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Probabilistic Model of Back Order Policy on *Dodol Pulut* Raw Material Inventory Control with Linear Programming

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Abstract

The high production costs of *Dodol Pulut Raya* further emphasize the importance of implementing a systematic, measurable, and effective inventory control strategy to maintain profitability, operational efficiency, and long-term business stability. This study applies the Probabilistic Back Order Policy Model combined with the Simplex Linear Programming method to determine the most optimal raw material management policy, taking into account demand uncertainty, supply variability, and the risk of stock shortages that could disrupt production smoothness. This approach comprehensively integrates precise safety stock calculations to minimize stockouts, reduce production delays, and ensure raw material availability so that customer needs can be consistently, timely, and sustainably met. The analysis results show that the implementation of this model can significantly reduce annual production costs from IDR 509,941,040 to IDR 338,719,300, resulting in cost efficiency of IDR 171,221,740 per year. This finding demonstrates the model's effectiveness in minimizing total inventory costs while enhancing competitiveness and operational sustainability. Additionally, this study provides a practical framework for small and medium-sized enterprises facing fluctuating demand, balancing service levels, safety stock, and overall cost optimization.

Keywords: Inventory control, probabilistic back order, linear programming, safety stock, cost optimization
MSC2020: 90B05,90C05

Abstrak

Biaya produksi yang tinggi pada usaha Dodol Pulut Raya semakin menegaskan pentingnya penerapan strategi pengendalian persediaan yang sistematis, terukur, dan efektif guna menjaga profitabilitas, efisiensi operasional, serta stabilitas bisnis dalam jangka panjang. Penelitian ini mengaplikasikan Model Probabilistik Back Order Policy yang dipadukan dengan metode Program Linear Simpleks untuk menentukan kebijakan pengelolaan bahan baku yang paling optimal dengan mempertimbangkan ketidakpastian permintaan, variasi pasokan, serta risiko kekurangan stok yang dapat menghambat kelancaran produksi. Pendekatan ini secara komprehensif mengintegrasikan perhitungan safety stock secara cermat untuk meminimalkan kekosongan persediaan, mengurangi keterlambatan produksi, dan memastikan ketersediaan bahan baku agar kebutuhan pelanggan dapat terpenuhi secara konsisten, tepat waktu, dan berkelanjutan. Hasil analisis menunjukkan bahwa penerapan model ini mampu menurunkan total biaya produksi tahunan secara signifikan dari Rp 509.941.040 menjadi Rp 338.719.300, yang berarti terjadi efisiensi biaya sebesar Rp 171.221.740 per tahun. Temuan ini membuktikan efektivitas model dalam meminimalkan biaya total persediaan sekaligus meningkatkan daya saing serta keberlanjutan operasional. Selain itu, penelitian ini memberikan kerangka kerja

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praktis bagi usaha kecil dan menengah yang menghadapi permintaan berfluktuasi, dengan menyeimbangkan tingkat layanan, safety stock, dan optimalisasi biaya menyeluruh.

Kata kunci: Pengendalian persediaan, probabilistik back order, program linear, safety stock, optimasi biaya
MSC2020: 90B05,90C05

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Introduction

Inventory control organizes production to be efficient, with safety stock to anticipate supplier delays without burdening storage costs [1]. Inventory is important for decision making, buffering, determining economic lots, and anticipating future needs [2]. Service and manufacturing companies need the right inventory so that demand is met without losing profit opportunities [3].

Dodol Pulut Raya Shop, a culinary business on Jl. Lintas Sumatera, Langkat, has been producing various types of *dodol pulut* for decades. The store faces inventory problems as a result of it does not have an optimal inventory and safety stock policy, causing high costs and unmet demand. The solution can use methods such as Linear Programming, EOQ, Optimal Control, Dynamic Programming, and Probabilistic Back Order Policy Model.

The Q-backorder model on probabilistic inventory determines the optimal policy to reduce costs while maintaining the satisfaction of consumers who are willing to wait for goods [4]. To optimize inventory that aims to reduce inventory costs, a Linear Programming approach is used.

