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# Optimization of Potato and Carrot Production in Karo District Using the Beale Method

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**Abstract**

This study aims to optimize land allocation for potato and carrot production in Karo Regency, North Sumatra, using a quadratic programming approach using the Beale method. Secondary data from the Karo Regency Agriculture Office for 2018-2024 are used to construct a quadratic objective function using the least squares method. The Beale method is applied iteratively to solve optimization problems with linear constraints. The results show that the optimal land allocation of 0.5382 hectares for carrots and 0.4005 hectares for potatoes provides a maximum yield of 1.6239 tons. These findings demonstrate the effectiveness of the mathematical optimization approach in improving land use efficiency and supporting data driven decision making in the horticultural agricultural sector.

*Keywords: Optimization, Quadratic Programming, Beale Method, Potatoes, Carrots*
*MSC2020: 65K10, 90C20*

***Abstrak***

*Penelitian ini bertujuan untuk mengoptimalkan alokasi lahan produksi kentang dan wortel di Kabupaten Karo, Sumatera Utara, dengan pendekatan pemrograman kuadratik menggunakan metode Beale. Data sekunder dari Dinas Pertanian Kabupaten Karo tahun 2018-2024 digunakan untuk membentuk fungsi objektif kuadratik melalui metode kuadrat terkecil. Metode Beale diterapkan secara iteratif untuk menyelesaikan permasalahan optimisasi dengan kendala linear. Hasil penelitian menunjukkan bahwa alokasi lahan optimal sebesar 0.5382 hektar untuk wortel dan 0.4005 hektar untuk kentang memberikan hasil panen maksimum sebesar 1.6239 ton. Temuan ini menunjukkan efektivitas pendekatan optimisasi matematis dalam meningkatkan efisiensi penggunaan lahan dan mendukung pengambilan keputusan berbasis data dalam sektor pertanian hortikultura.*

*Kata kunci: Optimisasi, Pemrograman Kuadratik, Metode Beale, Kentang, Wortel*
*MSC2020: 65K10, 90C20*

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## Introduction

The agricultural sector plays a strategic role in economic development and food security, especially in developing countries like Indonesian[1]. Horticultural commodities such as potatoes (Solanum tuberosum L.) and carrots (Daucus carota L.) have high added value due to their nutritional content and stable market demand[2], [3]. Based on data from the Central Statistics Agency (2023),

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national potato production reaches more than 1 million tons per year, while carrots are around 600 thousand tons, with a significant contribution from the North Sumatra region[4], [5].

Unlike previous studies, which generally focus on descriptive analysis or conventional optimization techniques, this research applies the Beale method within a quadratic programming framework to determine the optimal allocation of land for potato and carrot cultivation in Karo Regency. This approach not only enhances precision in resource allocation but also offers a reproducible decision support model that can be adapted for other horticultural commodities. Therefore, the study contributes both methodologically by demonstrating the practical application of the Beale method in agricultural optimization and empirically by providing region specific recommendations that address local production constraints and market potential.

Karo Regency is one of the leading areas in potato and carrot production due to its fertile volcanic soil conditions and supportive highland climate[6]. Data from the Karo Regency Agriculture Service (2024) shows an increasing trend in harvested area and production from 2018 to 2024, but this has not been matched by increased efficiency in land allocation[7], [8]. This indicates the need for a quantitative approach to maximize productivity[9].

In this context, quadratic programming is one of the effective mathematical optimization methods for handling nonlinear objective functions with linear constraints[10], [11]. According to Nocedal and Wright (2000), quadratic programming is widely used in agricultural economics, financial portfolios, and industrial planning because it is able to provide optimal solution in complex system[12]. Beale method, which is an interactive method for solving constrained quadratic programming, was chosen in this study because of its efficiency in finding local optimum points that satisfy the constraints conditions[13], [14].

This study aims to develop a mathematical model representing the relationship between land area potato and carrot yields, and to identify optimal land allocation that yield maximum yield. Using historical production data and the Beale method, it is hoped that the results of this study will contribute to data driven decision making at the farmer and policy making levels, as well as support more productive and sustainable agricultural practices in Karo Regency.

