# Bellman-Ford Algorithm for Optimizing Drinking Water Distribution by Perumda Air Minum Tirta Raharja in Cicalengka

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#### **Abstract**

Access to clean water in Cicalengka District, Bandung Regency, remains limited, with current coverage reaching only 5.08% of the population. Perumda Air Minum Tirta Raharja has set a target to expand drinking water service to 44% by 2030. To support this goal, this study investigates the optimization of the water distribution pipe network in Cicalengka. The research applies the Bellman-Ford algorithm to model the distribution network as a weighted, undirected, and connected graph, where customer houses are represented as vertices, pipe connections as edges, and pipe lengths as weights. Using data from the existing network and customer locations, the algorithm was implemented to identify optimal distribution paths. The results yielded two shortest path alternatives between the specified source and destination nodes. These findings demonstrate the potential of graph-based optimization in improving distribution planning and can serve as a reference for the development and management of future water supply infra.

Keywords: Bellman-Ford algorithm, optimization, the shortest path, water distribution MSC2020: 05C85, 05C90

#### **Abstrak**

Akses air bersih di Kecamatan Cicalengka, Kabupaten Bandung, masih terbatas dengan cakupan layanan baru mencapai 5,08% dari total penduduk. Perumda Air Minum Tirta Raharja menargetkan peningkatan layanan hingga 44% pada tahun 2030. Penelitian ini bertujuan mengoptimalkan jaringan pipa distribusi air minum di wilayah Cicalengka. Metode yang digunakan adalah pemodelan jaringan pipa distribusi sebagai graf berbobot, tak berarah, dan terhubung, di mana rumah pelanggan direpresentasikan sebagai simpul, pipa sebagai sisi, serta panjang pipa sebagai bobot. Algoritma Bellman-Ford diterapkan untuk menentukan jalur distribusi optimal berdasarkan data jaringan eksisting dan posisi rumah pelanggan. Hasil penelitian menunjukkan terdapat dua jalur terpendek antara titik awal dan akhir yang ditentukan. Temuan ini menunjukkan bahwa optimasi berbasis graf dapat mendukung perencanaan dan pengelolaan jaringan pipa distribusi, serta menjadi referensi dalam pengembangan infrastruktur penyediaan air di masa mendatang.

Kata kunci: Algoritma Bellman-Ford, Optimisasi, Lintasan Terpendek, Distribusi Air MSC2020: 05C85, 05C90

#### Introduction

Perumda Air Minum Tirta Raharja, as the operator of drinking water distribution, has undertaken various efforts to improve services within its service area. However, in Cicalengka District, Bandung Regency, the coverage of drinking water services remains very limited, reaching only about 5.08% of the total population. The local government has set a target to expand this coverage to 44% by 2030 [1]. Achieving this ambitious goal requires comprehensive planning and improvements to the water distribution pipe network, from the source to customer endpoints. This situation presents both an opportunity and a challenge for research on optimizing water distribution as part of efforts to improve service delivery.

Previous studies have demonstrated the application of optimization methods to infrastructure networks, such as research conducted in collaboration with PT. PLN UP3 Cimahi to optimize the electricity distribution network in Cimahi City [2]. In the context of water supply, data on the existing pipe network and customer locations obtained from Perumda Air Minum Tirta Raharja can be modeled as a weighted, undirected, and connected graph, where pipes are represented as edges, customer houses as vertices, and pipe lengths as weights. To ensure optimization runs effectively, the initial graph model may be simplified through engineering adjustments.

The Bellman-Ford algorithm has been widely applied across diverse domains. Studies have shown its use in symbolic optimal control for software synthesis [3], in optimizing electric vehicle routes where negative energy costs may be considered [4], in medical image watermarking to enhance security [5], and in fraud detection through graph analysis [6]. One of its main advantages is the ability to handle negative edge weights, making it versatile for a range of optimization problems.

