equations of the form (1), with initial and boundary conditions (2), which can be expressed as

$$Lu + Nu = f(x,t) \tag{3}$$

with the condition $C\left(u, \frac{\partial u}{\partial t}\right) = 0$, where $Lu = u_{tt}, Nu = -c^2 u_{xx} + (\alpha + \beta)u_t + \alpha\beta F(u)$ and $f(x,t)$ is the source term. As can be seen from the foregoing, L is the linear operator and N the nonlinear operator which may include $f(x,t)$ and some linear functions. The algorithm of the SAIM is represented by the flowchart in Figure 1 below. The first step in the implementation of the SAIM is to find the initial approximation by solving

$$L[u_0(x,t)] - f(x,t) = 0 \text{ with } C\left(u_0, \frac{\partial u_0}{\partial t}\right) = 0. \quad (4)$$

It should be noted that

$$u_0(x,t) = u(x,0) + tu_t(x,t) = g_1(x) + tg_2(x).$$

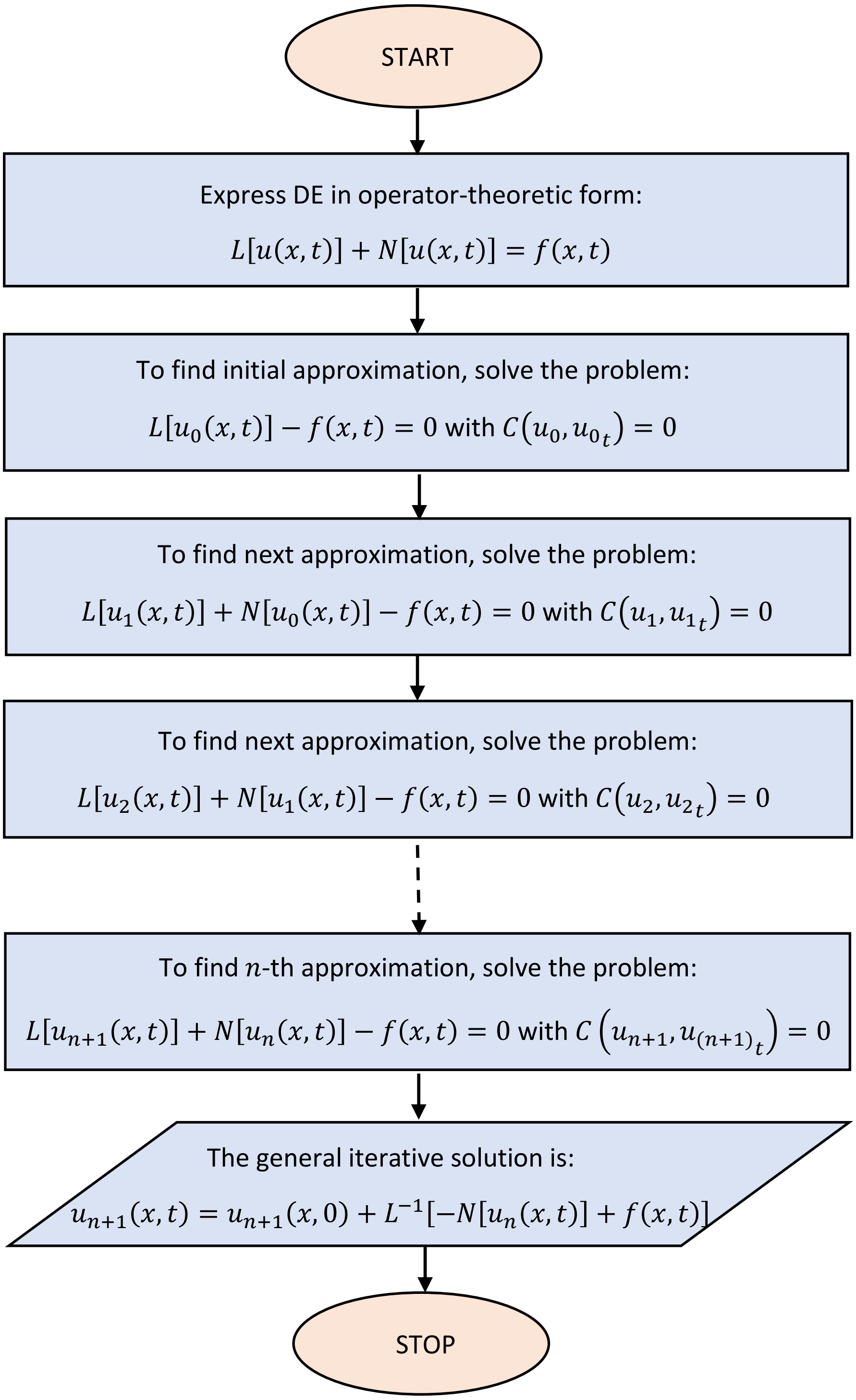


Figure 1. Flowchart of the SAIM algorithm

The next iteration to the solution can be obtained by solving

$$L[u_1(x,t)] + N[u_0(x,t)] - f(x,t) = 0 \text{ with } C\left(u_1, \frac{\partial u_1}{\partial t}\right) = 0. \quad (5)$$

After several iterations we obtain the general form of the SAIM solution which is

$$L[u_{n+1}(x,t)] + N[u_n(x,t)] - f(x,t) = 0 \text{ with } C\left(u_{n+1}, \frac{\partial u_{n+1}}{\partial t}\right) = 0 \quad (6)$$

from which the general iterative formula for solving the telegraph equation (1) is

$$u_{n+1}(x,t) = u_{n+1}(x,0) + L^{-1}[-N[u_n(x,t)] + f(x,t)], \quad (7)$$

where $L^{-1} = \int_0^t \int_0^t (.)dsds$. Each iteration of the function $u_n(x,t)$ effectively represents a complete solution for equation (3). If $f(x,t) = 0$, then the telegraph equation is homogeneous, otherwise it is nonhomogeneous.