## Methods

This study uses secondary data from the Karo Regency Agriculture Service for 2018-2024, covering land area potato and carrot production yields. The basic assumptions used are: (1) the relationship between land area and production yield is nonlinear, (2) the land is considered homogeneous, and (3) there are no significant external influences during the study period.

The model used is quadratic programming, because the relationship between input (land) and output (production) is not linear. The objective function was formed using the least squares method based on historical data, resulting in two quadratic production function for potatoes and carrots. The model constraints were the maximum planting are limits, namely 35,042 ha for carrots and 28,694 ha for potatoes.

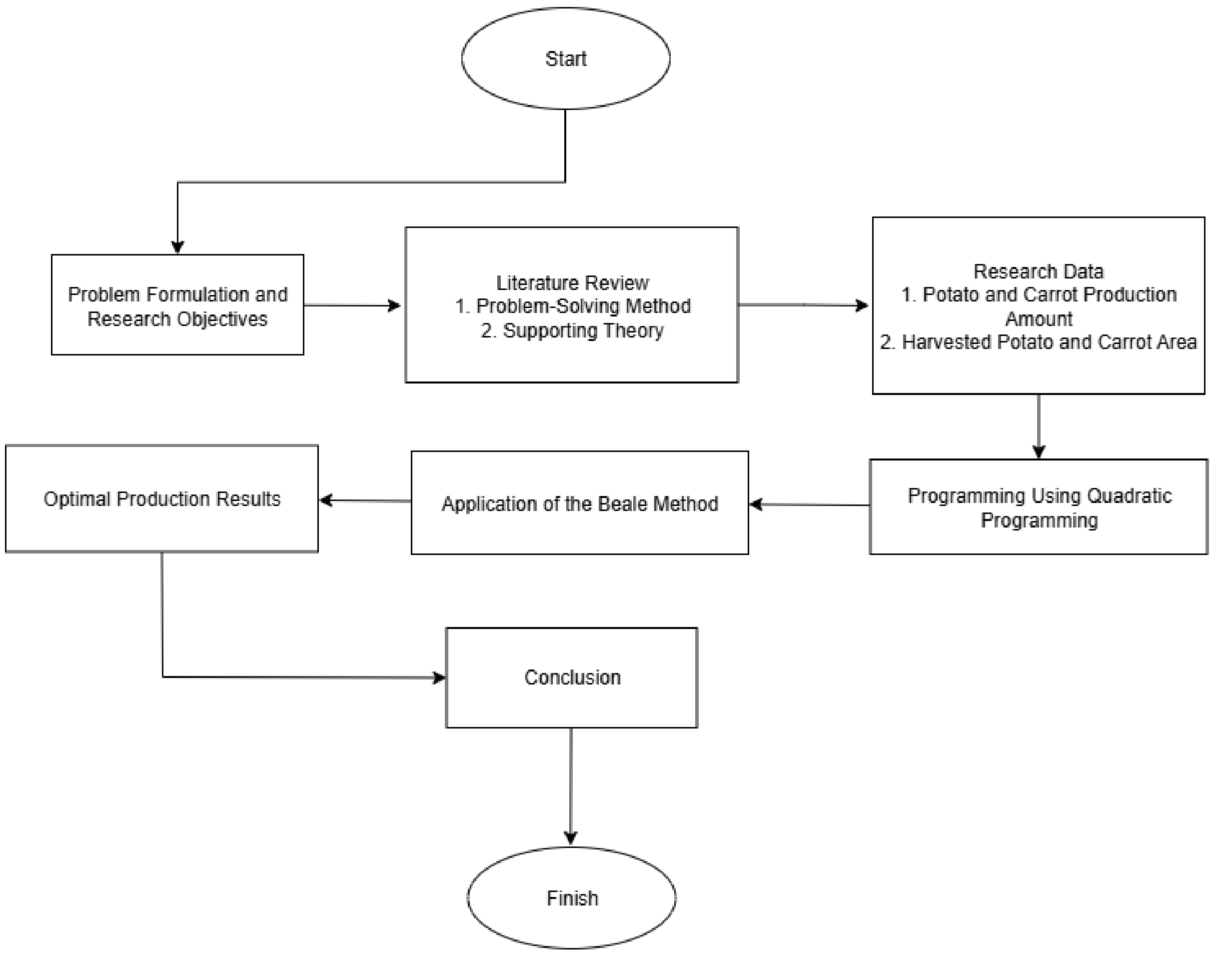


**Figure 1.** Research Procedure

**Data Collection**

The data used was obtained from the Karo Regency Agriculture Service, which includes data on harvested land area potato and carrot production for 2018-2024.

**Table 1 [7][8].** Data on Harvested Area and Average Production of Carrots and Potatoes for 2018-2024.

| No | Year | Carrot | | Potato | |
|---|---|---|---|---|---|
| | | Harvested Land Area | Production (tons) | Harvested Land Area | Production (tons) |
| 1 | 2018 | 2369 | 51209 | 3306 | 57413 |
| 2 | 2019 | 3853 | 91992 | 3953 | 72308 |
| 3 | 2020 | 3883 | 93247 | 3676 | 70368 |
| 4 | 2021 | 5210 | 133825 | 4568 | 96691 |
| 5 | 2022 | 5817 | 142079 | 4597 | 97385 |
