https://journal.uinsgd.ac.id/index.php/kubik/index
Vol. 10, No. 1, 2025, pp. 20-33
e-ISSN: 2686-0341 p-ISSN: 2338-0896

Optimization of Multi-Product Distribution with Modification of The Modified Exponential Approach Method

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Abstract

In the distribution of several different products, there is a condition where product allocation is not appropriate or there is a multi-product transportation problem. As a result, it is necessary to analyze the appropriate product allocation with minimum shipping costs. In solving this problem, a multi-product transportation problem model and modification of the Modified Exponential Approach method are proposed to obtain an allocation for each product with minimum shipping costs. In the case of the distribution of sweet snack products and spicy/salty snacks at the MSME Giu Store. Delivery is carried out using delivery services (J&T Express, Surya Cargo) and private delivery. Therefore, this problem is a form of transshipment problem. This problem is transformed into a multi-product transportation problem and solved by modifying the Modified Exponential Approach method. The calculation results show that private delivery is inefficient. It is recommended that products be distributed by J&T Express with delivery to the Sambas, Sekadau, and Sintang areas. Surya Cargo delivers to the Sanggau, Mempawah, and Singkawang areas. With this allocation, a minimum distribution cost of Rp 1,378,398 is obtained.

Keywords: Modified Exponential Approach Method, Allocation; intermediary; transshipment issues.
MSC2020: 90B06, 15A03

Abstrak

Pada pendistribusian beberapa produk yang berbeda terdapat suatu kondisi pengalokasian produk yang tidak tepat atau masalah transportasi multi produk. Oleh karena itu, perlu dilakukan analisis alokasi produk yang tepat dengan biaya pengiriman minimum. Dalam penyelesaian masalah tersebut, diusulkan model masalah transportasi multi produk dan modifikasi metode Modified Exponential Approach untuk memperoleh alokasi masing-masing produk dengan biaya pengiriman minimum. Pada kasus pendistribusian produk camilan manis dan camilan pedas/asin di UMKM Giu Store Pengiriman dilakukan dengan menggunakan jasa pengiriman (I&T Express, Surya Cargo) dan pengiriman pribadi. Oleh karena itu, permasalahan ini merupakan bentuk dari masalah transshipment. Permasalahan ini ditransformasikan menjadi masalah transportasi multi produk dan diselesaikan dengan memodifikasi metode Modified Exponential Approach. Hasil perhitungan menunjukkan bahwa pengiriman pribadi tidak efisien. Namun, disarankan agar produk didistribusikan oleh J&T Express dengan pengiriman ke area Sambas, Sekadau, dan Sintang. Surya Cargo melakukan pengiriman ke area Sanggau, Mempawah, dan Singkawang. Dengan alokasi tersebut diperoleh biaya pengiriman minimum sebesar Rp 1.378.398.

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Received: 21-10-2024, Accepted: 02-04-2025, Published: 30-05-2025

Kata kunci: Metode Modified Exponential Approach, Pengalokasian, Perantara, Masalah transshipment
MSC2020: 90B06, 15A03

Citation: D. Pajriah, M. Pasaribu, and M. Kiftiah, "Optimization of Multi-Product Distribution with Modification of The Modified Exponential Approach Method", *KUBIK J. Publ. Ilm. Mat.*, Vol. 10, No. 1, pp. 1-14, 2025.

Introduction

The transshipment problem is related to a condition where product distribution does not always go directly to the final destination, but can go through several intermediaries (transshipment nodes) [1][2][3]. Solving transshipment problems is carried out using a linear programming approach using the transportation method [4]. The transportation method is a method that regulates the distribution of the same product from source to destination optimally with minimum costs [5][6]. One method of transportation is the Modified Exponential Approach method. The Modified Exponential Approach method is a direct method which consists of three stages of the transportation method, namely initial feasible solutions, optimality tests, and table revisions [7] .The Modified Exponential Approach method is based on subtracting the minimum cost entries in rows and columns, then determining the exponential penalty associated with the minimum shipping costs. The Modified Exponential Approach method was originally proposed by [8]. However, [7] revised the section on determining the optimal solution. This method is a refinement of the Improved Exponential Approach method. Previously, the Improved Exponential Approach method was an improvement on the Exponential Approach method. The Exponential Approach method was first proposed by Vannan and Rekha [9]. The Improved Exponential Approach method is proposed because the Exponential Approach method cannot provide an optimal solution to unbalanced transportation problems [10][11]. However, according to [8] the Improved Exponential Approach method prioritizes allocating exponential penalties without paying attention to the minimum cost entry.

In solving transportation problems, it usually takes the form of distributing only one product and the transportation method used only provides allocation results for that one product. Meanwhile, in distribution it cannot be denied that the products distributed can be several different products [12]. This type of distribution is carried out to minimize shipping costs without having to determine time/facility priorities. This form is a multi-product transportation problem. Therefore, proper analysis is needed to obtain an allocation of each product with minimum costs. The solution to multi-product transportation problems was proposed by Santoso, et al. [12]. However, the model used is not in line with the problem given. Apart from that, modifications were made to the indirect method so that in solving the problem the process of determining the initial solution, revising tables and optimality testing had to be carried out. Therefore, a multi-product transportation problem model is proposed by utilizing the vector concept. In addition, problem solving is proposed by modification of the Modified Exponential Approach method as a direct method. The problem is modeled in vector form to make it easier to interpret and apply, especially to multi-product transportation problems. Meanwhile, modification of the Modified Exponential Approach method aims to obtain an allocation of each product with minimum shipping costs in mass distribution. In this way, the allocation of each product from source to destination is clearer and in accordance with the respective amounts of supply and demand.

One case of multi-product distribution occurred at the MSME Giu Store. MSME Giu Store is engaged in selling sweet snack products and spicy/salty snacks both offline and online. In online sales, delivery of both products is carried out using a delivery service and can also be done directly by private car. Distribution of sweet snack products and spicy/salty snack products is carried out on a mixed basis without paying attention to shipping costs for each product. In fact, there is an analysis carried out by MSMEs regarding the number of sweet snack products and spicy/salty snacks allocated to anticipate product stocks. Thus, in this case it is necessary to find an allocation of sweet snack products and spicy/salty snack products so that delivery costs are minimum with the right type of delivery. One alternative solution suggested is to model the problem in the form of a multi-product transportation problem and apply a modification of the Modified Exponential Approach method.

Methods

Transshipment Problems

The transshipment problem is the problem of sending goods from a source through an intermediary before reaching the final destination [13]. Resolving transshipment problems aims to ensure minimum shipping costs [14][15]. In solving the transshipment problem, it is necessary to transform it into a transportation problem as follows [16].

