

Comparative Evaluation of RK4 and ABM4 Methods for Underdamped Transient Response Analysis of RLC Circuits

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Abstract

The transient response of series RLC circuits in the underdamped regime displays oscillatory behavior that requires numerical method capable of preserving both amplitude and phase accuracy. This study aims to compare the numerical performance of two fourth-order integration methods, namely the classical Runge-Kutta method (RK4) and the Adams-Bashforth-Moulton predictor-corrector method (ABM4), in simulating the transient response of a series RLC circuit. The circuit dynamics are formulated as a system of first-order ordinary differential equations derived from Kirchhoff's Voltage Law and solved numerically using both methods. The simulations are performed using practical circuit parameters representative of low-power AC systems and zero initial conditions. Numerical simulation is evaluated in terms of computational efficiency, global accuracy measured by root mean square error relative to the analytical solution, and local accuracy assessed through peakwise relative error at the first oscillation peak. The results show that ABM4 consistently requires less computational time than RK4 for the same number of time steps due to its multi-step formulation. However, RK4 provides smaller global and peakwise errors, indicating superior phase accuracy. These results highlight a trade-off between computational efficiency and numerical accuracy in underdamped RLC transient simulations.

Keywords: Ordinary Differential Equation, numerical simulation, Runge-Kutta, Adams-Bashforth-Moulton

MSC2020: 34-04, 65L05, 65L06

Abstrak

Respons transien rangkaian RLC seri pada kondisi underdamped menunjukkan perilaku osilatori yang memerlukan metode numerik yang mampu mempertahankan akurasi amplitudo dan fasa. Penelitian ini bertujuan untuk membandingkan kinerja numerik dua metode integrasi orde empat, yaitu metode Runge-Kutta klasik (RK4) dan metode predictor-korektor Adams-Bashforth-Moulton (ABM4), dalam mensimulasikan respons transien rangkaian RLC seri. Dinamika rangkaian diformulasikan sebagai sistem persamaan diferensial biasa orde satu yang diturunkan dari Hukum Tegangan Kirchhoff dan diselesaikan secara numerik menggunakan kedua metode tersebut. Simulasi dilakukan dengan parameter rangkaian yang merepresentasikan sistem AC berdaya rendah serta kondisi awal nol. Kinerja simulasi numerik dievaluasi berdasarkan efisiensi komputasi, akurasi global yang diukur menggunakan root mean square error terhadap solusi analitik, serta akurasi lokal yang dinilai melalui galat relatif pada puncak osilasi pertama. Hasil penelitian menunjukkan bahwa ABM4 secara konsisten membutuhkan waktu komputasi lebih rendah dibandingkan RK4 untuk jumlah langkah waktu yang sama karena sifatnya sebagai metode multi-langkah. Namun, RK4 menghasilkan galat global dan galat puncak yang lebih kecil, yang mengindikasikan akurasi fasa yang lebih baik. Hasil ini

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menegaskan adanya trade-off antara efisiensi komputasi dan akurasi numerik dalam simulasi respons transien RLC underdamped. Rangkaian.

Kata kunci: Persamaan Diferensial Biasa, Simulasi Numerik, Runge-Kutta, Adams-Bashforth-Moulton
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Introduction

The combination of a resistor (R), an inductor (L), and a capacitor (C), known as RLC circuits, are fundamental components in various engineering applications such as signal processing [1, 2], design of filtering devices [3], medical instruments [4, 5], communication systems [6], etc. The dynamic behavior of such circuits is typically modeled by second-order ordinary equations (ODEs) based on Kirchhoff's Voltage Law (KVL), which often demonstrates an underdamped response when the damping factor is small. In this regime, the circuit shows oscillatory transient behavior before reaching steady state, achieving accurate numerical simulation fundamental for both teaching and design.

Analytical solutions of RLC circuits can be obtained through Laplace transform or characteristic equation [7], [8]. However, these solutions become less practical in real-world scenarios involving parameter variations, non-linear components, or other factors that complicate the analysis [9].

Previous studies compare classical explicit numerical schemes, such as Euler, Heun, and the fourth-order Runge-Kutta (RK4), for standard ODEs or for simple electrical circuits [10]–[13]. These studies show expected behavior that lower-order schemes accumulate phase and amplitude errors, while RK4 achieves high accuracy with moderate computational cost. However, existing comparative analyses rarely extend beyond these basic explicit schemes, and most do not examine their performance specifically on underdamped oscillatory systems, where phase fidelity and long-horizon stability are critical [14, 15].