| 6 | 2023 | 6158 | 136080 | 4170 | 84491 |
| 7 | 2024 | 7752 | 163576 | 4424 | 82244 |

**Forming a Mathematical Model**

A Quadratic Programming model was developed, with an objective function representing the relationship between land area and crop yield. The least square method was used to estimate the coefficient of the quadratic function [11, 15, 17, 18].

**Beale Method**

The Beale method is an extension of the simplex method in linear programming. This method is used to solve the problem of minimizing a convex quadratic function with the linear function constraints.

$$Q(x) = Q(x_1, x_2, \dots, x_n) \tag{1}$$

with constraint

$$Ax = b \; dan \; x \geq 0 \tag{2}$$

where:

Q(x) = convex quadratic function

A = matrix of size $m \times n$

m = number of constraint equation

n = number of objective function variables,

and $m \leq n$

Beale method is and iterative optimization technique, applied to solve Quadratic Programming models that do not satisfy the Karush Khun Tucker conditions[14], [15].

**Constraints**

The model is constrained by the maximum land available for potatoes (28,694 ha) and carrots (35,042 ha), as set by office of agriculture.

**Optimization**

The process of achieving optimal results under certain conditions, the ultimate goal of this activity is to minimize effort or maximize results. The Beale method is used repeatedly to determine the optimal land allocation that maximizes production, ensuring that the total land area does not exceed the available area[12], [16].