1. Analysis of the balance between the amount of inventory and the amount of demand.
   a_i represents the number of products available at source i and b_j is the amount requested at destination j with $i = 1,2,\dots,m$ and $j = 1,2,..m$. If $\sum_i a_i = \sum_j b_j$ then the problem is balanced. However, if they are not the same then the problem is not balanced. if $\sum_i a_i > \sum_j b_j$ then add a pseudo goal of $\sum_i a_i - \sum_j b_j$. If $\sum_i a_i < \sum_j b_j$ then add a pseudo source of $\sum_j b_j - \sum_i a_i$.
2. Determination pure source poins, pure destination points and intermediary point
   a. Pure source point (S_i) is a point that only sends goods, with $i = 1,2,\dots,m$.
   b. Pure destination point (D_j) is a point that only receives goods, with $j = 1,2,\dots,n$.
   c. Intermediary point (X_l) as a point that receives and sends goods, with $l = 1,2,\dots,r$.
   Group the sources and destinations. Source (S_i) as a combination of pure source points (S_i) and intermediate points (X_l) while destination (D_j) as a combination of pure destination points (D_j) with intermediate points (X_l). Note $i = 1,2,\dots m + l$ and $j = 1,2,..n + l$.
3. Determination of pure source inventory, namely a_i. Pure destination point request b_j. The supply of intermediate points is $P_i = a_i + T$ and the demand for intermediate points is $P_j = b_j + T$ with $T = \sum_i a_i = \sum_j b_j$. Where $i = 1,2,\dots m$ and $j = 1,2,..n$, with $m = m + l$ and $n = n + l$.
4. Determination of shipping costs based on the fact that if the direct path is a_i to b_j then it has a value of c_{ij}. If $i = j$ then it has a value of 0. However, if it is not on the direct path of a_i to b_j then it has a value of ∞ (denoted by M). $i = 1,2,\dots m$ and $j = 1,2,..n$ with $m = m + l$ and $n = n + l$.

The obtained amounts of inventory, demand, and shipping costs are formed into tables and transportation problem models.

Multi Product Transportation Problems

Multi-product transportation problems occur when the product distributed from source to destination consists of several different products. Solving multi-product transportation problems aims to minimize product delivery costs by meeting the supply of each source and the demand of each destination. Multi-product transportation problems can be formed into the multi-product transportation problem table presented in Table 1.

Table 1. Multi product transportation problem table

Source	Destination				Supply
	1	2	…	n	
1	$\boldsymbol{x}_{11}$ $\boldsymbol{c}_{11}$	$\boldsymbol{x}_{12}$ $\boldsymbol{c}_{12}$	…	$\boldsymbol{x}_{1n}$ $\boldsymbol{c}_{1n}$	$\boldsymbol{a}_1$
2	$\boldsymbol{x}_{21}$ $\boldsymbol{c}_{21}$	$\boldsymbol{x}_{22}$ $\boldsymbol{c}_{22}$	…	$\boldsymbol{x}_{2n}$ $\boldsymbol{c}_{2n}$	$\boldsymbol{a}_2$
⋮	…	…	…	…	⋮
m	$\boldsymbol{x}_{m1}$ $\boldsymbol{c}_{m1}$	$\boldsymbol{x}_{m2}$ $\boldsymbol{c}_{m2}$	…	$\boldsymbol{x}_{mn}$ $\boldsymbol{c}_{mn}$	$\boldsymbol{a}_m$
Demand	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	…	$\boldsymbol{b}_n$	

Based on Table 1 the transportation problem can be formulated as follows.

$$Minimum\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{c}_{ij}^{T} \cdot \boldsymbol{x}_{ij} \qquad [1]$$

$$= \boldsymbol{c}_{11}^{T} \cdot \boldsymbol{x}_{11} + \boldsymbol{c}_{12}^{T} \cdot \boldsymbol{x}_{12} + \boldsymbol{c}_{13}^{T} \cdot \boldsymbol{x}_{13} + \cdots + \boldsymbol{c}_{mn}^{T}\boldsymbol{x}_{mn}$$

$$= \begin{pmatrix} c_{11}^1 & c_{11}^2 & \cdots & c_{11}^k \end{pmatrix} \cdot \begin{pmatrix} x_{11}^1 \\ x_{11}^2 \\ \vdots \\ x_{11}^k \end{pmatrix} + \cdots + \begin{pmatrix} c_{mn}^1 & c_{mn}^2 & \cdots & c_{mn}^k \end{pmatrix} \cdot \begin{pmatrix} x_{mn}^1 \\ x_{mn}^2 \\ \vdots \\ x_{mn}^k \end{pmatrix}$$

Where $\boldsymbol{c}_{\boldsymbol{ij}}^{T} \cdot \boldsymbol{x}_{\boldsymbol{ij}}$ is the dot product operation between the transposed cost vector ($\boldsymbol{c}_{\boldsymbol{ij}}^{T}$) and the decision vector ($\boldsymbol{x}_{ij}$).

Constraints:

$$\sum_{j=1}^{n} \boldsymbol{x}_{\boldsymbol{ij}} \preccurlyeq \boldsymbol{a}_i, \qquad for\ i = 1,2,\dots,m \qquad [2]$$

$$\sum_{i=1}^{m} \boldsymbol{x}_{ij} \succcurlyeq \boldsymbol{b}_j, \qquad for\ j = 1,2,\dots,n \qquad [3]$$

$$\boldsymbol{x}_{ij} \succcurlyeq \boldsymbol{0}, \qquad for\ i = 1,2,\dots,m;\ j = 1,2,\dots,n$$

With $\sum_{j=1}^{n} \boldsymbol{x}_{\boldsymbol{ij}} \preccurlyeq \boldsymbol{a}_i$ if and only if $\sum_{j=1}^{n} x_{ij}^{p} \leq a_i^{p}$ for every i, p. $\sum_{j=1}^{n} \boldsymbol{x}_{\boldsymbol{ij}} \succcurlyeq \boldsymbol{b}_j$ if and only if $\sum_{i=1}^{m} x_{ij}^{p} \geq b_j^{p}$ for every j, p. $\boldsymbol{x}_{ij} \succcurlyeq \boldsymbol{0}$ if only if $x_{ij}^{p} \geq 0$ for every p, i, j.

Information:

Z : Total transportation costs.

x_{ij}^{p} : Number of deliveries of product p from source i to destination j, with $i = 1,2,\dots,m$, $j = 1,2,\dots,n$ and $p = 1,2,\dots,k$.
x_{ij}^{p} is a component in the column vector $\boldsymbol{x}_{ij}$.

$$\boldsymbol{x}_{ij} = \begin{pmatrix} x_{ij}^1 \\ x_{ij}^2 \\ \vdots \\ x_{ij}^k \end{pmatrix}$$

c_{ij}^{p} : The cost of shipping product p from source i to destination j with $i = 1,2,\dots,m$, $j = 1,2,\dots,n$ dan $p = 1,2,\dots,k$.

c_{ij}^{p} is a component in the column vector $\boldsymbol{c}_{ij}$.

$$\boldsymbol{c}_{ij} = \begin{pmatrix} c_{ij}^{1} \\ c_{ij}^{2} \\ \vdots \\ c_{ij}^{k} \end{pmatrix}$$

a_i^p : Inventory capacity of product p from source i, with $i = 1,2, \dots, m$ and $p = 1,2, \dots, k$.

a_i^p is a component in the column vector $\boldsymbol{a}_i$.

$$\boldsymbol{a}_i = \begin{pmatrix} a_i^1 \\ a_i^2 \\ \vdots \\ a_i^k \end{pmatrix}$$

b_j^p : The demand capacity of product p from destination j, with $j = 1,2, \dots, n$ and $p = 1,2, \dots, k$.

b_j^p is a component in the column vector $\boldsymbol{b}_j$.

$$\boldsymbol{b}_j = \begin{pmatrix} b_j^1 \\ b_j^2 \\ \vdots \\ b_j^k \end{pmatrix}$$