Results and Discussion

In this section we present some numerical examples illustrating the applicability of the SAIM for solving linear and nonlinear telegraph equations. All the computations associated with these examples were performed using a Samsung Series 3 PC with an Intel Celeron CPU 847 at 1.10GHz with 6.0GB internal memory and 64-bit operating system (Windows 8). Unless otherwise stated, a fixed $t = 0.01$ was used throughout and the figures were constructed using MATLAB R2016a. The results are presented in tables and figures accompanying the discussion.

Example 1. Consider the linear homogeneous telegraph equation [13],[14]:

$$u_{tt} + u_t + u = u_{xx}, u(x,0) = e^x, u_t(x,0) = -e^x, -2 \le x \le 3, \quad (8)$$

having exact solution $u(x,t) = e^{x-t}$.

To solve (8) using the SAIM, we need to rewrite it as

$$Lu + Nu = 0,$$

where $Lu = u_{tt}$ and $Nu = -u_{xx} + u_t + u$. The primary problem is to find the initial approximation by solving the equation

$$L[u_0(x,t)] = 0 \text{ with } u(x,0) = e^x, u_t(x,0) = -e^x. \quad (9)$$

Using the initial conditions, the solution of the primary problem is

$$u_0(x,t) = u(x,0) + tu_t(x,0) = e^x - te^x = e^x(1-t).$$

The general recursive relation for solving (8) is

$$L[u_{n+1}(x,t)] = -N[u_n(x,t)] \text{ with } u_{n+1}(x,0) = e^x, u_{(n+1)_t}(x,0) = -e^x, \tag{10}$$

i.e.,

$$u_{n+1}(x,t) = u_{n+1}(x,0) + \int_0^t \int_0^t [u_{n_{xx}} - u_{n_s} - u_n] ds ds. \tag{11}$$

Using this recursive relation, we have the approximations

$$u_0(x,t) = e^x(1-t)$$
$$u_1(x,t) = e^x(1-t) + \int_0^t \int_0^t [u_{0_{xx}} - u_{0_s} - u_0] ds ds = e^x\left(1 - t + \frac{t^2}{2}\right)$$
$$u_2(x,t) = e^x(1-t) + \int_0^t \int_0^t [u_{1_{xx}} - u_{1_s} - u_1] ds ds = e^x\left(1 - t + \frac{t^2}{2} - \frac{t^3}{6}\right)$$

and so on. Thus, as $n \to \infty$,

$$u(x,t) = e^x e^{-t} = e^{x-t},$$

which is the exact solution of the given telegraph equation. This result was also obtained using the Adomian decomposition (ADM) and Taylor Matrix methods (TMM) [13] and the homotopy perturbation method (HPM) [14]. The results are shown in Table 1 and Figures 2, 3 and 4 which compare the exact and SAIM solutions. It is evident from Figure 3 that there is a general reduction in the voltage $u(x,t)$ with time.

Table 1. Comparison of SAIM and exact solutions for Example 1 ($t = 0.01$)

x	$u(x,t)$	$u_{SAIM}(x,t)$
0	0.990049833749	0.990049833749
0.2	1.209249597657	1.209249597657
0.4	1.491824697641	1.491824697641
0.6	1.803988415398	1.803988415398
0.8	2.203396426256	2.203396426256
1.0	2.691234472349	2.691234472349

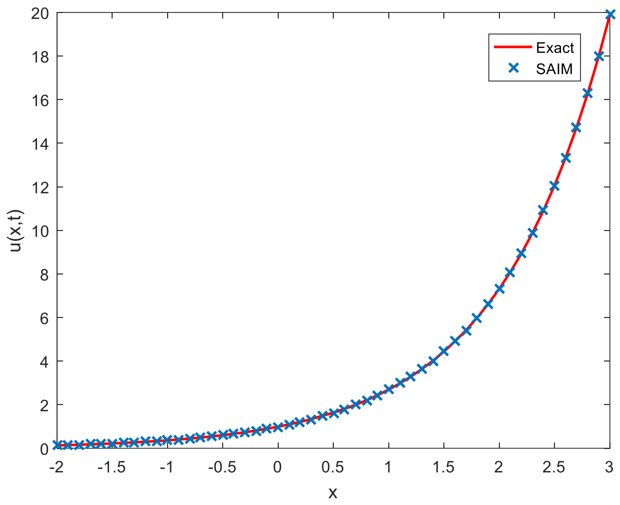


Figure 2. Plot of SAIM and exact solutions for $-2 \leq x \leq 3$ and $t = 0.01$ for the linear telegraph equation in Example 1

Example 2. Consider the linear nonhomogeneous telegraph equation [14]:

$$u_{tt} + u_t + u = u_{xx} + x^2 + t - 1, u(x,0) = x^2, u_t(x,0) = 1, -2 \leq x \leq 3, \tag{12}$$

whose exact solution is $u(x,t) = x^2 + t$.