Building on this background, the central research question of this study is: *How can the Bellman-Ford algorithm be applied to identify the most efficient distribution paths in the drinking water pipe network of Cicalengka District, which is planned for expansion?* Accordingly, the objectives of this research are to construct a weighted graph model of the drinking water distribution system using real-world data, to apply the Bellman-Ford algorithm to determine the optimal distribution paths, and to provide insights that can inform practical decision-making by Perumda Air Minum Tirta Raharja in expanding service coverage and improving efficiency. This study is the first to utilize the drinking water pipe network database in Cicalengka District for optimization using a graph-theoretical approach. The findings are expected to contribute not only to the development of graph theory applications in infrastructure distribution but also to provide practical value for stakeholders in the planning and management of water distribution systems in Indonesia. This aligns with previous research emphasizing the importance of improving water service strategies for customers [7], [8], [9] and ensuring the performance of water provision for communities [10], [11].

# Methods

The initial graph model was constructed using water pipe network data from the Cicalengka subdistrict, Bandung Regency, West Java. The drinking water distribution system managed by Perumda Air Minum Tirta Raharja was represented as a weighted, undirected, and connected graph. In this model, customer houses serve as vertices, the connecting pipes as edges, and the pipe lengths as edge weights.

The optimization process was carried out using the Bellman-Ford algorithm. The graph modeling approach followed methodologies previously applied in distribution network studies, such as logistics [12], delivery services [13], [14], and electricity distribution [2]. The choice of the Bellman-Ford algorithm was based on its effectiveness in solving shortest path problems, as demonstrated in route

optimization research conducted in logistics distribution [15], LPG gas distribution [16], and fuzzy image environments [17].

In the context of drinking water supply, previous studies by Nurviana et al. and Sinaga et al. applied the Prim algorithm to analyze water distribution networks [18], [19]. Distinct from those approaches, this study applies the Bellman-Ford algorithm to address the drinking water distribution challenges in Cicalengka. The methodological steps of this research are detailed in the following subsections.

## **Construct the Initial Model Graph**

The initial model graph contains cycles but is not a complete graph. This is due to practical considerations in installing water pipes between customer houses. For example, pipes cannot be laid across buildings or private gardens, as this would interfere with customer privacy. Instead, pipes are installed along the edges of roads, which means not every pair of customer houses can be directly connected by a single pipe.

On the left side of Figure 1 is a map of the drinking water distribution network along Jalan Raya Cicalengka, West Java. The red lines represent water pipes, while the green dots indicate customer houses. These red lines do not directly connect the green dots; rather, they show that pipes follow the sides of the road and, where possible, avoid crossing the main road. Crossing the main road could obstruct traffic and would require significant repairs in case of damage. To reflect these constraints, the edges of the initial model graph were designed to follow the pipe layout shown by the red lines.

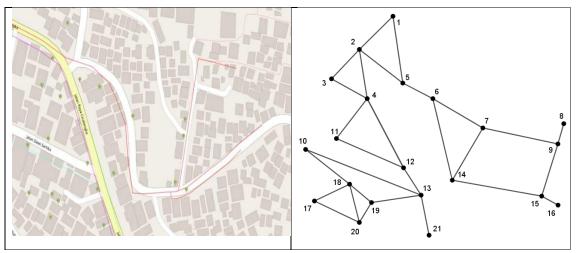


Figure 1. Location Map and Initial Model Graph G

The right side of Figure 1 shows the resulting initial model graph, which reflects both construction and engineering adjustments to match real conditions. The graph consists of 21 vertices and 28 edges, representing 21 customer houses and 28 connecting pipe segments. Because of the installation rules, the graph is not complete, though it remains connected. The numbering of vertices serves only as identifiers and does not indicate the direction of water flow. Starting and ending vertices can be selected as needed. The goal of this study is to determine the shortest path as the optimal water distribution route between selected vertices, using the Bellman-Ford algorithm.

## The Weight of The Edges

Table 1 presents the weight of each edge in the initial model graph. In this context, the weight represents the actual length of the water pipe in meters that connects one customer's house to another. Since each pipe segment is unique, the graph does not contain multiple or parallel edges; there is only one pipe connecting any two houses. Likewise, there are no loop edges, meaning no edge directly connects a vertex to itself. With these properties, the network can be classified as a *simple graph*.