Linear programming is a mathematical approach with objective functions and linear constraints to maximize profits or minimize costs, solved using the simplex method to determine the optimal solution. This Linear Programming is suitable for a variety of applications such as business management, production planning, and resource allocation [5].

The advantages of the Probabilistic Back Order Policy Model with Linear Programming are effective for overcoming fluctuating demand, determining safety stock, maintaining smooth operations, and optimizing raw material inventory efficiently according to existing constraints [4, 16, 17]. The author wants to apply the Probabilistic Back Order Policy Model with Linear Programming at *Dodol Pulut Raya* Shop to determine the optimal inventory policy and safety stock to reduce total costs.

Methods

This research is applied research, the results of which can be directly applied by industry players to find solutions to the problems they face. This research was conducted at Toko Dodol Pulut Raya in December 2024 to improve operational efficiency through inventory control. Secondary data from the production department were analyzed using the Back Order Policy Model with Excel software and the Simplex Linear Programming method with POM-QM software to determine the production quantities of four types of dodol to minimize costs. The variables of focus in this study include ordering costs, storage purchases, shortages, and raw material constraints. Data normality was tested using SPSS software.

Probabilistic Model Q-backorder

The Q model is a development of a simple probabilistic model with the main feature of a fixed order lot size, where orders are placed when inventory reaches the reorder point. Before calculating the inventory policy using Model Q with backorders, first calculate the total inventory cost (B_T) as follow:

$$B_T = D_p + \frac{AD}{q_0} + \left(\frac{q_0}{2} + r - DL\right) + Cu\frac{D}{q_0}N \quad (1)$$

where:

B_T = total inventory cost

B_b = purchase price

B_p = procurement price

B_s = storage price

B_k = inventory shortage price (backorder cost)

D = expected quantity of goods purchased

p = item price/unit

A = procurement cost per order

q_0 = order quantity

T = duration between orders

h = annual storage price/unit

r = reorder point

L = (lead time)

C_u = stockout inventory price/unit of goods

N = number of annual shortages [4]

Linear Programming

In Operations Research, Linear Programming is used as a method to solve linear problems and find optimal solutions. Various calculation models, including graphical methods and simplex methods, can be applied in this process [6]. According to [7] linear programming are a method to achieve efficient use of limited resources.

The linear program model consists of three main interrelated elements including decision variables, objective functions, and constraint functions. Linear programs require clear objectives, the existence of alternatives that are compared, limited resources, can be formulated quantitatively, and have Linear programs model objective functions and constraints linearly to maximize profits and reduce costs, solved by the simplex method to determine the optimal resource allocation and output [9]. The linear programming problem aims to optimize (maximize or minimize) a variable that is a linear function of independent variables considering a number of linear constraints [10]. Here is the general form of a linear programming:

Objective function (Max or Min):

$$Z_{max} = C_1X_1 + C_2X_2 + \cdots + C_nX_n \quad (2)$$

Constraint Function:

$$\begin{gathered} a_1X_1 + a_2X_2 + \cdots + a_nX_n \leq b_1 \\ a_{21}Xa_{21} + a_{22}X_{22} + \cdots + a_{2n}X_{2n} \leq b_2 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}X_{m1} + a_{m2}X_{m2} + \cdots + a_{mn}X_{mn} \leq b_m \end{gathered} \quad (3)$$

Description:

$C_1, C_2, \ldots C_n$ = objective function coefficient

$X_1, X_2, \ldots X_n$ = decision variable

$a_{21}, a_{22}, \ldots\ a_{2n}$= coefficient of constraint function

$Xa_{21}, Xa_{22}, \ldots Xa_{2n}$= number of constraint functions [11]

Simplex Method

In the simplex method of linear programming, developed by George B. Dantzing (1947), finding the optimal solution through iterative calculations based on Gauss-Jordan elimination [12]. The simplex method is an important technique in linear programs that iteratively searches for the optimal solution of the objective function by moving between points in the solution space until the best result is obtained [13]. The advantage of this method is that it is able to handle the calculation of two or more decision variables, while the graphical method is only limited to two variables [14]. The following are the stages of the simplex method:

1. Create an initial table by converting all inequalities into equations through the addition of slack variables.
2. Form a simplex table containing objective function coefficients, constraints, and basic variables.
3. Run iterations to obtain the $Z_j - C_j$ values of all non-base variables.
4. Selecting the outgoing base variable by finding the row with the smallest ratio between b_i and the coefficient of the incoming variable.
5. Replacing the outgoing base variable with the incoming base variable [15].