Rewriting (12) as

$$Lu + Nu = f(x,t),$$

where $Lu = u_{tt}$ and $Nu = -u_{xx} + u_t + u$ and $f(x,t) = x^2 + t - 1$, the general recursive relation is given by

$$L[u_{n+1}(x,t)] = -N[u_n(x,t)] + f(x,t) \text{ with } u_{n+1}(x,0) = x^2, u_{(n+1)_t}(x,0) = 1. \tag{13}$$

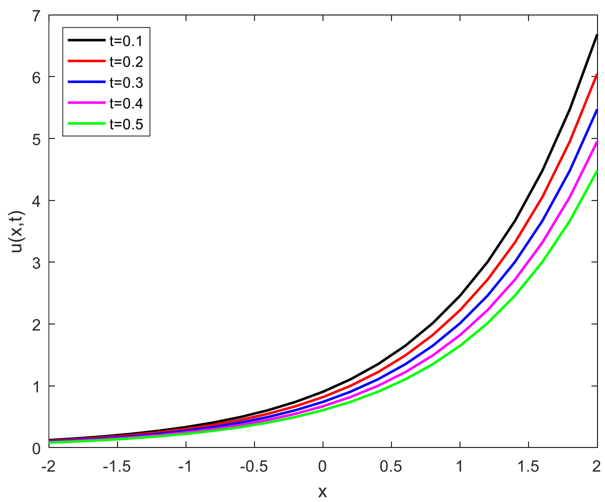
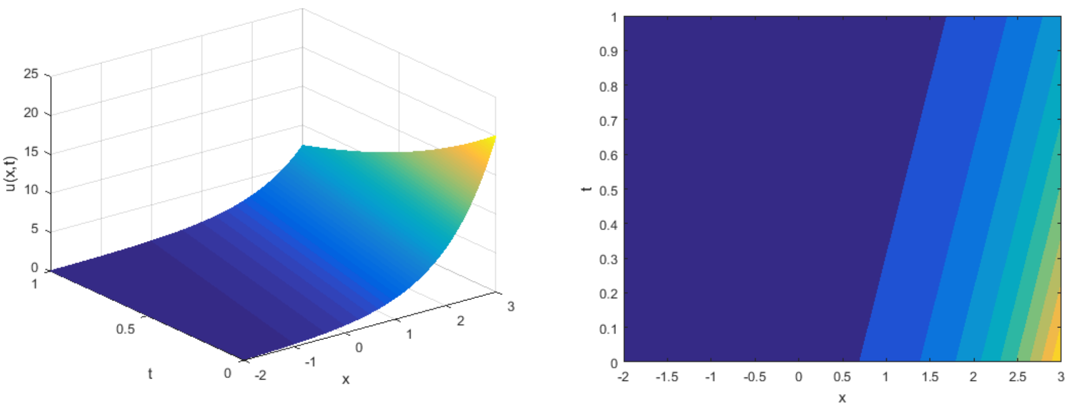


Figure 3. SAIM solutions for $-2 \leq x \leq 2$ at different time values for the linear telegraph equation in Example 1



(a) (b)

Figure 4. (a) Surface plot of SAIM solution for $-2 \leq x \leq 3$ and $0 \leq t \leq 1$ for the linear telegraph equation in Example 1; (b) Contour diagram of SAIM solution for Example 1

We use the iteration

$$u_{n+1}(x,t) = u_{n+1}(x,0) + \int_0^t \int_0^t [u_{n_{xx}} - u_{n_s} - u_n + x^2 + s - 1] dsds \tag{14}$$

to obtain the successive approximations

$$u_0(x,t) = x^2 + t$$
$$u_1(x,t) = x^2 + t + \int_0^t \int_0^t [u_{0_{xx}} - u_{0_s} - u_0 + x^2 + s - 1] dsds = x^2 + t$$
$$\vdots$$
$$u_{n+1}(x,t) = x^2 + t, n \geq 1$$

which is the exact solution that was also obtained using the homotopy perturbation method [14]. Table 2 shows the results for $t = 0.01$ and $0 \leq x \leq 1$, and Figures 5 and 6 show the results for $-2 \leq x \leq 3$.