The table illustrates how pipe lengths vary across the network, ranging from short connections such as edge (18, 19) with a length of only 7.5 meters, to longer segments like edge (10, 13) measuring 64.5 meters. These variations in edge weights reflect real-world construction constraints and the different distances between houses along the distribution line. The presence of both short and long pipe sections highlights the importance of optimizing the network, as inefficient path selection could lead to significant increases in total pipe length and installation costs. By assigning weights to each edge, the graph model provides a structured representation of the water distribution system in Cicalengka. This weighted graph serves as the foundation for applying the Bellman-Ford algorithm to determine the shortest path, which ultimately supports efforts to design an optimal water distribution network.

edge	weight	edge	weight	edge	weight	edge	weight
(1, 2)	27	(4, 12)	49,5	(9, 15)	30	(14, 15)	52,5
(1, 5)	39	(5, 6)	18,72	(10, 13)	64,5	(15, 16)	10,5
(2, 3)	18	(6, 7)	34,5	(10, 18)	37,5	(17, 18)	20,25
(2, 4)	24	(6, 14)	57	(11, 12)	37,5	(17, 20)	25,5
(2, 5)	27	(7, 9)	45	(12, 13)	19,5	(18, 19)	7,5
(3, 4)	18	(7, 14)	40,5	(13, 19)	34,5	(18, 20)	16,5
(4, 11)	30	(8, 9)	12	(13, 21)	25,5	(19, 20)	12

Table 1. The Weight of The Edges

#### The Bellman-Ford Algorithm

The Bellman-Ford algorithm is a well-known method for finding the shortest path from a source vertex to all other vertices in a weighted graph. Unlike Dijkstra's algorithm, Bellman-Ford can handle graphs containing negative edge weights. However, in the case of drinking water distribution, all edge weights are non-negative, which makes the algorithm steps more straightforward.

The core principle of Bellman-Ford is iterative edge relaxation. In each iteration, the algorithm evaluates whether the current shortest distance to a vertex can be improved by passing through one of its adjacent vertices. Formally, let w[i,v] represent the shortest distance from the source vertex to vertex v after the i-th iteration. Then:

$$w[i, v] = \min\{w[i-1, v], (w[i-1, n] + c(n, v))\},\$$

where

*i* is the iteration number (starting from 1).

v is the target vertex whose distance is being calculated.

n is adjacent to a vertex v

w[i-1,v] is the shortest distance from the starting vertex to the vertex v in the previous iteration.

w[i-1,n] is the shortest distance from the starting vertex to the vertex n in the previous iteration.

c(n, v) is the weight of the edge from the vertex n to vertex v.

In practice, this means that at each step, the algorithm attempts to update the shortest distance to v by checking whether traveling through a neighbor nnn provides a shorter route. This process, known as *edge relaxation*, is repeated for all edges across multiple iterations until no further improvements can be made.

## **Application for Optimizing Drinking Water Distribution**

The application of the Bellman-Ford algorithm begins with determining the source and destination vertices. In this study, vertex 16 was chosen as the starting point and vertex 17 as the endpoint. This selection was made because the distance between these two vertices is estimated to be the farthest compared to other pairs, making the path between them representative for analyzing the shortest path problem in this area.

The next step is simplifying the initial graph model G. The simplification focuses on retaining only the vertices that are likely to be part of the paths connecting vertex 16 and vertex 17. Leaf vertices, which cannot lie on such paths, are removed. In the initial graph, three vertices act as leaves: vertex 8, vertex 16, and vertex 21. Since vertex 16 is designated as the starting point, it must remain in the graph. Therefore, only vertices 8 and 21 are removed. The simplified graph is denoted as  $G_1$ .

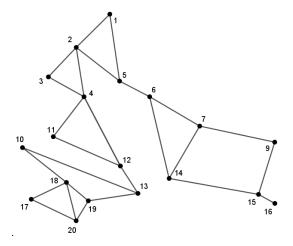


Figure 2. The Initial Model Graph G<sub>1</sub>

## **Results and Discussion**

This study applied the Bellman–Ford algorithm to determine the shortest path within the simplified pipe network graph (G1) in Cicalengka. The primary objective was to identify the optimal distribution route between vertex 16 (starting point) and vertex 17 (destination). The algorithm iteratively relaxes the edges of the graph to minimize path weights, and the detailed relaxation process is presented in Tables 2 and 3.