Results and Discussion

Dodol production requires various production factors. Information regarding the availability of raw materials for *dodol* production every month as can be seen in the table below:

Table 1. Raw Material Demand/Year (Year 2024)

Month	Granulated sugar (Kg)	Palm sugar (Kg)	Coconut (Butir)	Sticky rice flour (Kg)	Durian	Sesame	Pandan
Jan	424	64	704	320	16	16	16
Feb	424	64	704	320	16	16	16
Mar	424	64	704	320	16	16	16
Apr	371	56	616	280	14	14	14
May	424	64	704	320	16	16	16
June	371	56	616	280	14	14	14
Jul	371	56	616	280	14	14	14
Agust	318	48	536	240	12	12	12
Sept	318	48	536	240	12	12	12
Okt	371	56	616	280	14	14	14
Nov	424	64	704	320	16	16	16
Dec	424	64	704	320	16	16	16
Total	4.664	704	7.760	3.520	176	176	176

Source: Dodol Pulut Raya Business

Data Normality Test

Normality testing is done with SPSS software using the Kologorov-Smirnove test. The assessment criteria indicate that the data is considered normal if the significance value> 0.05, while the value < 0.05 indicates abnormal distribution. The test results state that the raw material demand data for Dodol Pulut Raya in 2024 meets the assumption of normality. The results are shown in the following figure:

One-Sample Kolmogorov-Smirnov Test

			Jan	Feb	Mar	Apr	Mei	Jun	Jul	Agus	Sep	Okto	Nov	Des
N			7	7	7	7	7	7	7	7	7	7	7	7
Normal Parameters[a,b]	Mean		222.86	222.86	222.86	195.00	222.86	195.00	195.00	168.29	168.29	195.00	222.86	222.86
	Std. Deviation		269.258	269.258	269.258	235.601	269.258	235.601	235.601	204.334	204.334	235.601	269.258	269.258
Most Extreme Differences	Absolute		.294	.294	.294	.294	.294	.294	.294	.293	.293	.294	.294	.294
	Positive		.294	.294	.294	.294	.294	.294	.294	.293	.293	.294	.294	.294
	Negative		-.221	-.221	-.221	-.221	-.221	-.221	-.221	-.222	-.222	-.221	-.221	-.221
Test Statistic			.294	.294	.294	.294	.294	.294	.294	.293	.293	.294	.294	.294
Asymp. Sig. (2-tailed)[c]			.068	.068	.068	.068	.068	.068	.068	.069	.069	.068	.068	.068
Monte Carlo Sig. (2-tailed)[d]	Sig.		.067	.067	.067	.067	.067	.067	.067	.068	.068	.067	.067	.067
	99% Confidence Interval	Lower Bound	.060	.060	.060	.060	.060	.060	.060	.061	.061	.060	.060	.060
		Upper Bound	.073	.073	.073	.073	.073	.073	.073	.074	.074	.073	.073	.073

a. Test distribution is Normal.
b. Calculated from data.
c. Lilliefors Significance Correction.
d. Lilliefors' method based on 10000 Monte Carlo samples with starting seed 2000000.