Table 2. Comparison of SAIM and HPM solutions with the exact solution for Example 2 ($t = 0.01$)

x	$u(x,t)$	$u_{SAIM}(x,t)$	$u_{HPM}(x,t)$
0	0.01	0.01	0.01
0.2	0.05	0.05	0.05
0.4	0.16	0.16	0.16
0.6	0.37	0.37	0.37
0.8	0.65	0.65	0.65

1.0	1.01	1.01	1.01

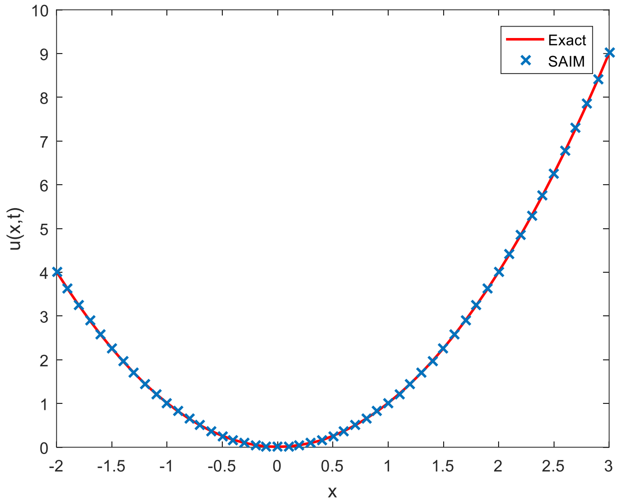


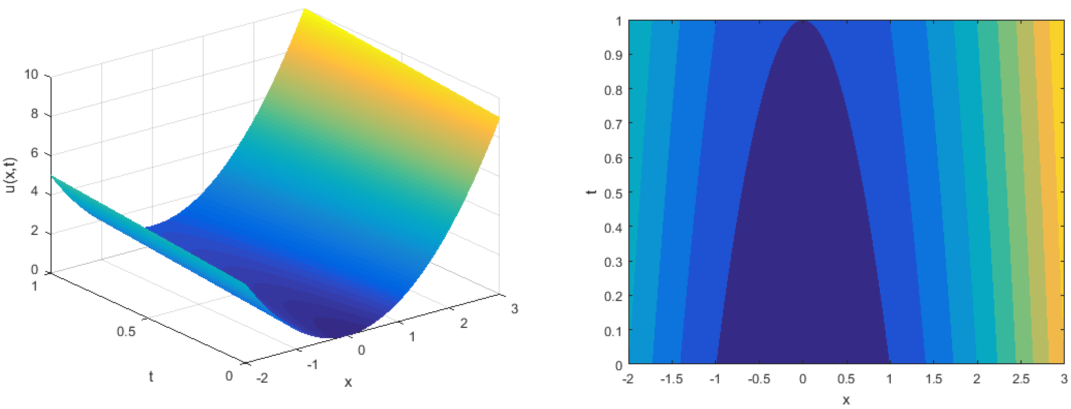
Figure 5. Plot of SAIM and exact solutions for $-2 \leq x \leq 3$ and $t = 0.01$ for the linear telegraph equation in Example 2

Example 3. Consider the linear nonhomogeneous telegraph equation [1], [13]:

$$u_{tt} + 8u_t + 4u = u_{xx} - 2e^{-t}\sin x, \tag{15}$$
$$u(x,0) = \sin x\,, u_t(x,0) = -\sin x\,, 0 \leq x \leq \pi,$$
$$u(0,t) = u(\pi,t) = 0, 0 \leq t \leq 1,$$

with exact solution $u(x,t) = e^{-t}\sin x$. Here, $Lu = u_{tt}$, $Nu = -u_{xx} - 8u_t - 4u$ and $f(x,t) = -2e^{-t}\sin x$. Since the primary problem $Lu_0 = 0$, with $u_0(x,0) = \sin x\,, u_t(x,0) = -\sin x$, has a solution $u_0(x,t) = (1-t)\sin x$, equation (15) can be solved using the general iterative scheme

$$u_{n+1}(x,t) = u_{n+1}(x,0) + \int_0^t \int_0^t \left[u_{n_{xx}} - 8u_{n_s} - 4u_n - 2e^{-s}\sin x\right] ds ds. \tag{16}$$



(a) (b)

Figure 6. (a) Surface plot of SAIM solution for $-2 \le x \le 3$ and $0 \le t \le 1$ for the linear telegraph equation in Example 2; (b) Contour diagram of SAIM solution for Example 2

Thus, the first three approximations are

$$u_0(x,t) = (1-t)\sin x$$
$$u_1(x,t) = (1-t)\sin x + \int_0^t \int_0^t [u_{n_{xx}} - 8u_{n_s} - 4u_n - 2e^{-s}\sin x] ds ds$$
$$= \left(3 - 3t + \frac{3}{2}t^2 + \frac{5}{6}t^3 - 2e^{-t}\right)\sin x$$
$$u_2(x,t) = (1-t)\sin x + \int_0^t \int_0^t [u_{n_{xx}} - 8u_{n_s} - 4u_n - 2e^{-s}\sin x] ds ds$$
$$= \left(9 - 9t + \frac{9}{2}t^2 - \frac{3}{2}t^3 - \frac{55}{24}t^4 - \frac{5}{24}t^5 - 8e^{-t}\right)\sin x$$

and so on. Table 3 and Figures 7, 8 and 9 compare the results from the SAIM with the exact solution. Figure 7 shows that for negative x values the voltage $u(x,t)$ increases with time, while for positive x values it reduces with time. Figure 10 shows the absolute errors of the SAIM for $0 \le x \le 1$ and $t = 0.01$ which, at fixed t, increase with x but with a reduced rate of increase.