Table 2 illustrates the iteration process for finding the shortest path between vertices 16 and 17. At the first iteration, the distance from vertex 16 to itself is set to zero, while the distances to other vertices are initialized as infinite. During the second iteration, the algorithm updates the edge weight from vertex 16 to vertex 15, which has a distance of 10.5 meters. This establishes the initial path segment 16–15. Subsequent iterations expand the path by evaluating connected vertices. For example, in the third iteration, the edge from vertex 15 to vertex 9 with a weight of 40.5 meters is identified, while in the fourth iteration the alternative path 15–14 with a weight of 63 meters is considered. The algorithm consistently chooses the edge with the smaller weight, resulting in the path extension 16–15–9.

3 Itertion 16 15 14 6 1 0  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$ 2 0 10.5  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$ 3 - 410.5 0 40.5 63  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$ 5 0 10.5 40.6 63 85.5  $\infty$  $\infty$  $\infty$  $\infty$  $\infty$ 6 0 10.5 40.7 63 85.5 120 7 0 10.5 40.8 63 85.5 120 138.72  $\infty$  $\infty$  $\infty$ 8 - 910.5 40.9 138.72 177.72 0 63 85.5 120 165.72  $\infty$ 10 0 10.5 40.10 63 85.5 120 138.72 177.72 165.72 183.72 11 - 120 10.5 40.11 120 138.72 177.72 183.72 63 85.5 165.72 13 0 10.5 40.12 85.5 138.72 177.72 165.72 183.72 63 120 14 - 150 10.5 40.13 63 85.5 120 138.72 177.72 165.72 183.72 16 - 1710.5 40.14 85.5 120 138.72 177.72 165.72 183.72 0 63 18 0 10.5 40.15 85.5 120 138.72 177.72 165.72 183.72 63

Table 2. Iteration of The Shortest Path between Vertices 16 and 3

The relaxation process continues across 18 iterations, as required by the Bellman–Ford algorithm (V– 1 iterations for a graph with V vertices). The final result is the shortest path T1:

with a total weight of 320.97 meters.

Table 3 shows the relaxation process for another segment of the graph involving vertices 4 and 17. Interestingly, during iteration 6, the algorithm identifies two alternative sub-paths with the same weight of 120 meters:

- Path A: 16 − 15 − 9 − 7 − 6
- Path B: 16 15 14 6

This branching leads to the discovery of a second shortest path, T2, with the same total weight as T1:

Thus, the Bellman–Ford algorithm produces two equally optimal solutions, T1 and T2, both with a path length of 320.97 meters (Figure 3).

Iteration	4	11	12	13	19	10	18	20	17
1	<sub>∞</sub>	∞							
2	∞	∞	∞	∞	∞	∞	∞	∞	∞
3 – 4	<sub>∞</sub>	œ	<sub>∞</sub>	<sub>∞</sub>	<sub>∞</sub>	<sub>∞</sub>	<sub>∞</sub>	œ	∞
5	∞	×	∞	∞	∞	∞	∞	×	∞
6	<sub>∞</sub>	∞							
7	×	<sub>∞</sub>	$\infty$	$\infty$	$\infty$	∞	×	<sub>∞</sub>	$\infty$
8 – 9	×	∞	∞	∞	∞	∞	×	×	$\infty$
10	189.72	×	∞	∞	∞	∞	×	×	$\infty$
11 – 12	189.72	219.72	239.22	∞	∞	$\infty$	$\infty$	$\infty$	$\infty$
13	189.72	219.72	239.22	258.72	∞	∞	×	×	$\infty$
14 – 15	189.72	219.72	239.22	258.72	293.22	357.72	×	×	$\infty$
16 – 17	189.72	219.72	239.22	258.72	293.22	357.72	300.72	305.22	$\infty$
18	189.72	219.72	239.22	258.72	293.22	357.72	300.72	305.22	320.97

Table 3. Iteration of the Shortest Path between Vertices 4 and 17

However, if relaxation is performed at iteration 6, then there are other paths that have the same weight of 120 meters, namely path 16 - 15 - 9 - 7 - 6 and path 16 - 15 - 14 - 6, as illustrated below.