## Results and Discussion

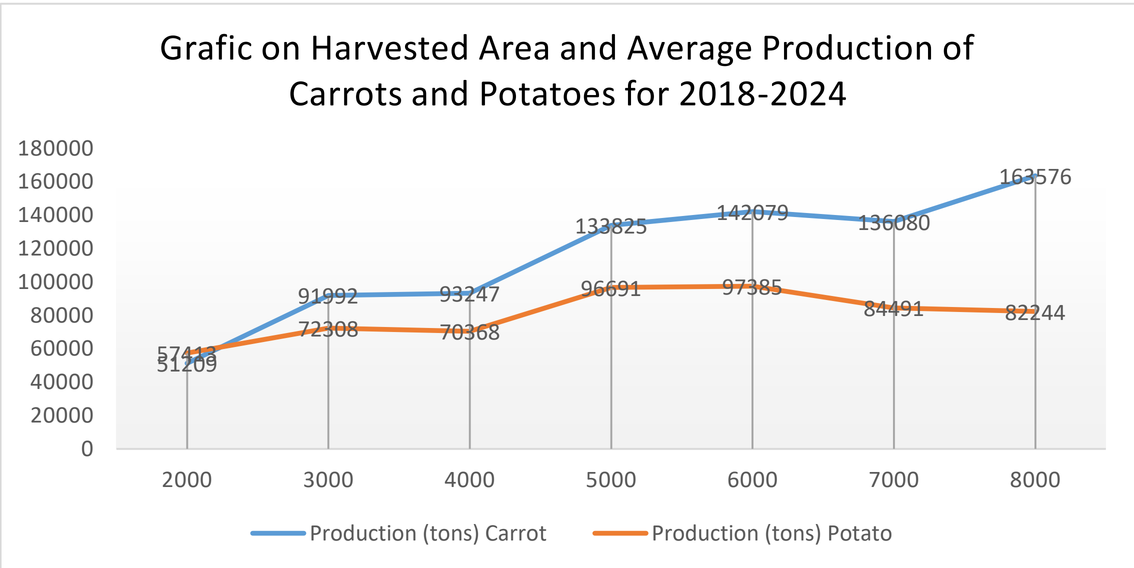


**Figure 2.** Graph of Land Area vs. Production Yield

Based on Figure 2, the analysis results show that the most significant factor affecting production trends is the fluctuation in harvested area, where an increase in harvested area generally leads to higher production, particularly for carrots. However, for potatoes, production tends to decline despite variations in harvested area, indicating possible constraints such as soil conditions, crop diseases, or inefficient farming practices that limit yield growth over time.

**Forming a Mathematical Model for Average Carrot and Potato Production**

In the objective function of this problem, the average production is defined as the agricultural output per hectare. which is measured by mass in tons. The harvested area is estimated as the sum of the plants harvested after reaching sufficient age are also included in the category of failed harvests. Because it is not possible to calculate each plant one by one, the total plants are considered the same as the area of plant land calculated in hectares.

The average overall production is a combination of the average yields of carrots and potatoes. Thus, the objective function to be compiled can be formulated as the sum of the average yields of carrots and potatoes.

$$S(x,y) = S(x) + S(y) = \beta_1 x_i^2 + \beta_2 y_i^2 + \beta_3 x_i + \beta_4 y_i + \beta_5 \tag{3}$$

The variables used are as follows:

$x_i$ = Harvested area of carrots in the i – th years, in heactares (ha)

$y_i$= Harvested area of potatoes in the i – th years, in heactares (ha)

$S_i$= Average production data in the i – th years, in tons

$i = 1,2,3, \dots, n$; n = number of data points

$B_j$= Objective function parameters

It is known that the average production function of carrots and potatoes is

$$S(x,y) = S(x) + S(y) = \beta_1 x_i^2 + \beta_2 x_i + \beta_3 \tag{4}$$

$$S(x,y) = S(x) + S(y) = \beta_1 y_i^2 + \beta_2 y_i + \beta_3 \tag{5}$$

To determine the parameters of the objective function, the least squares method is used, namely by solving the following system of linear equations:

$$A\beta = Y \tag{6}$$

$$A'\beta = Y \tag{7}$$

where

$$A = \begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & n \end{bmatrix} \text{ dan } A' = \begin{bmatrix} \sum y_i^4 & \sum y_i^3 & \sum y_i^2 \\ \sum y_i^3 & \sum y_i^2 & \sum y_i \\ \sum y_i^2 & \sum y_i & n \end{bmatrix}$$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}, Y = \begin{bmatrix} \sum x_i^2 S_i \\ \sum x_i S_i \\ \sum S_i \end{bmatrix} \text{ dan } Y' = \begin{bmatrix} \sum y_i^2 S_i \\ \sum y_i S_i \\ \sum S_i \end{bmatrix}$$

Solution $\beta$ from the equation (4) and (5) is obtained by

$$\beta = (A)^{-1} Y \tag{8}$$

$$\beta = (A')^{-1} Y \tag{9}$$

The system of 3rd order linear equations can be solved analytically. If solved analytically, then by using Cramer's rule, namely by finding the parameter or constant solution $\beta_1, \beta_2, \text{ dan } \beta_3$. If Ax=b is a system of n linear equations with n variables such that $\det(A) \neq 0$ ,then the solution is

$$x_1 = \frac{\det(A_1)}{\det(A)}, x_2 = \frac{\det(A_2)}{\det(A)}, \dots, x_n = \frac{\det(A_n)}{\det(A)}$$