Table 3. Comparison of SAIM and exact solutions for Example 3 ($t = 0.01$)

x	$u(x,t)$	$u_{SAIM}(x,t)$	e_{SAIM}
0	0	0	0
0.2	0.196692537925	0.196692532624	5.301×10^{-9}
0.4	0.389418342309	0.389418342309	1.039×10^{-8}
0.6	0.559024186912	0.559024171848	1.507×10^{-8}
0.8	0.710218278534	0.710218259395	1.914×10^{-8}
1.0	0.833098208614	0.833098186163	2.245×10^{-8}

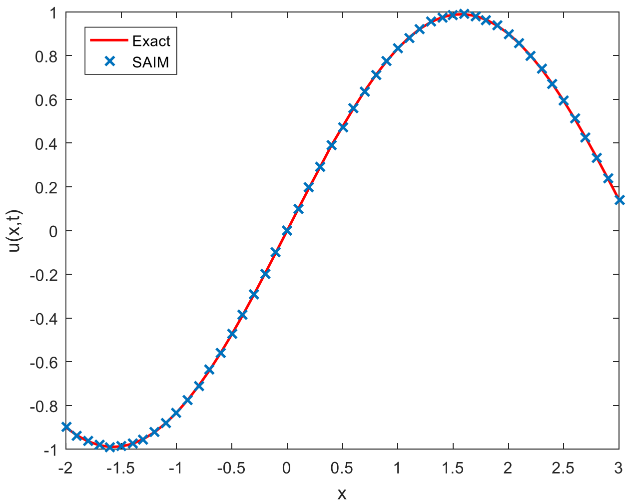


Figure 7. Plot of SAIM and exact solutions for $-2 \le x \le 3$ and $t = 0.01$ for the linear telegraph equation in Example 3

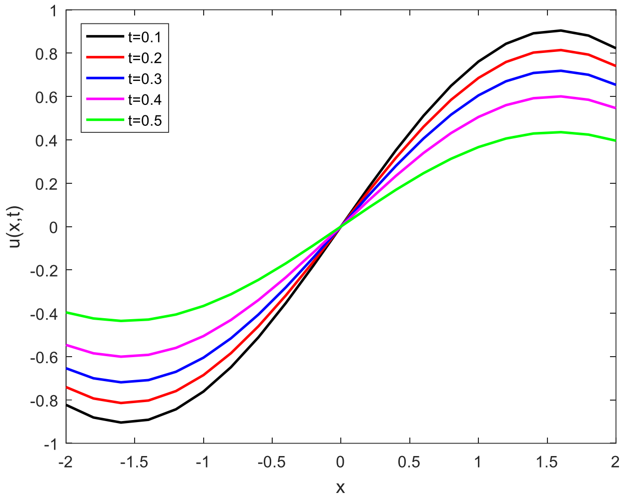
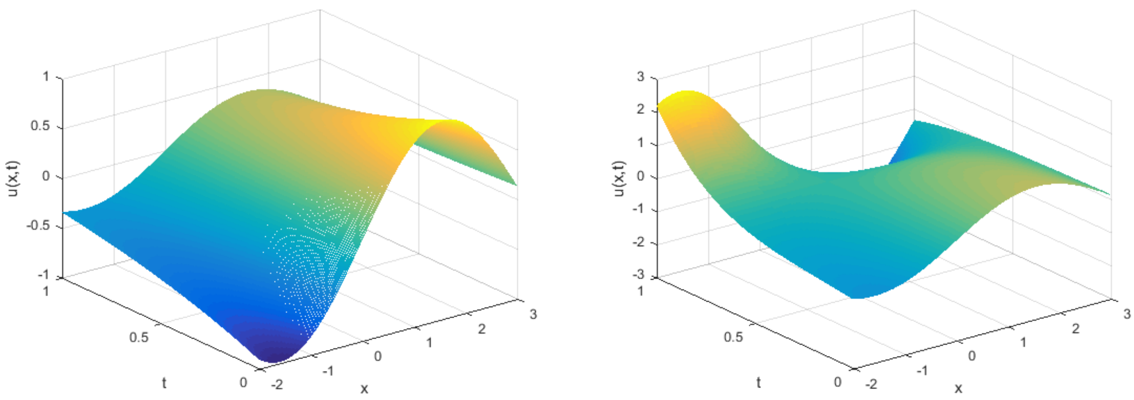


Figure 8. SAIM solutions for $-2 \leq x \leq 2$ at different time values for the linear telegraph equation in Example 3



(a) (b)

Figure 9. Surface plot of (a) exact solution and (b) SAIM solution for $-2 \leq x \leq 3$ and $0 \leq t \leq 1$ for the linear telegraph equation in Example 3

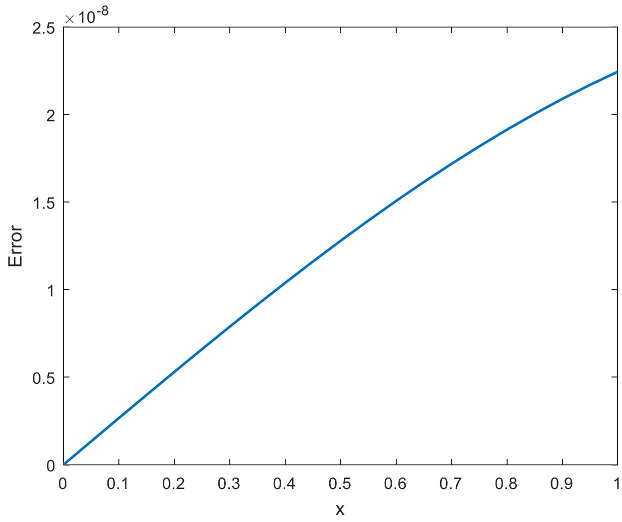


Figure 10. Absolute errors of the SAIM for Example 3 for 0≤ $x \leq 1$ and $t = 0.01$