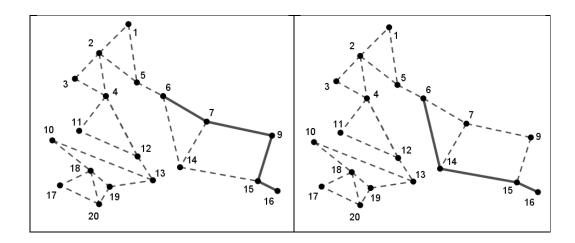


Figure 2. The Two Path with Equal Weight

So, in addition to the shortest path T1, the shortest path T2 is also obtained, namely 16-15-14-6-5-2-4-12-13-19-18-17 with a total weight of 320.97 meters. The two shortest paths are shown in the following figures.

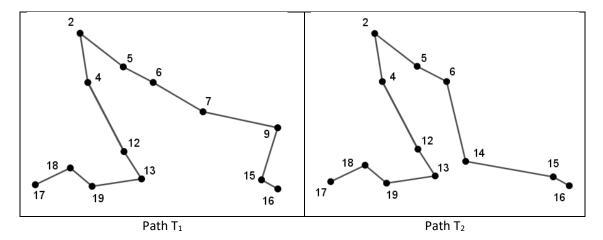


Figure 3. The Shortest Path Generated by Bellman-Ford Algorithm

The non-uniqueness of the solution highlights one of the important characteristics of Bellman–Ford: depending on the connectivity of the graph, multiple shortest paths of equal weight may exist. This is consistent with the theoretical properties of shortest-path algorithms. Moreover, simplifying the initial graph G into G1 proved advantageous, as it reduced the number of iterations required (from 20 to 18), thereby improving computational efficiency.

Further structural analysis of graph G1 reveals the presence of six bridges: vertices 2, 4, 5, 6, 12, and 13. According to graph theory, a bridge is an edge whose removal disconnects the graph. The inclusion of these vertices in both shortest paths (T1 and T2) indicates their strategic importance in maintaining network connectivity. Consequently, they represent critical points in the water distribution network where failures could significantly disrupt service. According to [20], an edge in a connected graph G, which, when this edge is deleted, results in the graph G becoming a disconnected graph, then this edge is called a bridge. In graph  $G_L$  there are 6 bridges, namely, vertices 2, 4, 5, 6, 12, and 13.

Overall, the results demonstrate that the Bellman–Ford algorithm can be effectively applied to optimize water distribution routes. The identification of multiple shortest paths and critical bridge vertices provides valuable insights for both network planning and resilience analysis.

#### Conclusion

The optimization of the drinking water distribution network in the Cicalengka area, managed by Perumda Air Minum Tirta Raharja, was carried out using the initial model graph G1 as input for the Bellman-Ford algorithm. From this process, two alternative shortest paths with identical weights were obtained. The first path, T1, follows the sequence 16-15-9-7-6-5-2-4-12-13-19-18-17, while the second path, T2, follows 16-15-14-6-5-2-4-12-13-19-18-17. These results demonstrate that the algorithm is capable of identifying multiple optimal routes with equivalent efficiency, which may provide flexibility for decision-makers in planning and managing water distribution.

Looking ahead, further research can extend this work by comparing the Bellman-Ford algorithm with other shortest path algorithms, particularly Dijkstra's algorithm. A comparative study would enable a deeper evaluation of efficiency, performance, and suitability in the context of water distribution optimization. For instance, prior research [21] has conducted a comparative analysis between Dijkstra and Bellman-Ford, focusing on two main aspects: (1) the structural modeling of the problem, and (2) the resulting shortest paths produced by each algorithm. Conducting a similar comparative study in the case of Cicalengka could enrich the understanding of algorithmic performance, highlight trade-offs between computation time and accuracy, and support the selection of the most effective method for practical implementation.

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