Multi Product Transportation Problems

Modification of the Modified Exponential Approach method is a proposed development of the Modified Exponential Approach method. This modification was made in order to provide allocation of each product with minimum shipping costs. In addition, this modification is used in multi-product transportation problems with mass shipments. The modification of Modified Exponential Approach method is a direct method where the process of determining initial feasibility, table revision and optimality testing is carried out in one stage. From the transportation problem model created, it is then solved using a modification of the Modified Exponential Approach method. The steps for modification of the Modified Exponential Approach method are as follows:

1. Transport problems are formed in the transport model (table). If the quantity demanded is equal to the quantity supplied for each product or $\sum_{j=1}^{n} \boldsymbol{b}_j = \sum_{i=1}^{m} \boldsymbol{a}_i$ then the table is balanced. With $\sum_{j=1}^{n} \boldsymbol{b}_j = \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $\sum_{j=1}^{n} b_j^p = \sum_{i=1}^{m} a_i^p$ for each product p. The process can be continued to Step 3. However, if $\sum_{j=1}^{n} \boldsymbol{b}_j \neq \sum_{i=1}^{m} \boldsymbol{a}_i$ then there is at least one product p such that $\sum_{j=1}^{n} b_j^p \neq \sum_{i=1}^{m} a_i^p$. Next, provisions are obtained with several possibilities that must be chosen, including:
   a. Comparison of the total with the condition that if $\sum_{j=1}^{n} \boldsymbol{b}_j \prec \sum_{i=1}^{m} \boldsymbol{a}_i$ then add a dummy destination vector equal to if $\sum_{i=1}^{m} \boldsymbol{a}_i - \sum_{j=1}^{n} \boldsymbol{b}_j$.. With $\sum_{j=1}^{n} \boldsymbol{b}_j \prec \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $\sum_{j=1}^{n} b_j^p < \sum_{i=1}^{m} a_i^p$ in each product p. In addition, if $\sum_{j=1}^{n} \boldsymbol{b}_j \succ \sum_{i=1}^{m} \boldsymbol{a}_i$ then add a dummy source vector of $\sum_{j=1}^{n} \boldsymbol{b}_j - \sum_{i=1}^{m} \boldsymbol{a}_i$. With $\sum_{j=1}^{n} \boldsymbol{b}_j \succ \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $\sum_{j=1}^{n} b_j^p > \sum_{i=1}^{m} a_i^p$ in each product p. Thus, both of these possibilities suggest that the table is not completely balanced. The process continues to Step 2.
   b. Comparison of the mixture for each product p with the condition that if $\sum_{j=1}^{n} b_j^p < \sum_{i=1}^{m} a_i^p$ then product p is not balanced so a dummy goal of $\sum_{i=1}^{m} a_i^p - \sum_{j=1}^{n} b_j^p$ is added to product p.

However, if $\sum_{j=1}^{n} b_j^p > \sum_{i=1}^{m} a_i^p$ then the product p is also unbalanced so a dummy source of $\sum_{j=1}^{n} b_j^p - \sum_{i=1}^{m} a_i^p$ is added. The process continues to Step 2.

2. If a dummy column (row) is added then subtract the $\boldsymbol{c}_{ij}$ component from the minimum $\boldsymbol{c}_{ij}$ component in the respective column (row). In the $\boldsymbol{c}_{ij}$ component the dummy column (row) is replaced with the largest $\boldsymbol{c}_{ij}$ component from the reduced table. If previously a dummy column was added then go to Step 3 then go to Step 5. However, if the opposite is true for the row then go to Step 4 then Step 5.
3. Subtraction of the $\boldsymbol{c}_{ij}$ component in the row with the minimum $\boldsymbol{c}_{ij}$ component of each row.
4. Reducing the $\boldsymbol{c}_{ij}$ components in the columns with the minimum $\boldsymbol{c}_{ij}$ components of each column.
5. Check each row and column to see if it has a zero vector. If yes, then do Step 6. But if not, note that if it is in a row then do Step 3 then go to Step 6. However, if it is in a column then do Step 4 then Step 6.
6. Check whether the $\boldsymbol{b}_j$ component in the column is less than or equal to the $\boldsymbol{a}_i$ component in the row. The check is carried out by looking at the column whose $\boldsymbol{c}_{ij}$ component is reduced to the zero vector. Then, conversely, pay attention to whether each component $\boldsymbol{a}_i$ is less than or equal to component $\boldsymbol{b}_j$ by looking at the row whose component cij is reduced to the zero vector. Where $\boldsymbol{b}_j \prec \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $b_j^p < \sum_{i=1}^{m} a_i^p$ at every j, p. $\boldsymbol{b}_j = \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $b_j^p = \sum_{i=1}^{m} a_i^p$ in each j, p. $\boldsymbol{b}_j \preccurlyeq \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $b_j^p \leq \sum_{i=1}^{m} a_i^p$ for every j, p where there is at least $b_j^p < \sum_{i=1}^{m} a_i^p$ for some j, p. Do the opposite analysis. If both conditions are met then the process continues to Step 9. However, if it is not met then proceed to Step 7.
7. Draw the minimum possible horizontal and vertical lines in each row and column that has a reduced $\boldsymbol{c}_{ij}$ vector of zero. At the with draw al stage note that the $\boldsymbol{c}_{ij}$ not met in Step 6 is not covered.
8. Selection of the smallest $\boldsymbol{c}_{ij}$ component that is not affected by the line. Subtraction of all $\boldsymbol{c}_{ij}$ components that are not affected by the line with the selected $\boldsymbol{c}_{ij}$ components. Add the selected $\boldsymbol{c}_{ij}$ component to all $\boldsymbol{c}_{ij}$ components located at the intersection of the two lines. Process returns to Step 6.
9. Determining the penalty exponent denoted by 0_n, n represents the number of reduced $\boldsymbol{c}_{ij}$ zero vectors from each row i and column j. Note that this determination does not include $\boldsymbol{c}_{ij}$ for which the penalty value will be determined. Do the same for all reduced $\boldsymbol{c}_{ij}$ zero vectors.
10. Allocation of cell values with the maximum possible number of supply rows and demand columns is let as $\boldsymbol{x}_{ij}$, with $\boldsymbol{x}_{ij} = \min(\boldsymbol{a}_i, \boldsymbol{b}_j)$.The things to pay attention to in the allocation include:
    a. The $\boldsymbol{c}_{ij}$ component has a vector value of zero with an exponent penalty of 1.
    b. The $\boldsymbol{c}_{ij}$ component has a vector value of zero with an exponent penalty of 2.
    c. Select the cell that has the smallest $\boldsymbol{c}_{ij}$ component. Allocate row i or column j to the reduced shipping costs against the penalty exponent $\boldsymbol{c}'_{ij}$ based on the smallest $\boldsymbol{c}_{ij}$ component. Except, the dummy $\boldsymbol{c}'_{ij}$ component.
11. Note that $\boldsymbol{a}_i = \boldsymbol{0}$ and $\boldsymbol{b}_j = \boldsymbol{0}$ in the selected cells. If yes then the process continues to Step 12. However, if not then check whether $\boldsymbol{a}_i = \boldsymbol{0}$. If yes then allocate $\boldsymbol{x}_{ij} = \min(\boldsymbol{a}_i, \boldsymbol{b}_j)$ to the zero vector element of column j. However, if not then allocate $\boldsymbol{x}_{ij} = \min(\boldsymbol{a}_i, \boldsymbol{b}_j)$ to the zero vector element of row i. The process continues to Step 12.
12. Check whether $\boldsymbol{a}_i = \boldsymbol{0}$ for every i and $\boldsymbol{b}_j = \boldsymbol{0}$ for every j. If yes, then the solution is optimal and continues to Step 13. However, if not then the process continues at Step 5.