Example 4. Consider the following nonlinear nonhomogeneous telegraph equation [20]:

$$u_{tt} + 2u_t + u^2 - u_{xx} = e^{2x+4t} - e^{x-2t}, \tag{17}$$
$$u(x,0) = e^x, u_t(x,0) = -2e^x, 0 \leq x \leq 1,$$
$$u(0,t) = e^{-2x}, u(1,t) = e^{1-2t}, 0 \leq t \leq 1.$$

This equation has exact solution $u(x,t) = e^{x-2t}$ and can be rewritten as:

$$Lu + Nu = f(x,t)$$

with $Lu = u_{tt}, Nu = 2u_t + u^2 - u_{xx}$ and $f(x,t) = e^{2x+4t} - e^{x-2t}$. The initial problem yields the solution $u_0(x,t) = e^x - 2te^x = e^x(1-2t)$, so that the first two iterations give the approximations

$$u_0(x,t) = e^x(1-2t)$$
$$u_1(x,t) = e^x(1-2t) + \int_0^t \int_0^t \left[u_{0_{xx}} - 2u_{0_s} - u_0^2 + e^{2x+4s} - e^{x-2s}\right] ds ds$$
$$= \left(\frac{5}{4} - \frac{5}{2}t + \frac{5}{2}t^2 - \frac{1}{2}t^3\right)e^x - \left(\frac{1}{16} + \frac{1}{4}t + \frac{1}{2}t^2 - \frac{2}{3}t^3 + \frac{1}{3}t^4\right)e^{2x} + \frac{1}{16}e^{2x+4t} - \frac{1}{4}e^{x-2t}$$

This approximates the exact solution $u(x,t) = e^{x-2t}$. These results are comparable to those obtained by Arslan [20] using the hybrid method (HM) which is a blend of the differential transform and finite difference methods as shown in Table 4 and Figure 11. More iterations would result in smaller errors, hence better accuracy of the SAIM. Figure 12 shows absolute errors of the SAIM for Example 4 for 0≤ $x \leq 1$ and $t = 0.01$ for this example.

Table 4. Comparison of SAIM and HM solutions with exact solutions for Example 4 ($t = 0.01$)

x	$u(x,t)$	$u_{SAIM}(x,t)$	$u_{HM}(x,t)$	e_{SAIM}	e_{HM}
0	0.980198673	0.980201168	0.980198673	2.495×10^{-6}	0

0.2	1.197217363	1.197220772	1.197217363	3.409×10^{-6}	0
0.4	1.462284589	1.462289292	1.462284590	4.703×10^{-6}	1×10^{-9}
0.6	1.786038431	1.786044979	1.786038431	6.549×10^{-6}	0
0.8	2.181472265	2.181481464	2.181472265	9.199×10^{-6}	0
1.0	2.664456242	2.664469268	2.664456241	1.303×10^{-5}	1×10^{-9}

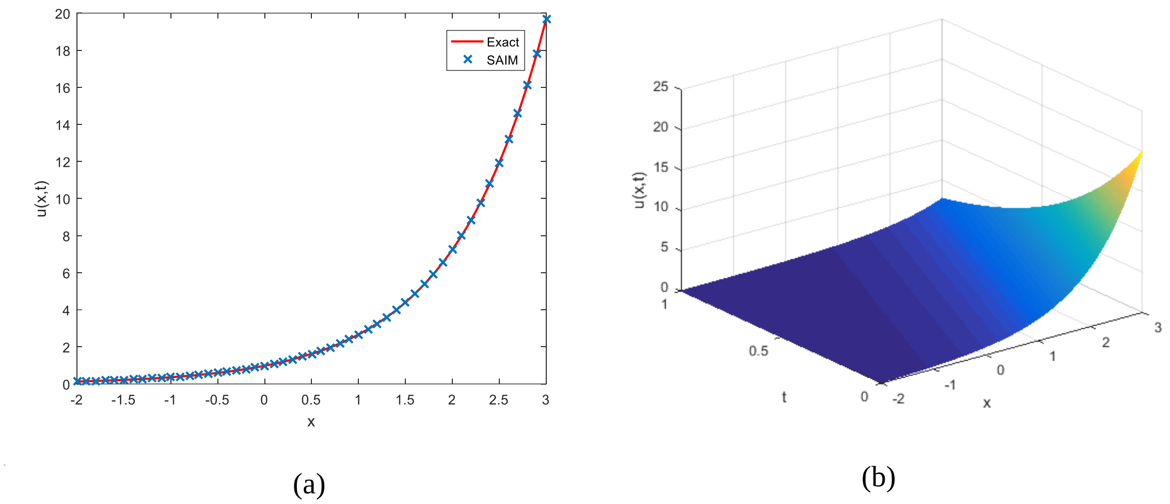


Figure 11. (a) Plot of SAIM and exact solutions for $-2 \leq x \leq 3$ and $t = 0.01$ for the nonlinear telegraph equation in Example 4; (b) Surface plot of SAIM solution for Example 4