1. Finding det(A) and det(A')

$$\det(A) = \begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & n \end{bmatrix} = 7{,}822E + 21$$

$$\det(A') = \begin{bmatrix} \sum y_i^4 & \sum y_i^3 & \sum y_i^2 \\ \sum y_i^3 & \sum y_i^2 & \sum y_i \\ \sum y_i^2 & \sum y_i & n \end{bmatrix} = 1{,}9197E + 18$$

2. Finding the parameters of the objective function of carrots and potatoes
   Due to $\det(A) \neq 0$, then the parameters $\beta_1, \beta_2$ dan $\beta_3$ can be searched in the following

$$\sum x_i^2 S_i = 1{,}5796E + 14$$

$$\sum x_i S_i = 2{,}8454E + 10$$

$$\sum S_i = 812008$$

$$\beta_1 = \frac{\det(A_1)}{\det(A)}$$

$$\beta_1 = \frac{\begin{bmatrix} \sum x_i^2 S_i & \sum x_i^3 & \sum x_i^2 \\ \sum x_i S_i & \sum x_i^2 & \sum x_i \\ \sum S_i & \sum x_i & n \end{bmatrix}}{\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & n \end{bmatrix}} = -1{,}8931(1{,}0E + 10)$$

$$\beta_2 = \frac{\det(A_2)}{\det(A)}$$

$$\beta_2 = \frac{\begin{bmatrix} \sum x_i^4 & \sum x_i^2 S_i & \sum x_i^2 \\ \sum x_i^3 & \sum x_i S_i & \sum x_i \\ \sum x_i^2 & \sum S_i & n \end{bmatrix}}{\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & n \end{bmatrix}} = 2{,}0377(1{,}0E + 10)$$

$$\beta_3 = \frac{\det(A_3)}{\det(A)}$$

$$\beta_2 = \frac{\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 S_i \\ \sum x_i^3 & \sum x_i^2 & \sum x_i S_i \\ \sum x_i^2 & \sum x_i & \sum S_i \end{bmatrix}}{\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & n \end{bmatrix}} = 4{,}9283(1{,}0E + 10)$$

The purpose of carrots is to

$$S(x) = -1{,}8931x^2 + 2{,}0377x + 4{,}9283 \tag{10}$$

$$\sum y_i^2\, S_i = 6{,}6763E + 13$$

$$\sum y_i S_i = 1{,}6094E + 10$$

$$\sum S_i = 560900$$

$$\beta_1 = \frac{\det(A_1)}{\det(A')}$$

$$\beta_1 = \frac{\begin{bmatrix} \sum y_i^2\, S_i & \sum y_i^3 & \sum y_i^2 \\ \sum y_i S_i & \sum y_i^2 & \sum y_i \\ \sum S_i & \sum y_i & n \end{bmatrix}}{\begin{bmatrix} \sum y_i^4 & \sum y_i^3 & \sum y_i^2 \\ \sum y_i^3 & \sum y_i^2 & \sum y_i \\ \sum y_i^2 & \sum y_i & n \end{bmatrix}} = -2{,}7086(1{,}0E + 10)$$

$$\beta_2 = \frac{\det(A_2)}{\det(A')}$$

$$\beta_2 = \frac{\begin{bmatrix} \sum y_i^4 & \sum y_i^2\, S_i & \sum y_i^2 \\ \sum y_i^3 & \sum y_i S_i & \sum y_i \\ \sum y_i^2 & \sum S_i & n \end{bmatrix}}{\begin{bmatrix} \sum y_i^4 & \sum y_i^3 & \sum y_i^2 \\ \sum y_i^3 & \sum y_i^2 & \sum y_i \\ \sum y_i^2 & \sum y_i & n \end{bmatrix}} = 2{,}1695(1{,}0E + 10)$$

$$\beta_3 = \frac{\det(A_3)}{\det(A')}$$

$$\beta_3 = \frac{\begin{bmatrix} \sum y_i^4 & \sum y_i^3 & \sum y_i^2\, S_i \\ \sum y_i^3 & \sum y_i^2 & \sum y_i S_i \\ \sum y_i^2 & \sum y_i & \sum S_i \end{bmatrix}}{\begin{bmatrix} \sum y_i^4 & \sum y_i^3 & \sum y_i^2 \\ \sum y_i^3 & \sum y_i^2 & \sum y_i \\ \sum y_i^2 & \sum y_i & n \end{bmatrix}} = -4{,}2871(1{,}0E + 10)$$