13. Optimum cost calculation and completion.

Multi-Product Distribution Problems

One form of multi-product distribution occurs at the MSME Giu Store which distributes sweet snack products and spicy/salty snacks. Product delivery is carried out using goods delivery services and can also be sent directly. The data used is online sales data in 2023 obtained directly from the MSME Giu Store. The initial data obtained summarizes the number of deliveries for each product. In addition, the shipping cost per kg with a delivery service is the rate for sending the product to the destination area per kg. The results of product delivery recaps from delivery services are briefly presented in Table 2.

Table 2. Data on product delivery by delivery service in 2023

Regency	Number of Product Shipments (kg)						Shipping Cost (Rp/kg)	
	Sweet Snack			Spicy/salty Snack				
	J&T Express	Surya Cargo	Total	J&T Express	Surya Cargo	Total	J&T Express	Surya Cargo
Sambas	8,25	1,75	10	9,75	0,75	10,5	10.000	10.000
Sanggau	10,25	4	14,25	11,75	2	13,75	10.900	7.200
Sekadau	1,25	4	5,25	2,75	20	22,75	10.200	9.600
Mempawah	4,50	12,5	17	4,5	18	22,5	11.000	3.000
Sintang	0,25	2,5	2,75	0,75	9,5	10,25	15.000	17.600
Singkawang	1,25	8	9,25	1,75	8	9,75	11.000	6.800
Total	25,75	32,75	58,5	31,25	58,25	89,5	68.100	54.200

In delivering products to the MSME Giu Store, the amount of inventory is the same as the amount of demand because any excess goods will be optimized for offline sales and product shortages are avoided by supplying inventory regularly. Apart from delivery services, delivery of goods can be done directly by private vehicle. Direct delivery costs are costs obtained from dividing the distance traveled to the destination area by the total vehicle distance per liter and multiplied by the fuel price per liter. The shipping costs from Giu Store to J&T Express are zero because J&T Express takes the products directly to MSME Giu Store so there are no fees charged. The shipping costs if using direct delivery by private vehicle are presented in Table 3.

Table 3. Personal shipping cost data

Source	Destination	Shipping Costs (Rp)
Giu Store (Pontianak)	Sambas	169.630
	Sanggau	137.037
	Sekadau	200.000
	Mempawah	56.296
	Sintang	255.556
	Singkawang	111.852
	Surya Cargo	2.074
	J&T Express	0
J&T Express	Surya Cargo	2.815
	Total	935.259

This distribution is classified as a transshipment problem because it uses an intermediary in the form of a shipping service. The product delivery network at the MSME Giu Store is as follows.

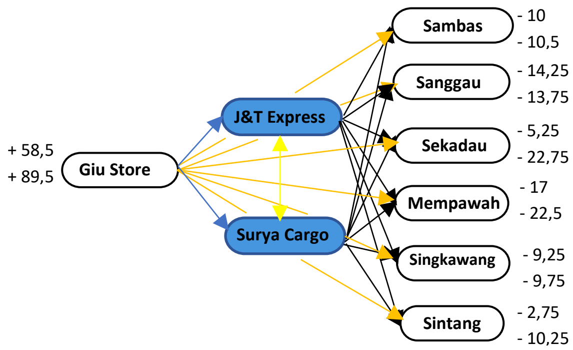

Figure 1. Product delivery network in MSME Giu Store

Results and Discussion

Problem Modeling in MSME Giu Store

In solving this problem, it is necessary to transform the transshipment problem into a transportation problem as follows.

1. Analysis of supply and demand balance. In Table 2, the initial a_i^p and initial b_j^p for $p = 1$ (sweet snack products) are $a_i^1 = 58{,}5$ and $b_j^1 = 58{,}5$ so that $a_i^1 = b_j^1 = 58{,}5$. In addition, $p = 2$ (spicy salty snack products) are $a_i^2 = 89{,}5$ and $b_j^2 = 89{,}5$. So that $a_i^2 = b_j^2 = 89{,}5$. Thus, this transshipment problem is balanced and there is no need to add pseudo sources/destinations.
2. Based on Figure 1, the pure source point, pure destination point, and intermediate point are obtained, namely:
   a. The pure source point (S_i) is MSME Giu Store.
   b. The pure destination points (D_j) are Sambas, Sanggau, Sekadau, Mempawah, Singkawang, and Sintang.
   c. Intermediary points (X_l) are J&T Express and Surya Cargo.

   Thus, those classified as sources (S_i) are MSME Giu Store (S_1), J&T Express (S_2), Surya Cargo (S_3). Destinations (D_j) are J&T Express (D_1), Surya Cargo (D_2), Sambas (D_3), Sanggau (D_4), Sekadau (D_5), Mempawah (D_6), Singkawang (D_7), and Sintang (D_8).

   J&T Express and Surya Cargo can function as destination and source points, because they act as intermediary points in the distribution system.
3. Determine the amount of supply and demand.
   a. The amount of pure source point inventory is a_i^p, so for example at MSME Giu Store the inventory obtained is $a_1^1 = 58{,}5$ (sweet snack products) and $a_1^2 = 89{,}5$ (spicy/salty snack products)
   b. The number of pure destination point requests is b_j^p. If in Sambas, the demand obtained is $b_3^1 = 10$ (sweet snack products) and $b_3^2 = 10{,}5$ (spicy/salty snack products).
   c. The supply and demand capacity at the intermediate point is $T^p = \sum a_i^p = \sum b_j^p$ so that for sweet snack products it is $T^1 = \sum a_i^1 = \sum b_j^1 = 58{,}5$ and for spicy/salty snack products it is $T^2 = \sum a_i^2 = \sum b_j^2 = 89{,}5$. The intermediate point supply is $P_i = a_i^p + T^p$ and the

intermediate point demand is $P_j = b_j^p + T^p$ so that if you use J&T Express, the supply of sweet snack products is $0 + 58.5 = 58.5$ and the destination demand is $0 + 58.5 = 58.5$.

4. Determination of product shipping costs. Based on Figure 1, for example, shipping from J&T Express to Sanggau is a direct route so that the shipping cost is Rp 10.900.

Conduct analysis on each source and destination. Then form it into a multi-product transportation problem table. The table is presented in Table 4.