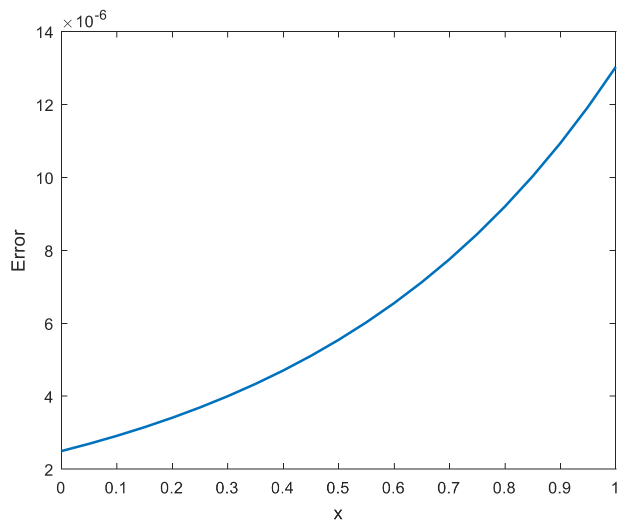


Figure 12. Absolute errors of the SAIM for Example 4 for $0 \leq x \leq 1$ and $t = 0.01$

Example 5. Consider the nonlinear nonhomogeneous telegraph equation [21]:

$$u_{tt} - u_{xx} + 2u_t + u^2 = e^{-2t} \cosh^2 x - 2e^{-t} \cosh x, \tag{18}$$

having the following initial conditions and exact solution, respectively,

$$u(x,0) = \cosh x\,, u_t(x,0) = -\cosh x,$$
$$u(x,t) = e^{-t}\cosh x.$$

We rewrite (18) as

$$Lu + Nu = f(x,t),$$

where $Lu = u_{tt}, Nu = -u_{xx} + 2u_t + u^2$ and $f(x,t) = e^{-2t}\cosh^2 x - 2e^{-t}\cosh x$. The first few iterations of the SAIM are

$$u_0(x,t) = \cosh x - t\cosh x = (1-t)\cosh x$$
$$u_1(x,t) = (1-t)\cosh x + \int_0^t\int_0^t \left[u_{0_{xx}} - 2u_{0_s} - u_0^2 + e^{-2s}\cosh^2 x - 2e^{-s}\cosh x\right]dsds$$
$$= \left(3 - 3t + \frac{3}{2}t^2 - \frac{1}{6}t^3 - 2e^{-t}\right)\cosh x - \left(\frac{1}{4} - \frac{1}{2}t + \frac{1}{2}t^2 - \frac{1}{3}t^3 + \frac{1}{12}t^4 - \frac{1}{4}e^{-2t}\right)\cosh^2 x$$

and so on. This is an approximation to the exact solution $u(x,t) = e^{-t}\cosh x$. The results are shown in Table 5 and Figures 13, 14 and 15. It can be observed from Figure 14 that the voltage in the wire $u(x,t)$ generally reduces as time increases. The absolute errors of the SAIM for $0 \le x \le 1$ and $t = 0.01$ for this example are shown in Figure 16 from which it can be noted that at fixed time, the absolute errors increase with x and have an increasing rate of increase.

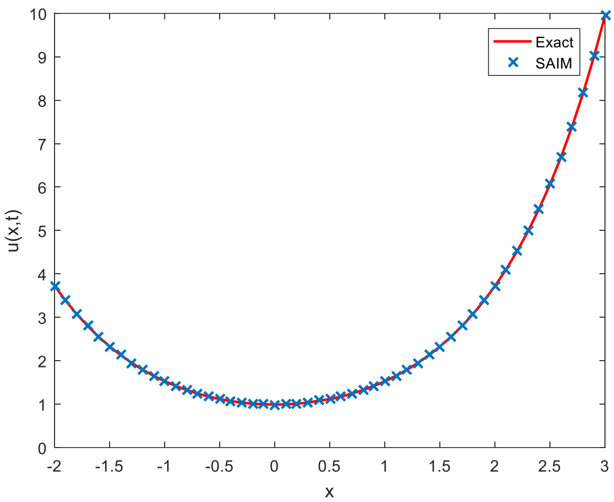


Figure 13. Plot of SAIM and exact solutions for $-2 \le x \le 3$ and $t = 0.01$ for the nonlinear telegraph equation in Example 5

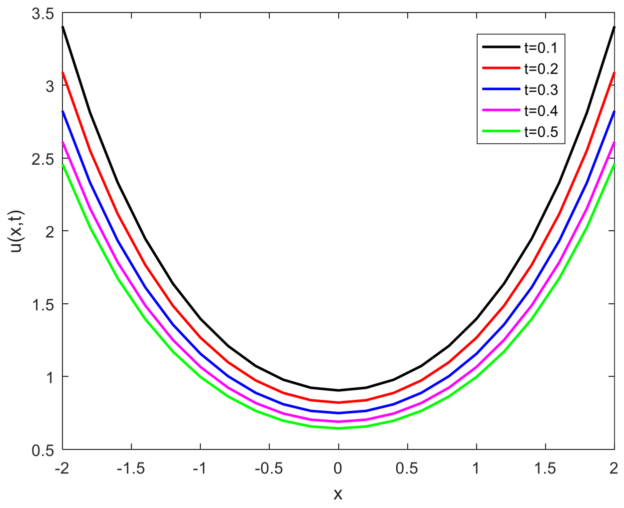


Figure 14. SAIM solutions for $-2 \leq x \leq 2$ at different time values for the nonlinear telegraph equation in Example 5