The purpose of the potatoes function is

$$S(y) = -2{,}7086y^2 + 2{,}1695y - 4{,}2871 \tag{11}$$

The average sum of these two function is

$$S(x, y) = -1{,}8931x^2 - 2{,}7086y^2 + 2{,}0377x + 2{,}1695y + 0{,}6412 \tag{12}$$

**Forming Constraint Functions**

Based on from the data from the Karo Regency Agriculture Service, the planting area from 2018-2024 for carrots, the maximum planting area is 35,042 ha and potatoes are 28,694 ha. So that the constraints obtained

$$x \leq 5006 \tag{13}$$

$$y \leq 4099{,}1 \tag{14}$$

$$x, y \leq 0$$

**Optimization Problem Solving Using Beale's Method**

1. Forming a quadratic programming model given with linear constraints by adding slack or surplus variables.
   Maximize
$$S(x,y) = -1{,}8931x^2 - 2{,}7086y^2 + 2{,}0377x + 2{,}1695y + 0{,}6412 \tag{15}$$
   Subject to :
$$x + S_1 \leq 5006 \tag{16}$$
$$y + S_2 \leq 4099{,}1 \tag{17}$$
$$x, y, S_1, S_2 \geq 0 \tag{18}$$
2. Choose any a basic variable and a non-basic variable.
$$X_B = \{s_1, s_2\}$$
$$X_{NB} = \{x, y\}$$
   1) Iteration 1
$$S_1 = 5006 - x \tag{19}$$
$$S_2 = 4099{,}1 - y \tag{20}$$
$$S(x,y) = -1{,}8931x^2 - 2{,}7086y^2 + 2{,}0377x + 2{,}1695y + 0{,}6412 \tag{21}$$
$$\frac{\partial f}{\partial x} = -3{,}7862x + 2{,}0377$$
$$\frac{\partial f}{\partial y} = -5{,}4172y + 2{,}1695$$
      with,
$$X_{NB} = \{x, y\} = 0 \text{ ,so}$$
$$\frac{\partial f}{\partial x} = -3{,}7862(0) + 2{,}0377$$
$$\frac{\partial f}{\partial x} = 2{,}0377, and$$
$$\frac{\partial f}{\partial y} = -5{,}4172(0) + 2{,}1695$$
$$\frac{\partial f}{\partial y} = 2{,}1695$$
      Because $\frac{\partial f}{\partial y} = 2{,}1695$ is the largest value, so y is selected as the input variable. Determine the output variable, by finding the value of
$$min\left\{\frac{\alpha_{10}}{|\alpha_{12}|}, \frac{\alpha_{20}}{|\alpha_{22}|}, \frac{\gamma_{20}}{|\gamma_{22}|}\right\}$$
      where :
      $\alpha_{10}$ : Constant value in equation (19)
      $\alpha_{12}$ : Coefficient of the entering variable in equation (19)
      $\alpha_{20}$ : Constant value in equation (20)
      $\alpha_{22}$ : Coefficient of the entering variable in equation (20)
      $\gamma_{20}$ : Constant value from the partial derivative with respect to y
      $\gamma_{22}$ : Coefficient of the entering variable from the partial derivative with respect to y
$$min\left\{\frac{\alpha_{10}}{|\alpha_{12}|}, \frac{\alpha_{20}}{|\alpha_{22}|}, \frac{\gamma_{20}}{|\gamma_{22}|}\right\}$$
$$min\left\{\frac{5006}{|0|}, \frac{4099{,}1}{|-1|}, \frac{2{,}1695}{|-5{,}4172|}\right\} = 0{,}4005 \; corresponds\ to\ \frac{\gamma_{20}}{|\gamma_{22}|}$$
      By adding non-basic independent variable

$$u_1 = \frac{1}{2}\frac{\partial f}{\partial y}$$
$$u_1 = \frac{1}{2}(-5{,}4172y + 2{,}1695)$$
$$u_1 = -2{,}7086y + 1{,}0848 \quad (22)$$