Table 4. Multi product transportation problem table in MSME Giu Store

Source	Destination								Supply
	J&T Express	Surya Cargo	Sambas	Sanggau	Sekadau	Mempawah	Sintang	Singkawang	
Giu Store	$\binom{0}{0}$ $\binom{x_{11}^1}{x_{11}^2}$	$\binom{2.074}{2.074}$ $\binom{x_{12}^1}{x_{12}^2}$	$\binom{169.630}{169.630}$ $\binom{x_{13}^1}{x_{13}^2}$	$\binom{137.037}{137.037}$ $\binom{x_{14}^1}{x_{14}^2}$	$\binom{200.000}{200.000}$ $\binom{x_{15}^1}{x_{15}^2}$	$\binom{56.296}{56.296}$ $\binom{x_{16}^1}{x_{16}^2}$	$\binom{255.556}{255.556}$ $\binom{x_{17}^1}{x_{17}^2}$	$\binom{111.852}{111.852}$ $\binom{x_{18}^1}{x_{18}^2}$	$\binom{58,5}{89,5}$
J&T Express	$\binom{0}{0}$ $\binom{x_{21}^1}{x_{21}^2}$	$\binom{2.815}{2.815}$ $\binom{x_{22}^1}{x_{22}^2}$	$\binom{10.000}{10.000}$ $\binom{x_{23}^1}{x_{23}^2}$	$\binom{10.900}{10.900}$ $\binom{x_{24}^1}{x_{24}^2}$	$\binom{10.200}{10.200}$ $\binom{x_{25}^1}{x_{25}^2}$	$\binom{11.000}{11.000}$ $\binom{x_{26}^1}{x_{26}^2}$	$\binom{15.000}{15.000}$ $\binom{x_{27}^1}{x_{27}^2}$	$\binom{11.000}{11.000}$ $\binom{x_{28}^1}{x_{28}^2}$	$\binom{58,5}{89,5}$
Surya Cargo	$\binom{2.815}{2.815}$ $\binom{x_{31}^1}{x_{31}^2}$	$\binom{0}{0}$ $\binom{x_{32}^1}{x_{32}^2}$	$\binom{10.000}{10.000}$ $\binom{x_{33}^1}{x_{33}^2}$	$\binom{7.200}{7.200}$ $\binom{x_{34}^1}{x_{34}^2}$	$\binom{9.600}{9.600}$ $\binom{x_{35}^1}{x_{35}^2}$	$\binom{3.000}{3.000}$ $\binom{x_{36}^1}{x_{36}^2}$	$\binom{17.600}{17.600}$ $\binom{x_{37}^1}{x_{37}^2}$	$\binom{6.800}{6.800}$ $\binom{x_{38}^1}{x_{38}^2}$	$\binom{58,5}{89,5}$
Demand (kg)	$\binom{58,5}{89,5}$	$\binom{58,5}{89,5}$	$\binom{10}{10,5}$	$\binom{14,25}{13,75}$	$\binom{5,25}{22,75}$	$\binom{17}{22,5}$	$\binom{2,75}{10,25}$	$\binom{9,25}{9,75}$	

The transformation results are cacapitalized into a transportation problem model as follows.

Decision variable :

$$\boldsymbol{x}_{ij} = \begin{pmatrix} x_{ij}^1 \\ x_{ij}^2 \end{pmatrix}$$

$\boldsymbol{x}_{ij}$ is a vector of the sum of shipments of al product p from source i to destination j.

x_{ij}^p is the number of shipments of all products p from source i to destination j.

With $i = 1,2,3;\ j = 1,2,3,4,5,6,7,8$, and $p = 1$ (sweet snack), 2 (Salty/spicy snack).

Objective Function:

The aim of the problem of distributing sweet snack products and spicy/salty snacks at the MSME Giu Store is to minimize shipping costs. Thus the objective function is denoted as follows.

$$Minimize\ Z = \sum_{i=1}^{3}\sum_{j=1}^{8} \boldsymbol{c}_{ij}^T \cdot \boldsymbol{x}_{ij}$$

$$= \boldsymbol{c}_{11}^T \cdot \boldsymbol{x}_{11} + \boldsymbol{c}_{12}^T \cdot \boldsymbol{x}_{12} + \boldsymbol{c}_{13}^T \cdot \boldsymbol{x}_{13} + \boldsymbol{c}_{14}^T \cdot \boldsymbol{x}_{14} + \cdots + \boldsymbol{c}_{38}^T \boldsymbol{x}_{38}$$

$$= (0 \quad 0) \cdot \begin{pmatrix} x_{11}^1 \\ x_{11}^2 \end{pmatrix} + (2.074 \quad 2.074) \cdot \begin{pmatrix} x_{12}^1 \\ x_{12}^2 \end{pmatrix} + \cdots + (200.000 \quad 200.000) \cdot \begin{pmatrix} x_{38}^1 \\ x_{38}^2 \end{pmatrix}.$$

Constraints:

The function of constraints on supply is:

$$\begin{pmatrix} x_{11}^1 \\ x_{11}^2 \end{pmatrix} + \begin{pmatrix} x_{12}^1 \\ x_{12}^2 \end{pmatrix} + \begin{pmatrix} x_{13}^1 \\ x_{13}^2 \end{pmatrix} + \begin{pmatrix} x_{14}^1 \\ x_{14}^2 \end{pmatrix} + \begin{pmatrix} x_{15}^1 \\ x_{15}^2 \end{pmatrix} + \begin{pmatrix} x_{16}^1 \\ x_{16}^2 \end{pmatrix} + \begin{pmatrix} x_{17}^1 \\ x_{17}^2 \end{pmatrix} + \begin{pmatrix} x_{18}^1 \\ x_{18}^2 \end{pmatrix} \leqslant \begin{pmatrix} 58.5 \\ 89.5 \end{pmatrix}$$

$$\begin{pmatrix} x_{21}^1 \\ x_{21}^2 \end{pmatrix} + \begin{pmatrix} x_{22}^1 \\ x_{22}^2 \end{pmatrix} + \begin{pmatrix} x_{23}^1 \\ x_{23}^2 \end{pmatrix} + \begin{pmatrix} x_{24}^1 \\ x_{24}^2 \end{pmatrix} + \begin{pmatrix} x_{25}^1 \\ x_{25}^2 \end{pmatrix} + \begin{pmatrix} x_{26}^1 \\ x_{26}^2 \end{pmatrix} + \begin{pmatrix} x_{27}^1 \\ x_{27}^2 \end{pmatrix} + \begin{pmatrix} x_{28}^1 \\ x_{28}^2 \end{pmatrix} \leqslant \begin{pmatrix} 58.5 \\ 89.5 \end{pmatrix}$$

$$\begin{pmatrix} x_{31}^1 \\ x_{31}^2 \end{pmatrix} + \begin{pmatrix} x_{32}^1 \\ x_{32}^2 \end{pmatrix} + \begin{pmatrix} x_{33}^1 \\ x_{33}^2 \end{pmatrix} + \begin{pmatrix} x_{34}^1 \\ x_{34}^2 \end{pmatrix} + \begin{pmatrix} x_{35}^1 \\ x_{35}^2 \end{pmatrix} + \begin{pmatrix} x_{36}^1 \\ x_{36}^2 \end{pmatrix} + \begin{pmatrix} x_{37}^1 \\ x_{37}^2 \end{pmatrix} + \begin{pmatrix} x_{38}^1 \\ x_{38}^2 \end{pmatrix} \preccurlyeq \begin{pmatrix} 58.5 \\ 89.5 \end{pmatrix}$$