Double-click to activate

Figure 1. Normality Test Results of *Dodol Pulut Raya* Raw Material Demand Data

Estimation of Costs Used

- Order Cost (A), Purchase Cost (p), and Storage Cost (h)

Table 2. Order Cost, Purchase Cost, and Storage Cost

Raw Material	Order Cost (IDR)	Purchase Cost (IDR)	Storage Cost (IDR)/0.5%
Granulated sugar	480.000	83.952.000	419.760
Palm sugar	480.000	14.080.000	70.400
Coconut	672.000	62.080.000	310.400
Sticky rice flour	480.000	70.400.000	352.000
Durian	2.400.000	12.320.000	61.600
Sesame	480.000	10.560.000	52.800
Pandan	288.000	2.640.000	1.3200
TOTAL	**4.800.000**	**256.032.000**	**1.280.160**

Source: Dodol Pulut Raya Business

- Shortage cost (Cu)
  1. Lost sales cost
     Profit margin/Kg x amount of unmet demand/Kg = 10 kg x IDR 18,200 = IDR182,000
  2. Production delay cost
     Paying the wages of unemployed workers, the wages incurred by the owner once the production is IDR 280,000.
  3. Emergency procurement cost
     Coconut needs (8 grains)/once production
     = (IDR 10,000 / grain - IDR 8,000 / grain) x 8
     = IDR 2,000 x 8
     = IDR 16,000
     Durian needs (0.5 kg)/once production
     = (IDR80,000/kg – IDR 70,000/kg) x 0.5
     = IDR 10,000 x 0.5
     = IDR 5000
  4. Cost of loss of production efficiency
     The total cost of shortages (Cu) amounted to: IDR 182,000 + IDR 200,000 + IDR 16,000 + IDR 5000 = IDR 403,000
  5. Waiting Time/Lead Time (L)
     3 days with a total of 8 working days in 1 month

Calculation Using the Probabilistic Method Q-Back Order Model

Calculations using the probabilistic Q-backorder method using the Hadley-Within model, carried out manually and then continued with Excel assistance.

Q-Back Order Calculation for Granulated Sugar:

Table 3. Granulated Sugar Data

No	Cost	Unit	Price	Total Cost
1	Purchase Cost	761	IDR 18,000	IDR 13,698,000
2	Ordering Cost	96	IDR 48,0000	IDR 46,080,000
3	Storage Cost			IDR 411,120
4	Inventory Shortage Cost			IDR 403,000
	Total Inventory Cost			IDR 60,592,1220

1) Calculating value q_1

$$q_1 = \sqrt{\frac{2AD}{h}} \tag{4}$$

$$= \sqrt{\frac{2 \times 5000 \times 389}{411.200}} = \sqrt{\frac{730.560.000}{411.200}} = \sqrt{1.776,99} = 3,04 \approx 3$$

2) Calculating value $Z\alpha$

$$\alpha = \frac{hq_1}{CuD} \tag{5}$$

$$= \frac{411.120 \times 42}{403.000 \times 761} = \frac{17.267.040}{306.683.000} = 0,0008$$

$$Z\alpha \rightarrow \text{Seen From the distribusi normal}$$
$$Z\alpha = 2,407$$

3) Calculating value r_1

$$r_1 = DL + Z\alpha S\sqrt{L} \tag{6}$$

$$= (761 \times 0,00833) + \left(1.586 \times 10 \times \sqrt{0,00833}\right) = 6,341 + \left(1.586 \times 10 \times \sqrt{0,00833}\right)$$
$$= 6,341 + (1.586 \times 10 \times 0,091) = 6,341 + 1,443 = 12,25 \approx 12$$

4) Calculating value q_2

$$q_2 = \sqrt{\frac{2D[A - Cu\int_{\eta}^{\alpha}(x - r_1)f(x)dx}{h}} \tag{7}$$

N value is determined first

$$N = \int_{r_1}^{\alpha}(x - r_1)f(x)dx \quad \rightarrow \quad SL[f(Z\alpha - Z\alpha\psi(Z\alpha)]$$

Determining the N value

$$SL[f(Z\alpha - Z\alpha\psi(Z\alpha)]$$
$$SL = 0,0083 \times 10 = 0,0083$$
$$f(Z\alpha) \text{ dan } \psi(Z\alpha)(\text{determined from the indentity function tabel})$$

$$\text{Known: } Z\alpha = 1{,}586 \qquad \text{So: } f(Z\alpha) = 0{,}12 \quad \psi(Z\alpha) = 0{,}06$$
$$\text{So that: } N = SL[f(Z\alpha - Z\alpha\psi(Z\alpha)]$$
$$= 0{,}0833\left[0{,}12 - \left(1{,}586(0{,}06)\right)\right] = 0{,}0833[0{,}12 - 0{,}095] = 0{,}0020 \rightarrow 1$$