2) Iteration 2

$$X_B = \{s_1, s_2, y\}$$
$$X_{NB} = \{x, u_1\}$$
$$y = \frac{1{,}0848 - u_1}{2{,}7086}$$
$$y = 0{,}4005 - 0{,}3691u_1 \quad (23)$$
$$s_2 = 4099{,}1 - y$$
$$s_2 = 4099{,}1 - (0{,}4005 - 0{,}3691u_1)$$
$$s_2 = 4098{,}6995 - 0{,}3691u_1 \quad (24)$$
$$s_1 = 5006 - x \quad (25)$$
$$S(x,y) = -1{,}8931x^2 - 2{,}7086y^2 + 2{,}0377x + 2{,}1695y + 0{,}6412$$
$$S(x,y) = -1{,}8931x^2 - 2{,}7086(0{,}4005 - 0{,}3691u_1)^2 + 2{,}0377x + 2{,}1695(0{,}4005 - 0{,}3691u_1) + 0{,}6412$$
$$S(x,y) = -1{,}8931x^2 + 0{,}9997u_1^2 + 2{,}0377x - 0{,}8007u_1 + 1{,}0775 \quad (26)$$
$$\frac{\partial f}{\partial x} = -3{,}7862x + 2{,}0377$$
$$\frac{\partial f}{\partial u_1} = 1{,}9994u_1 - 0{,}8007$$

with,

$$X_{NB}\{x, u_1\} = 0, so$$
$$\frac{\partial f}{\partial x} = -3{,}7862(0) + 2{,}0377$$
$$\frac{\partial f}{\partial x} = 2{,}0377, and$$
$$\frac{\partial f}{\partial u_1} = 1{,}9994(0) - 0{,}8007$$
$$\frac{\partial f}{\partial u_1} = -0{,}80007$$

Because $\frac{\partial f}{\partial x} = 2{,}0377$ is the largest value, so x is selected as the input variable. Determine the output variable, by finding the value of:

$$min\left\{\frac{\alpha_{10}}{|\alpha_{12}|}, \frac{\alpha_{20}}{|\alpha_{22}|}, \frac{\alpha_{30}}{|\alpha_{32}|}, \frac{\gamma_{10}}{|\gamma_{11}|}\right\}$$

where:

$\alpha_{10}$ : Constant value in equation (25)
$\alpha_{12}$ : Coefficient of the entering variable in equation (25)
$\alpha_{20}$ : Constant value in equation (24)
$\alpha_{22}$ : Coefficient of the entering variable in equation (24)
$\alpha_{30}$ : Constant value in equation (23)
$\alpha_{32}$ : Coefficient of the entering variable in equation (23)
$\gamma_{20}$ : Constant value from the partial derivative with respect to x
$\gamma_{22}$ : Coefficient of the entering variable from the partial derivative with respect to x

$$min\left\{\frac{\alpha_{10}}{|\alpha_{12}|},\frac{\alpha_{20}}{|\alpha_{22}|},\frac{\alpha_{30}}{|\alpha_{32}|},\frac{\gamma_{10}}{|\gamma_{11}|}\right\}$$
$$min\left\{\frac{5006}{|0|},\frac{4099{,}1}{|-1|},\frac{0{,}4005}{|0|},\frac{2{,}0377}{|-3{,}7862|}\right\} = 0{,}5381\ corresponds\ to\ \frac{\gamma_{10}}{|\gamma_{11}|}$$

By adding no-basic variables

$$u_2 = \frac{1}{2}\frac{\partial f}{\partial x}$$
$$u_1 = \frac{1}{2}(-3{,}7862x + 2{,}0377)$$
$$u_1 = -1{,}8931x + 1{,}0189 \tag{27}$$