The function of constraints on demand is:

$$\begin{pmatrix} x_{11}^1 \\ x_{11}^2 \end{pmatrix} + \begin{pmatrix} x_{21}^1 \\ x_{21}^2 \end{pmatrix} + \begin{pmatrix} x_{31}^1 \\ x_{31}^2 \end{pmatrix} \succcurlyeq \begin{pmatrix} 58.5 \\ 89.5 \end{pmatrix}$$

$$\begin{pmatrix} x_{12}^1 \\ x_{12}^2 \end{pmatrix} + \begin{pmatrix} x_{22}^1 \\ x_{22}^2 \end{pmatrix} + \begin{pmatrix} x_{32}^1 \\ x_{32}^2 \end{pmatrix} \succcurlyeq \begin{pmatrix} 58.5 \\ 89.5 \end{pmatrix}$$

$$\begin{pmatrix} x_{13}^1 \\ x_{13}^2 \end{pmatrix} + \begin{pmatrix} x_{23}^1 \\ x_{23}^2 \end{pmatrix} + \begin{pmatrix} x_{33}^1 \\ x_{33}^2 \end{pmatrix} \succcurlyeq \begin{pmatrix} 10 \\ 10.5 \end{pmatrix}$$

$$\begin{pmatrix} x_{14}^1 \\ x_{14}^2 \end{pmatrix} + \begin{pmatrix} x_{24}^1 \\ x_{24}^2 \end{pmatrix} + \begin{pmatrix} x_{34}^1 \\ x_{34}^2 \end{pmatrix} \succcurlyeq \begin{pmatrix} 14{,}25 \\ 13{,}75 \end{pmatrix}$$

$$\begin{pmatrix} x_{15}^1 \\ x_{15}^2 \end{pmatrix} + \begin{pmatrix} x_{25}^1 \\ x_{25}^2 \end{pmatrix} + \begin{pmatrix} x_{35}^1 \\ x_{35}^2 \end{pmatrix} \succcurlyeq \begin{pmatrix} 5{,}25 \\ 22{,}75 \end{pmatrix}$$

$$\begin{pmatrix} x_{16}^1 \\ x_{16}^2 \end{pmatrix} + \begin{pmatrix} x_{26}^1 \\ x_{26}^2 \end{pmatrix} + \begin{pmatrix} x_{36}^1 \\ x_{36}^2 \end{pmatrix} \succcurlyeq \begin{pmatrix} 17 \\ 22{,}5 \end{pmatrix}$$

$$\begin{pmatrix} x_{17}^1 \\ x_{17}^2 \end{pmatrix} + \begin{pmatrix} x_{27}^1 \\ x_{27}^2 \end{pmatrix} + \begin{pmatrix} x_{37}^1 \\ x_{37}^2 \end{pmatrix} \succcurlyeq \begin{pmatrix} 2{,}75 \\ 20{,}25 \end{pmatrix}$$

$$\begin{pmatrix} x_{18}^1 \\ x_{18}^2 \end{pmatrix} + \begin{pmatrix} x_{28}^1 \\ x_{28}^2 \end{pmatrix} + \begin{pmatrix} x_{38}^1 \\ x_{38}^2 \end{pmatrix} \succcurlyeq \begin{pmatrix} 9{,}25 \\ 9{,}75 \end{pmatrix}$$

Non negative constraint $\boldsymbol{x}_{ij} \succcurlyeq \boldsymbol{0}$ for $i = 1{,}2{,}3$ and $j = 1{,}2{,}3{,}4{,}5{,}6{,}7{,}8$.

Solution with Modification of Modified Exponential Approach Method

Step 1. formation of the problem into a transportation model (table). In the case of distribution at UMKM Giu Store, it is a balanced transshipment problem. Therefore, the transformation process into a transportation problem is also balanced so that there is no need to add dummy sources and destinations. The transportation model (table) is presented in Table 4.

Step 2. This step not carried out because there were not dummy sources and destinations.

Step 3. Subtraction of the $\boldsymbol{c}_{ij}$ component of each row with the minimum $\boldsymbol{c}_{ij}$ component of the row. Based on Table 4, $\boldsymbol{c}_{ij}$ is obtained with the minimum components in each row, namely $\boldsymbol{c}_{11}, \boldsymbol{c}_{21}, \boldsymbol{c}_{32}$. Thus, the $\boldsymbol{c}_{ij}$ row is reduced by the minimum component.

Table 5. Reduction of the $\boldsymbol{c}_{ij}$ components in the row

Source	Destination								Supply (kg)
	J&T Express	Surya Cargo	Sambas	Sanggau	Sekadau	Mempawah	Sintang	Singkawang	
Giu Store	$\binom{0}{0}$	$\binom{2.074}{2.074}$	$\binom{169.630}{169.630}$	$\binom{137.037}{137.037}$	$\binom{200.000}{200.000}$	$\binom{56.296}{56.296}$	$\binom{255.556}{255.556}$	$\binom{111.852}{111.852}$	$\binom{58{,}5}{89{,}5}$
J&T Express	$\binom{0}{0}$	$\binom{2.815}{2.815}$	$\binom{10.000}{10.000}$	$\binom{10.900}{10.900}$	$\binom{10.200}{10.200}$	$\binom{11.000}{11.000}$	$\binom{15.000}{15.000}$	$\binom{11.000}{11.000}$	$\binom{58{,}5}{89{,}5}$
Surya Cargo	$\binom{2.815}{2.815}$	$\binom{0}{0}$	$\binom{10.000}{10.000}$	$\binom{7.200}{7.200}$	$\binom{9.600}{9.600}$	$\binom{3.000}{3.000}$	$\binom{17.600}{17.600}$	$\binom{6.800}{6.800}$	$\binom{58{,}5}{89{,}5}$
Demand (kg)	$\binom{58{,}5}{89{,}5}$	$\binom{58{,}5}{89{,}5}$	$\binom{10}{10{,}5}$	$\binom{14{,}25}{13{,}75}$	$\binom{5{,}25}{22{,}75}$	$\binom{17}{22{,}5}$	$\binom{2{,}75}{10{,}25}$	$\binom{9{,}25}{9{,}75}$	

Step 4. Subtraction of $\boldsymbol{c}_{ij}$ components in columns with the minimum $\boldsymbol{c}_{ij}$components of each column. Table 5 shows that the minimum $\boldsymbol{c}_{ij}$ components in the column are $\boldsymbol{c}_{11}, \boldsymbol{c}_{32}, \boldsymbol{c}_{33}, \boldsymbol{c}_{34}, \boldsymbol{c}_{35}, \boldsymbol{c}_{36}, \boldsymbol{c}_{27}, \boldsymbol{c}_{38}$ the other $\boldsymbol{c}_{ij}$ in the column is subtracted from the minimum $\boldsymbol{c}_{ij}$.

Table 6. Reduction of the $\boldsymbol{c}_{ij}$ components in the column

Source	Destination								Supply (kg)
	J&T Express	Surya Cargo	Sambas	Sanggau	Sekadau	Mempawah	Sintang	Singkawang	
Giu Store	$\binom{0}{0}$	$\binom{2.074}{2.074}$	$\binom{159.630}{159.630}$	$\binom{129.837}{129.837}$	$\binom{190.400}{190.400}$	$\binom{53.296}{53.296}$	$\binom{240.556}{240.556}$	$\binom{105.052}{105.052}$	$\binom{58,5}{89,5}$
J&T Express	$\binom{0}{0}$	$\binom{2.815}{2.815}$	$\binom{0}{0}$	$\binom{3.700}{3.700}$	$\binom{600}{600}$	$\binom{8.000}{8.000}$	$\binom{0}{0}$	$\binom{4.200}{4.200}$	$\binom{58,5}{89,5}$
Surya Cargo	$\binom{2.815}{2.815}$	$\binom{0}{0}$	$\binom{0}{0}$	$\binom{0}{0}$	$\binom{0}{0}$	$\binom{0}{0}$	$\binom{2.600}{2.600}$	$\binom{0}{0}$	$\binom{58,5}{89,5}$
Demand (kg)	$\binom{58,5}{89,5}$	$\binom{58,5}{89,5}$	$\binom{10}{10,5}$	$\binom{14,25}{13,75}$	$\binom{5,25}{22,75}$	$\binom{17}{22,5}$	$\binom{2,75}{10,25}$	$\binom{9,25}{9,75}$	

Step 5. Based on Table 6, it shows that each row and column has a zero vector so there is no need to subtract the rows and columns. The process can be continued directly to Step 6.