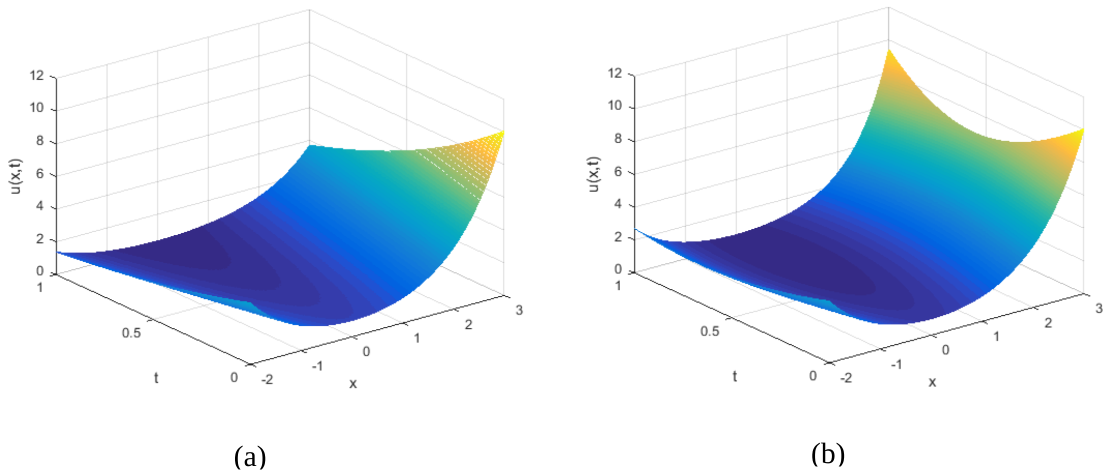


Figure 15. Surface plot of (a) exact solution and (b) SAIM solution for $-2 \leq x \leq 3$ and $0 \leq t \leq 1$ for the nonlinear telegraph equation in Example 5

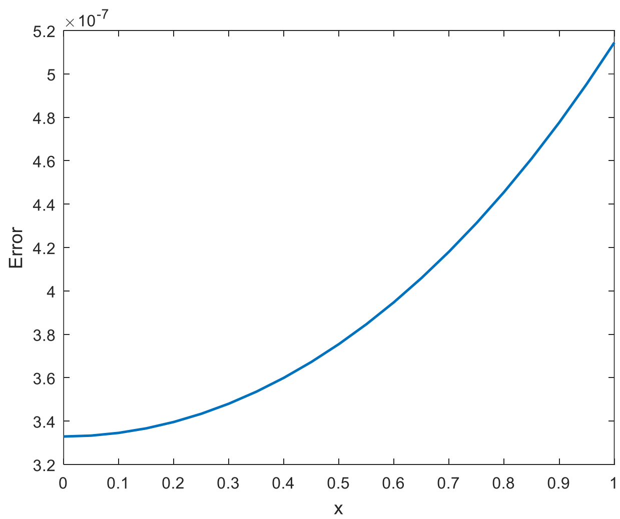


Figure 16. Absolute errors of the SAIM for Example 5 for 0$\leq x \leq 1$ and $t = 0.01$

Table 5. Comparison of SAIM and exact solutions for Example 5 ($t = 0.01$)

x	$u(x,t)$	$u_{SAIM}(x,t)$	e_{SAIM}
0	0.990049833749	0.990050166662	3.329×10^{-7}
0.2	1.009916921814	1.009917261424	3.396×10^{-7}
0.4	1.070315522009	1.070315881984	3.600×10^{-7}
0.6	1.173669642236	1.173670037074	3.395×10^{-7}
0.8	1.324127246239	1.324127691861	4.456×10^{-7}
1.0	1.527726725960	1.527727240364	5.144×10^{-7}

Conclusion

This study has demonstrated that the Semi-Analytic Iterative Method (SAIM) is an effective and accurate tool for solving both linear and nonlinear telegraph equations. Through five numerical examples, the method has shown high precision, efficiency, and suitability for a broad class of partial differential equations. While SAIM can also address stochastic differential equations with random excitations, its applicability is limited by the difficulty in integrating random functions. Future work may focus on enhancing SAIM through approaches such as the Discrete Temimi-Ansari Method (DTAM), polynomial-based approximations like Taylor or Chebyshev series, or numerical integration improvements using Boole's Rule. Overall, SAIM remains a simple, direct, and reliable method that achieves near-exact solutions with minimal iterations.

Acknowledgments

The author gratefully acknowledges the support of Mulungushi University and thanks the Editor and anonymous referees for their valuable comments. Special thanks are due to Chimuka Kasumo who offered invaluable assistance with flowchart design.

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