Determining the value q_2

$$q_2 = \sqrt{\frac{2D[A - Cu\int_{\eta}^{\alpha}(x - r_1)f(x)dx}{h}} = \sqrt{\frac{2 \times 761(80.800 - 403.000(1))}{411.120}} = \sqrt{285{,}06} = 2{,}93$$
$$\approx 3$$

5) Calculating value $Z\alpha'$

$$\alpha' = \frac{hq_2}{CuD} \qquad \rightarrow Z\alpha \tag{8}$$

$$\alpha' = \frac{411.120(17)}{403.000(761)} = \frac{6.989.040}{306.683.000} = 0{,}02279$$
$$Z\alpha' = 1{,}999$$

6) Calculating value r_2

$$\alpha = \int_{r'}^{\alpha} f(x)dx \qquad \rightarrow \quad r_2 = DL + Z\alpha S\sqrt{L} \tag{9}$$
$$r_2 = DL + Z\alpha S\sqrt{L}$$

$$r_2 = 761 \times 0{,}00833 + \left(1{,}999 \times 10 \times \sqrt{0{,}0083}\right) = 6{,}3391 + (1{,}999 \times 10 \times 0{,}0912)$$
$$= 6{,}3391 + 1{,}8230 = 12{,}25 \approx 12$$

7) Calculating the level of service (η)

$$\eta = \left(1 - \frac{N}{D_2}\right) \times 100\% \tag{10}$$

$$\eta = \left(1 - \frac{1}{761 \times 0{,}0833}\right) \times 100\% = (1 - 0{,}1577) \times 100\% = 0{,}8423 \times 100\% = 84{,}23\%$$

8) Calculating the total cost of inventory

$$OT = Dp + \frac{AD}{q_2} + h\left(\frac{1}{2}q_2 + r - DL\right)Cu\frac{D}{q_2}\int_{f}^{\alpha}(x - r)(x)dx \tag{11}$$
$$= 761(18.000) + \frac{480.000(761)}{17} + 411.120\left(\frac{1}{2}(17) + 8 - 6{,}3391\right) + 403.000\left(\frac{761}{17}\right) \times 1$$
$$= 13.698.000 + 21.487.058{,}8 + 4.177.349{,}21 + 19.248.821 = 64.212.038$$

Table 4. Calculation data for the year 2024

No	Name of raw material	D	EOQ	ROP	SS	OT(Old)	OT(New)	Difference OT
1	Granulated sugar	389	3	12	9	171.171.640	64.212.038	106.959.602
2	Palm sugar	59	3	2	1	6.216.600	1.500.106	4.762.494
3	Coconut	647	5	21	15	207.079.800	11.758.331	195.321.467
4	Sticky rice flour	239	3	9	7	109.879.000	48.544.533	1.477.660
5	Durian	15	3	0,4	0	4.777.000	3.299.340	8.352.671
6	Sesame	15	0,5	0,4	0	9.703.000	1.350.329	8.352.671
7	Pandan	15	3	1	0	1.114.000	86.019	1.027.981
					Total	509.941.040	130.1750.696	379.190.344

Dodol pulut inventory optimization is carried out with the Linear Programming simplex method, carried out manually and then continued with POM-QM software, using the Probabilistic Back Order Policy Model which is based on safety stock calculations.