Step 6. Check whether the $\boldsymbol{b}_j$ component is less or equal to the $\boldsymbol{a}_i$ component by looking at the column where the $\boldsymbol{c}_{ij}$ component is reduced to the zero vector. Do the opposite analysis. Where $\boldsymbol{b}_j \prec \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $b_j^p < \sum_{i=1}^{m} a_i^p$ at every j, p. $\boldsymbol{b}_j = \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $b_j^p = \sum_{i=1}^{m} a_i^p$ in each j, p. $\boldsymbol{b}_j \preccurlyeq \sum_{i=1}^{m} \boldsymbol{a}_i$ if and only if $b_j^p \leq \sum_{i=1}^{m} a_i^p$ for every j, p where there is at least $b_j^p < \sum_{i=1}^{m} a_i^p$ for some j, p based on the reduced $\boldsymbol{c}_{ij}$ zero vector. The checking results are as follows.

Checking $\boldsymbol{b}_j$ for $\boldsymbol{a}_i$

$$\boldsymbol{b}_1 \prec \boldsymbol{a}_1 + \boldsymbol{a}_2 \Leftrightarrow \binom{58,5}{89,5} \prec \binom{58,5}{89,5} + \binom{58,5}{89,5} \Leftrightarrow \binom{58,5}{89,5} \prec \binom{117}{179}$$

$$\boldsymbol{b}_2 = \boldsymbol{a}_3 \Leftrightarrow \binom{58,5}{89,5} = \binom{58,5}{89,5}$$

$$\boldsymbol{b}_3 \prec \boldsymbol{a}_2 + \boldsymbol{a}_3 \Leftrightarrow \binom{10}{10,5} \prec \binom{58,5}{89,5} + \binom{58,5}{89,5} \Leftrightarrow \binom{10}{10,5} \prec \binom{117}{179}$$

$$\boldsymbol{b}_4 \prec \boldsymbol{a}_3 \Leftrightarrow \binom{14,25}{13,75} \prec \binom{58,5}{89,5}$$

$$\boldsymbol{b}_5 \prec \boldsymbol{a}_3 \Leftrightarrow \binom{5,25}{22,75} \prec \binom{58,5}{89,5}$$

$$\boldsymbol{b}_6 \prec \boldsymbol{a}_3 \Leftrightarrow \binom{17}{22,5} \prec \binom{58,5}{89,5}$$

$$\boldsymbol{b}_7 \prec \boldsymbol{a}_2 \Leftrightarrow \binom{2,75}{10,25} \prec \binom{58,5}{89,5}$$

$$\boldsymbol{b}_8 \prec \boldsymbol{a}_3 \Leftrightarrow \binom{9,25}{9,75} \prec \binom{58,5}{89,5}$$

Checking $\boldsymbol{a}_i$ for $\boldsymbol{b}_j$

$$\boldsymbol{a}_1 = \boldsymbol{b}_1 \Leftrightarrow \binom{58,5}{89,5} = \binom{58,5}{89,5}$$

$$\boldsymbol{a}_2 \prec \boldsymbol{b}_1 + \boldsymbol{b}_3 + \boldsymbol{b}_7 \Leftrightarrow \binom{58,5}{89,5} \prec \binom{58,5}{89,5} + \binom{10}{10,5} + \binom{2,75}{10,25} \Leftrightarrow \binom{58,5}{89,5} \prec \binom{71,25}{110,25}$$

$$\boldsymbol{a}_3 \prec \boldsymbol{b}_2 + \boldsymbol{b}_3 + \boldsymbol{b}_4 + \boldsymbol{b}_5 + \boldsymbol{b}_6 + \boldsymbol{b}_8$$

$$\Leftrightarrow \binom{58,5}{89,5} \prec \binom{58,5}{89,5} + \binom{10}{10,5} + \binom{14,25}{13,75} + \binom{5,25}{22,75} + \binom{17}{22,5} + \binom{9,25}{9,75} \Leftrightarrow \binom{58,5}{89,5} \prec \binom{114,25}{168,75}$$

The results of the check show that both conditions have been met so there is no need to perform line analysis in Steps 7 and 8. The process continues to Step 9.

Steps 7 and 8: This step is not carried out because the conditions in step 6 are met.

Step 9. Determination of the exponent penalty (0_n). n represents the number of zero vectors in row i and column j. With the provisions of the determination, it does not include $\boldsymbol{c}_{ij}$ for which the exponential penalty will be determined. Based on Table 6, it is assumed that 0_n will be determined at $\boldsymbol{c}_{11}$, namely the source Giu Store to the destination J&T Express. The table shows that at the source Giu Store there is no reduced $\boldsymbol{c}_{ij}$ zero vector. However, there is one zero vector at the destination J&T Express, namely $\boldsymbol{c}_{21}$. Thus, the exponent penalty is 0_1. The other exponent penalty results are presented in Table 7.

Table 7. Determination of the exponent penalty

Source	Destination								Supply (kg)
	J&T Express	Surya Cargo	Sambas	Sanggau	Sekadau	Mempawah	Sintang	Singkawang	
Giu Store	0_1	$\binom{2.074}{2.074}$	$\binom{159.630}{159.630}$	$\binom{129.837}{129.837}$	$\binom{190.400}{190.400}$	$\binom{53.296}{53.296}$	$\binom{240.556}{240.556}$	$\binom{105.052}{105.052}$	$\binom{58,5}{89,5}$
J&T Express	0_3	$\binom{2.815}{2.815}$	0_3	$\binom{3.700}{3.700}$	$\binom{600}{600}$	$\binom{8.000}{8.000}$	0_2	$\binom{4.200}{4.200}$	$\binom{58,5}{89,5}$
Surya Cargo	$\binom{2.815}{2.815}$	0_5	0_6	0_5	0_5	0_5	$\binom{2.600}{2.600}$	0_2	$\binom{58,5}{89,5}$
Demand (kg)	$\binom{58,5}{89,5}$	$\binom{58,5}{89,5}$	$\binom{10}{10,5}$	$\binom{14,25}{13,75}$	$\binom{5,25}{22,75}$	$\binom{17}{22,5}$	$\binom{2,75}{10,25}$	$\binom{9,25}{9,75}$	

Step 10. Allocating cell values to the $\boldsymbol{c}_{ij}$ component of the vector zeros the exponential penalty. In Table 7 there is a penalty exponent 0_1 on $\boldsymbol{c}_{11}$ so allocate $\boldsymbol{x}_{ij} = \min(\boldsymbol{a}_i, \boldsymbol{b}_j)$ as follows

$$\boldsymbol{x}_{11} = \min(\boldsymbol{a}_1, \boldsymbol{b}_1) \Leftrightarrow \begin{pmatrix} x_{11}^1 \\ x_{11}^2 \end{pmatrix} = \min\left(\begin{pmatrix} 58,5 \\ 89,5 \end{pmatrix}, \begin{pmatrix} 58,5 \\ 89,5 \end{pmatrix}\right) = \begin{pmatrix} \min(58,5;\ 58,5) \\ \min(89,5;\ 89,5) \end{pmatrix} = \begin{pmatrix} 58,5 \\ 89,5 \end{pmatrix}.$$