3) Iteration 3

$$X_B = \{s_1, s_2, x, y\}$$
$$X_{NB} = \{u_1, u_2\}$$
$$x = \frac{1{,}0189 - u_2}{1{,}8931}$$
$$x = 0{,}5382 - 0{,}5282u_2 \tag{28}$$
$$s_2 = 4098{,}6995 - 0{,}3691u_1 \tag{29}$$
$$s_1 = 5006 - (0{,}5382 - 0{,}5282u_2)$$
$$s_1 = 5005{,}4618 + 0{,}5282u_2 \tag{30}$$
$$S = -1{,}8931x^2 + 0{,}9997u_1^2 + 2{,}0377x - 0{,}8007u_1 + 1{,}0775$$
$$S = -1{,}8931(0{,}5382 - 0{,}5282u_2)^2 + 0{,}9997u_1^2 + 2{,}0377(0{,}5382 - 0{,}5282u_2) - 0{,}8007u_1 + 1{,}0775$$
$$S = 0{,}9997u_1^2 + 0{,}9999u_2^2 - 0{,}8007u_1 - 1{,}0763u_2 + 1{,}637 \tag{31}$$
$$\frac{\partial f}{\partial u_1} = 0{,}9997u_1^2 - 0{,}8007u_1$$
$$\frac{\partial f}{\partial u_1} = 1{,}9994u_1 - 0{,}8007$$
$$\frac{\partial f}{\partial u_2} = 0{,}9999u_2^2 - 1{,}0763u_2$$
$$\frac{\partial f}{\partial u_2} = 1{,}9998u_2 - 1{,}0763$$

With

$$X_{NB}\{u_1, u_2\} = 0, obtained$$
$$\frac{\partial f}{\partial u_1} = -0{,}8007$$
$$\frac{\partial f}{\partial u_2} = -1{,}0763$$

Because the first partial derivative in equation (31) < 0, it can be said to be optimal. Based on these calculations, the results obtained are $s_1 = 5005{,}4618$; $s_2 = 4098{,}6995$; $y = 0{,}4005$; $x = 0{,}5382$. To obtain the maximum value sought, the x, y values are substituted into the initial objective function, namely:

$S(x, y) = -1{,}8931x^2 - 2{,}7086y^2 + 2{,}0377x + 2{,}1695y + 0{,}6412$
$S(x, y) = -1{,}8931(0{,}5382)^2 - 2{,}7086(0{,}4005)^2 + 2{,}0377(0{,}5382) + 2{,}1695(0{,}4005) + 0{,}6412$
$S(x, y) = 1{,}6239$

The optimal solution for potato and carrot harvests is the area of land harvested for carrots $x = 0,5382\ \mathrm{ha}$, area of potato harvest land $y = 0,4005\ \mathrm{ha}$ with a maximum harvest of 1,6239 ton.

## Conclusion

The model's outcome offers a practical solution for farmers in Karo Regency, suggesting that land area allocation should prioritize 0.5382 hectares for carrots and 0.4005 hectares for potatoes to achieve the highest production. This approach can help optimize the use of available land, reduce waste, and increase farm profitability. Moreover, the application of the Beale method demonstrates its potential in solving real-world agricultural optimization problems.

While the optimization model provides a clear solution for land allocation, it is important to note that real-world conditions such as market demand fluctuations, climate variability, and other external factors were not directly incorporated into this model. Future research could expand the model by including such factors to further enhance its applicability in dynamic agricultural environments. Additionally, the model assumes a constant relationship between land area and yield, which may vary due to changes in farming practices, crop varieties, or other local conditions.

Despite these limitations, the study contributes valuable insights into how mathematical optimization techniques like Quadratic Programming can be applied to agricultural planning. By offering a data-driven approach to land allocation, the research lays the foundation for more sustainable and productive agricultural practices in Karo Regency and potentially other similar regions. Incorporating sustainability principles into the optimization model by considering environmental impact and resource conservation in agricultural yield calculations.

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