Table 5. Production Needs of *Dodol Pulut Raya* (Year 2024)

No	Raw Materials	Production Needs						
		Original Dodol	Durian Dodol	Sesame Dodol	Pandan Dodol	Inventory Stock (Needs + Safety Stock)		
						Needs	Safety Stock	
1	Granulated sugar	440	1408	1408	1408	4664	9	4673
2	Palm sugar	704	0	0	0	704	1	705
3	Coconut	1940	1940	1940	1940	9765	15	9771
4	Sticky rice flour	880	880	880	880	3520	7	3527
5	Durian	0	176	0	0	176	1	177
6	Sesame	0	0	176	0	176	1	177
7	Pandan	0	0	0	176	176	1	177

In this study, the variables used in the Linear Programming Simplex Method are as follows:

- Decision variable

  X_1= Original Dodol X_3= Sesame Dodol

  X_2= Durian Dodol X_4= Pandan Dodol
- Objective function

$$Z_{min=}51.482.400X_1 + 83.155.200X_2 + 79.387.200X_3 + 70.879.200X_4$$

- Constraint function

$$440X_1 + 1408X_2 + 1408X_3 + 1408X_4 \geq 4673$$
$$704X_1 + 0X_2 + 0X_3 + 0X_4 \geq 705$$
$$1940X_1 + 1940X_2 + 1940X_3 + 1940X_4 \geq 9771$$
$$880X_1 + 880X_2 + 880X_3 + 880X_4 \geq 3527$$
$$0X_1 + 176X_2 + 0X_3 + 0X_4 \geq 177$$

$$0X_1 + 0X_2 + 176X_3 + 0X_4 \geq 177$$
$$0X_1 + 0X_2 + 0X_3 + 176X_4 \geq 177$$
$$X_1, X_2, X_3, X_4 \geq 0$$

1. Changing the Objective Function and Constraint Function

$$Z_{min=}51.482.400X_1 + 83.155.200X_2 + 79.387.200X_3 + 70.879.200X_4$$
$$440X_1 + 1408X_2 + 1408X_3 + 1408X_4 \geq 4673$$
$$704X_1 + 0X_2 + 0X_3 + 0X_4 \geq 705$$
$$1940X_1 + 1940X_2 + 1940X_3 + 1940X_4 \geq 9771$$
$$880X_1 + 880X_2 + 880X_3 + 880X_4 \geq 3527$$
$$0X_1 + 176X_2 + 0X_3 + 0X_4 \geq 177$$
$$0X_1 + 0X_2 + 176X_3 + 0X_4 \geq 177$$
$$0X_1 + 0X_2 + 0X_3 + 176X_4 \geq 177$$
$$X_1, X_2, X_3, X_4 \geq 0$$

Transformed in canonical from by adding slack, surplus and artificial variables

$$Z_{min=}51.482.400X_1 + 83.155.200X_2 + 79.387.200X_3 + 70.879.200X_4 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5 + 0S_6 + 0S_7 + MA_1 + MA_2 + MA_3 + MA_4 + MA_5 + MA_6 + MA_7$$
$$440X_1 + 1408X_2 + 1408X_3 + 1408X_4 - S_1 + A_1 \geq 4673$$
$$704X_1 + 0X_2 + 0X_3 + 0X_4 - S_2 + A_2 \geq 705$$
$$1940X_1 + 1940X_2 + 1940X_3 + 1940X_4 - S_3 + A_3 \geq 9771$$
$$880X_1 + 880X_2 + 880X_3 + 880X_4 - S_4 + A_4 \geq 3527$$
$$0X_1 + 176X_2 + 0X_3 + 0X_4 - S_5 + A_5 \geq 177$$
$$0X_1 + 0X_2 + 176X_3 + 0X_4 - S_6 + A_6 \geq 177$$
$$0X_1 + 0X_2 + 0X_3 + 176X_4 - S_7 + A_7 \geq 177$$
$$X_1, X_2, X_3, X_4, S_1, S_2, S_3, S_4, S_5, S_6, S_7, A_1, A_2, A_3, A_4, A_5, A_6, A_7 \geq 0$$