Table 8 Exponent penalty allocation

Source	Destination								Supply (kg)
	J&T Express	Surya Cargo	Sambas	Sanggau	Sekadau	Mempawah	Sintang	Singkawang	
Giu Store	0_1 $\binom{58,5}{89,5}$	$\binom{2.074}{2.074}$	$\binom{159.630}{159.630}$	$\binom{129.837}{129.837}$	$\binom{190.400}{190.400}$	$\binom{53.296}{53.296}$	$\binom{240.556}{240.556}$	$\binom{105.052}{105.052}$	$\binom{0}{0}$
J&T Express	0_3	$\binom{2.815}{2.815}$	0_3	$\binom{3.700}{3.700}$	$\binom{600}{600}$	$\binom{8.000}{8.000}$	0_2	$\binom{4.200}{4.200}$	$\binom{58,5}{89,5}$
Surya Cargo	$\binom{2.815}{2.815}$	0_5	0_6	0_5	0_5	0_5	$\binom{2.600}{2.600}$	0_2	$\binom{58,5}{89,5}$
Demand (kg)	$\binom{0}{0}$	$\binom{58,5}{89,5}$	$\binom{10}{10,5}$	$\binom{14,25}{13,75}$	$\binom{5,25}{22,75}$	$\binom{17}{22,5}$	$\binom{2,75}{10,25}$	$\binom{9,25}{9,75}$	

Step 11. Checking that satisfies the conditions $\boldsymbol{a}_i = \boldsymbol{0}$ and $\boldsymbol{b}_j = \boldsymbol{0}$ in the selected cell. Based on Table 8, the row and column of cell $\boldsymbol{x}_{11}$ shows that $\boldsymbol{a}_1 = \binom{0}{0}$ and $\boldsymbol{b}_1 = \binom{0}{0}$ so that supply and demand have been allocated to cell $\boldsymbol{x}_{11}$. The process continues directly to Step 12.

Step 12. Table 8 shows that only $\boldsymbol{a}_i = 0$ is met in $\boldsymbol{a}_1$ and $\boldsymbol{b}_j = 0$ is only met in $\boldsymbol{b}_1$ while other sources and destinations still have unallocated supplies and demands. So, the process continues to the next iteration until $\boldsymbol{a}_i = \boldsymbol{0}$ is satisfied for each i and $\boldsymbol{b}_j = \boldsymbol{0}$ for each j.

Step 13: Calculate the optimal cost. From three iterations, the final allocation result is obtained. The process is continued with the calculation of the optimal cost.

Table 9. Solution with modification of the Modified Exponential Approach method

Source	Destination								Supply (kg)
	J&T Express	Surya Cargo	Sambas	Sanggau	Sekadau	Mempawah	Sintang	Singkawang	
Giu Store	$\binom{0}{0}$ $\binom{58,5}{89,5}$	$\binom{2.074}{2.074}$	$\binom{169.630}{169.630}$	$\binom{137.037}{137.037}$	$\binom{200.000}{200.000}$	$\binom{56.296}{56.296}$	$\binom{255.556}{255.556}$	$\binom{111.852}{111.852}$	$\binom{0}{0}$
J&T Express	$\binom{0}{0}$	$\binom{2.815}{2.815}$ $\binom{40,5}{46}$	$\binom{10.000}{10.000}$ $\binom{10}{10,5}$	$\binom{10.900}{10.900}$	$\binom{10.200}{10.200}$ $\binom{5,25}{22,75}$	$\binom{11.000}{11.000}$	$\binom{15.000}{15.000}$ $\binom{2,75}{10,25}$	$\binom{11.000}{11.000}$	$\binom{58,5}{89,5}$
Surya Cargo	$\binom{2.815}{2.815}$	$\binom{0}{0}$ $\binom{18}{43,5}$	$\binom{10.000}{10.000}$	$\binom{7.200}{7.200}$ $\binom{14,25}{13,75}$	$\binom{9.600}{9.600}$	$\binom{3.000}{3.000}$ $\binom{17}{22,5}$	$\binom{17.600}{17.600}$	$\binom{6.800}{6.800}$ $\binom{9,25}{9,75}$	$\binom{58,5}{89,5}$
Demand (kg)	$\binom{0}{0}$	$\binom{58,5}{89,5}$	$\binom{10}{10,5}$	$\binom{14,25}{13,75}$	$\binom{5,25}{22,75}$	$\binom{17}{22,5}$	$\binom{2,75}{10,25}$	$\binom{9,25}{9,75}$	

$$
\begin{aligned}
Z &= \boldsymbol{c}_{11}^T \cdot \boldsymbol{x}_{11} + \boldsymbol{c}_{22}^T \cdot \boldsymbol{x}_{22} + \boldsymbol{c}_{23}^T \cdot \boldsymbol{x}_{23} + \boldsymbol{c}_{25}^T \cdot \boldsymbol{x}_{25} + \boldsymbol{c}_{27}^T \cdot \boldsymbol{x}_{27} + \boldsymbol{c}_{32}^T \cdot \boldsymbol{x}_{32} + \boldsymbol{c}_{34}^T \cdot \boldsymbol{x}_{34} + \boldsymbol{c}_{36}^T \\
&\quad \cdot \boldsymbol{x}_{36} + \boldsymbol{c}_{38}^T \cdot \boldsymbol{x}_{38} \\
&= (0 \quad 0) \cdot \binom{58,5}{89,5} + (2.815 \quad 2.815) \cdot \binom{40,5}{46} + (10.000 \quad 10.000) \cdot \binom{10}{10,5} + \\
&\quad (10.200 \quad 10.200) \cdot \binom{5,25}{22,75} + (15.000 \quad 15.000) \cdot \binom{2,75}{10,25} + (0 \quad 0) \cdot \binom{18}{43,5} \\
&\quad + (7.200 \quad 7.200) \cdot \binom{14,25}{13,75} + (3.000 \quad 3.000) \cdot \binom{17}{22,5} + (6.800 \quad 6.800) \cdot \binom{9,25}{9,75} \\
&= 1.378.398.
\end{aligned}
$$

Conclusion

From the research results, several conclusions can be drawn, including:

1. The use of multi-product transportation problem model and modification of the Modified Exponential Approach method is in line with the condition of multi-product distribution. With the modification of the Modified Exponential Approach method, the allocation obtained provides allocation results in all products at once in one process without having to separate and calculate each product in a different process.
2. The solution obtained by modification of the Modified Exponential Approach method shows that shipping with a shipping service is more appropriate than direct shipping with a private car. This is because the shipping costs for products incurred from direct shipping tend to be greater than with shipping services. Shipping allocation with J&T Express should be sent to the Sambas, Sekadau, and Sintang areas with respective allocations for sweet snack products, namely 10 kg, 5,25 kg, 2,75 kg while for spicy/salty snack products, namely 10,5 kg, 22,75 kg, 10,25 kg. In addition, shipping with Surya Cargo should be sent to the Sanggau, Mempawah, and Singkawang areas. with respective allocations for sweet snack products, namely 14,25 kg, 17 kg, 9,25 kg and spicy/salty snack products, namely 13,75 kg, 22,5 kg, 9,75 kg. The minimum shipping cost obtained is Rp 1,378,398. This cost is smaller than the total cost of all three types of shipping.
3. Allocation with a modification of the Modified Exponential Approach method provides an alternative delivery for MSMEs to ensure more accurate stock of goods and lower shipping costs compared to previously carried out in a mixed manner.

4. Future research can focus on developing modifications of the Modified Exponential Approach method to overcome varying shipping costs and overcome limitations in step 6, especially in mixing comparisons with results of " $<$ " and " $>$ " on components according to supply and demand.

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