In addition, higher-order and adaptive solvers, such as the fourth-order Adams-Bashforth-Moulton (ABM4) predictor-corrector family, have not been carefully evaluated in the context of RLC transient simulations. These solvers offer potential advantages in efficiency, accuracy, and step-size control, yet their comparative performance on oscillatory electrical circuits remains underexplored in the literature.

This study addresses these gaps by performing a focused comparison between three widely used higher-order numerical integrators, namely the classical fourth-order Runge-Kutta method (RK4), the fourth-order Adams-Bashforth-Moulton predictor-corrector method (ABM4). By isolating the effect of solver structure, namely single-step versus multi-step methods, on accuracy, stability, and computational efficiency when simulating the underdamped transient response of a series RLC circuit. The underdamped regime is particularly challenging because numerical methods must preserve oscillatory amplitude and phase accuracy over extended simulation times. We evaluate global accuracy (RMSE), peakwise relative error, computational cost (runtime and number of steps), and sensitivity to step-size or tolerance selection to validate the performance of each method.

Methods

Mathematical Formulation of The RLC Circuit

The series RLC circuit considered in this study, as shown in Figure 1, consists of a resistor (R), inductor (L), and Capacitor (C), connected in series and driven by a sinusoidal voltage source $V(t) = V_0 \sin \omega t$. Applying Krichhoff's Voltage Law (KVL) to the circuit produces the second-order differential equation (ODE):

$$L \frac{d^2 q(t)}{dt} + R \frac{dq(t)}{dt} + \frac{1}{C} q(t) = V_0 \sin \omega t \quad (1)$$

where:

- $q(t)$ is the charge on the capacitor at time t
- R is the resistance in ohm (Ω)
- L is the inductance in Henries (H)
- C is the capacitance in farads (F)
- V_0 is the amplitude of the input voltage
- ω is the angular frequency.

The equation (1) can be transformed into a system of first-order ODEs by introducing the current $i(t) = dq(t)/dt$. Thus, the system becomes:

$$\begin{cases} \frac{dq(t)}{dt} = i(t) \\ \frac{di(t)}{dt} = \frac{1}{L} \left(V_0 \sin \omega t - Ri(t) + \frac{1}{C} q(t) \right) \end{cases} \quad (2)$$

This system of equations establishes the basis for applying explicit numerical methods, such as Euler, Heun, and RK, which will be deployed to compute the time evolution of the current and charge in the circuits.

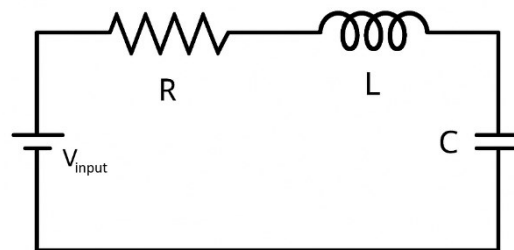


Figure 1. A simple circuit consists of a resistor (R), Inductor (L), and capacitor (C) connected in series, driven by an input voltage.

Numerical Method

This study considers three higher-order numerical schemes commonly used for the time integration of ordinary differential equations as mentioned above. These methods were selected to represent three distinct solver structures which are single-step fixed-step, multi-step fixed-step, and adaptive step-size control, while maintaining comparable order of accuracy. Equation (2) can be written in the vector form below:

$$\mathbf{y}(t) = \begin{bmatrix} q(t) \\ i(t) \end{bmatrix} \quad (3)$$

$$\mathbf{y}'(t) = \mathbf{f}(t, \mathbf{y}) = \begin{bmatrix} i(t) \\ \frac{1}{L} \left(V_0 \sin \omega t - Ri(t) + \frac{1}{C} q(t) \right) \end{bmatrix} \quad (4)$$

for time $t_n = t_0 + n\Delta t$ and $\mathbf{y}_n \approx \mathbf{y}(t_n)$. The explicit scheme for mentioned methods is derived as follows.

Fourth-order Runge-Kutta (RK4)

The classical fourth-order Runge-Kutta method (RK4) offers a stability between accuracy and computational cost. For the system introduced in equation (2), with state vector (3) and (4), the RK4 update from $\mathbf{y}_n \approx \mathbf{y}(t_n)$ to $\mathbf{y}_{n+1}$ is calculated as follows:

$$k_1 = \mathbf{f}(t_n, \mathbf{y}_n) \quad (5)$$

$$k_2 = \mathbf{f}\left(t_n + \frac{\Delta t}{2}, \mathbf{y}_n + \frac{\Delta t}{2} k_1\right) \quad (6)$$