2. Organizing Equations into a Picture

	Cj	51482400	83155200	79387200	70879200	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
Cbi	BV	X1	X2	X3	X4	S1	S2	S3	S4	S5	S6	S7	A1	A2	A3	A4	A5	A6	A7	NK
M	A1	440	1408	1408	1408	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	4673
M	A2	704	0	0	0	0	-1	0	0	0	0	0	9	1	0	0	0	0	0	705
M	A3	1940	1940	1940	1940	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	9771
M	A4	880	880	880	880	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	3527
M	A5	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	177
M	A6	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	177
M	A7	0	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	177
	Zj	**3964M**	**4404M**	**4404M**	**4404M**	**-M**	**-M**	**-M**	**-M**	**-M**	**-M**	**-M**	**M**	**M**	**M**	**M**	**M**	**M**	**M**	**19207M**
	Cj-Zj	51482400-3964M	83155200-4404M	79387200-4404M	70879200-4404M	M	M	M	M	M	M	M	0	0	0	0	0	0	0	

Figure 2. Organizing equations into tables

3. Selecting Key Columns

	Cj	51482400	83155200	79387200	70879200	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
Cbi	BV	X1	X2	X3	X4	S1	S2	S3	S4	S5	S6	S7	A1	A2	A3	A4	A5	A6	A7	NK
M	A1	440	1408	1408	1408	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	4673
M	A2	704	0	0	0	0	-1	0	0	0	0	0	9	1	0	0	0	0	0	705
M	A3	1940	1940	1940	1940	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	9771
M	A4	880	880	880	880	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	3527
M	A5	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	177
M	A6	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	177
M	A7	0	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	177
	Zj	**3964M**	**4404M**	**4404M**	**4404M**	**-M**	**-M**	**-M**	**-M**	**-M**	**-M**	**-M**	**M**	**M**	**M**	**M**	**M**	**M**	**M**	**19207M**
	Cj-Zj	51482400-3964M	83155200-4404M	79387200-4404M	70879200-4404M	M	M	M	M	M	M	M	0	0	0	0	0	0	0	

Figure 3. Selecting the key column

4. Determining the key row (Index = value in column NK / value in key column)

	Cj	51482400	83155200	79387200	70879200	0	0	0	0	0	0	0	M	M	M	M	M	M	M		
Cbi	BV	X1	X2	X3	X4	S1	S2	S3	S4	S5	S6	S7	A1	A2	A3	A4	A5	A6	A7	NK	Ratio
M	A1	440	1408	1408	1408	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	4673	3.31889
M	A2	704	0	0	0	0	-1	0	0	0	0	0	9	1	0	0	0	0	0	705	#DIV/0!
M	A3	1940	1940	1940	1940	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	9771	5.0366
M	A4	880	880	880	880	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	3527	4.00795
M	A5	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	177	1.00568
M	A6	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	177	#DIV/0!
M	A7	0	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	177	#DIV/0!
	Zj	**3964M**	**4404M**	**4404M**	**4404M**	**-M**	**-M**	**-M**	**-M**	**-M**	**-M**	**-M**	**M**	**M**	**M**	**M**	**M**	**M**	**M**	**19207M**	
	Cj-Zj	51482400-3964M	83155200-4404M	79387200-4404M	70879200-4404M	M	M	M	M	M	M	M	0	0	0	0	0	0	0		

Figure 4. Selecting the key row

5. Updating the values in the key row (Pivot row=key row/key number)

	Cj	51482400	83155200	79387200	70879200	0	0	0	0	0	0	0	M	M	M	M	M	M	M		
Cbi	BV	X1	X2	X3	X4	S1	S2	S3	S4	S5	S6	S7	A1	A2	A3	A4	A5	A6	A7	NK	Ratio
M	A1	440	1408	1408	1408	-1	0	0	0	0	0	0	1	0	0	0	0	0	0	4673	3.31889
M	A2	704	0	0	0	0	-1	0	0	0	0	0	9	1	0	0	0	0	0	705	#DIV/0!
M	A3	1940	1940	1940	1940	0	0	-1	0	0	0	0	0	0	1	0	0	0	0	9771	5.0366
M	A4	880	880	880	880	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	3527	4.00795
83155200	X2	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	0	177	1.00568
M	A6	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	177	#DIV/0!
M	A7	0	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	177	#DIV/0!
	Zj	3964M	4404M	4404M	4404M	-M	-M	-M	-M	-M	-M	-M	M	M	M	M	M	M	M	19207M	
	Cj-Zj	51482400-3964M	83155200-4404M	79387200-4404M	70879200-4404M	M	M	M	M	M	M	M	0	0	0	0	0	0	0		

Figure 5. Updating the values in the key row

6. Updating values other than key rows (BB=BL-C(NBBK))

	Cj	51482400	83155200	79387200	70879200	0	0	0	0	0	0	0	M	M	M	M	M	M	M		
Cbi	BV	X1	X2	X3	X4	S1	S2	S3	S4	S5	S6	S7	A1	A2	A3	A4	A5	A6	A7	NK	Ratio
M	A1	440	0	1408	1408	-1	0	0	0	8	0	0	1	0	0	0	-8	0	0	3257	#DIV/0!
M	A2	704	0	0	0	0	-1	0	0	0	0	0	9	1	0	0	0	0	0	705	#DIV/0!
M	A3	1940	0	1940	1940	0	0	-1	0	11.02272727	0	0	0	0	1	0	-11.02272727	0	0	7819.977273	#DIV/0!
M	A4	880	0	880	880	0	0	0	-1	5	0	0	0	0	0	1	-5	0	0	2642	#DIV/0!
83155200	X2	0	1	0	0	0	0	0	0	-0.005681818	0	0	0	0	0	0	0.005681818	0	0	177	177
M	A6	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	0	177	#DIV/0!
M	A7	0	0	0	176	0	0	0	0	0	0	-1	0	0	0	0	0	0	1	177	#DIV/0!
	Zj	51482400+3964M	83155200+4404M	79387200+4404M	70879200+4404M	-M	-M	-M	-M	-472321.536+24.02M	-M	-M	M	M	M	M	472321.536-24.02M	M	M	14718470400+14778M	
	Cj-Zj	51482400-3964M	4404M	79387200-4404M	70879200-4404M	M	M	M	M	472321.536-23.02M	M	M	0	0	0	0	-472321.536+24.02M	0	0		

Figure 6. Updating values other than the key row

Iteration continued with POM-QM software, resulting in 11 iterations to obtain the optimal solution, with details of the calculation shown below:

Linear Programming Results

PRODUKSI DODOL PULUT

	X1	X2	X3	X4		RHS
Minimize	51482400	83155200	79387200	70879200		
Gula Pasir	440	1408	1408	1408	>=	4673
Gula Aren	704	0	0	0	>=	705
Kelapa	1940	1940	1940	1940	>=	9771
Tepung Pulut	880	880	880	880	>=	3527
Durian	0	176	0	0	>=	177
Wijen	0	0	176	0	>=	177
Pandan	0	0	0	176	>=	177
Solution->	2.0196	1.0057	1.0057	1.0057		338719300

Variable	Status	Value
X1	Basic	2.0196
X2	Basic	1.0057
X3	Basic	1.0057
X4	Basic	1.0057
surplus 1	Basic	463.6031
surplus 2	Basic	716.765
surplus 3	NONBasic	0
surplus 4	Basic	905.2062
surplus 5	NONBasic	0
surplus 6	NONBasic	0
surplus 7	NONBasic	0
Optimal Value (Z)		338719300

Figure 7. Calculation Results with POM-QM

Then the optimal solution is obtained with the following variable values:

$$X_1 = 2.0196;\ X_2 = 1.0057;\ X_3 = 1.0057;\ X_4 = 1.0057$$

Then the value of Z_{min} = IDR 338,719,300

Conclusion

The application of the Probabilistic Back Order Policy Model using the Simplex Linear Programming Method at *Dodol Pulut Raya* optimizes raw material inventory, reduces annual production costs from IDR 509,941,040 to IDR 338,719,300 (savings of IDR 171,221,740), while also improving efficiency and profitability. This study serves as a practical reference for other SMEs in inventory policies under uncertain demand, with opportunities for enhancement through real-time information systems to improve forecasting and stock management. Therefore, further research is recommended to add sensitivity analysis to test capital resilience to changes in variables, so that the results obtained are more optimal.

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