$$k_3 = \mathbf{f}\left(t_n + \frac{\Delta t}{2}, \mathbf{y}_n + \frac{\Delta t}{2} k_2\right) \quad (7)$$

$$k_4 = \mathbf{f}(t_n + \Delta t, \mathbf{y}_n + \Delta t k_3) \quad (8)$$

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{\Delta t}{6} (k_1 + 2k_2 + 2k_3 + k_4) \quad (9)$$

where each k_j for $j = 1, 2, 3, 4$ contains two elements corresponding to $\frac{dq}{dt} = i$ and $\frac{di}{dt} = \frac{1}{L} (V_0 \sin \omega t - Ri + 1/C q)$. By combining multiple intermediate slopes, RK4 produces results that closely follow the true oscillatory behavior of the underdamped RLC circuit while maintaining stability even for relatively larger time steps.

Fourth-order Adams-Bashforth-Moulton Method (ABM4)

The ABM4 is a multi-step predictor-corrector scheme that combines an explicit Adams-Bashforth predictor with an implicit Adams-Moulton corrector. In this study, the fourth-order Adams-Bashforth method is used to predict the solution, followed by a fourth-order Adams-Moulton correction.

The predictor step is given by

$$\tilde{\mathbf{y}}_{n+1} = \mathbf{y}_n + \frac{\Delta t}{24} (55\mathbf{f}_n - 59\mathbf{f}_{n-1} + 37\mathbf{f}_{n-2} - 9\mathbf{f}_{n-3}), \quad (10)$$

while the corrector step is

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{\Delta t}{24} (9\mathbf{f}_{n+1} + 19\mathbf{f}_n - 5\mathbf{f}_{n-1} + \mathbf{f}_{n-2}), \quad (11)$$

where $\mathbf{f}_k = \mathbf{f}(t_k, \mathbf{y}_k)$.

Because ABM4 is a multi-step method, initial values for the first three steps are required. These starting values are generated using the RK4 method with the same step size. ABM4 reuses previously computed derivatives, which can improve computational efficiency for smooth solutions such as underdamped oscillatory responses.

Numerical Set-Up

The numerical methods were evaluated by simulating a RLC circuit with the following parameters:

Table 1. Circuit parameters for numerical simulation

Parameter	Symbol	Value	Unit
Resistance	R	10	Ω
Inductance	L	100	mH
Capacitance	C	100	μF
Input Voltage	$V(t)$	$10 \sin 1000t$	V
Initial charge	$q(0)$	0	C
Initial current	$i(0)$	0	A
Simulation time	T	0.1	s

These values are chosen to represent a practical low-power AC circuit operating in the kilohertz range, which is common in laboratory and educational settings. The initial conditions setting correspond to an uncharged capacitor and zero initial current through the inductor. The simulation runs for $T = 0.1 s$, which covers several oscillation cycles of the circuit response and allows the transient behavior to be observed clearly.

To investigate computational efficiency, particular attention is given to the relationship between number of time steps and computational time, following established practices in numerical method comparison studies. For the fixed-step methods (RK4 and ABM4), the simulation is repeated by varying the total number of time steps

$$N = \{1000, 3000, 5000, 7000, 10000\},$$

with the corresponding step size defined as $\Delta t = T/N$. This approach allows direct observation of how computational cost scales with increasing temporal resolution.

Because ABM4 is a multi-step method, the first three steps are initialized using RK4 with the same step size Δt . This start-up cost is included in the reported computational time to ensure a fair comparison.

Performance is assessed in terms of computational efficiency (computational time versus number of time steps), global accuracy measured by RMSE with respect to analytical solution, peakwise relative error at the first oscillation, and sensitivity to step size. This evaluation framework enables a consistent and transparent comparison of single-step, multi-step, and adaptive fourth-order integration strategies for underdamped RLC transient simulations. The RMSE is defined as

$$RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^N (y_n^{Num} - y_n^{An})^2} \quad (12)$$

and Relative Error, highlights the local accuracy at specific point (at $t = \text{first peak}$), expressed as:

$$\text{Peak Relative Error } (RE_{peak}) = \frac{|y_n^{Num} - y_n^{An}|}{y_n^{An}} \times 100\% \quad (13)$$

Analytical Solution

To provide a reference for precision assessment, the analytical solution of the RLC circuit is derived from (1). By applying the Laplace transform or solving the characteristic equation, the general

solution of the second-order ODE can be expressed as the sum of the transient and steady-state components. For the underdamped case $\left(\frac{R}{2L} < \sqrt{\frac{1}{LC}}\right)$, the current response is written as:

$$i(t) = Ae^{-\alpha t} \sin(\omega_d t + \phi) + i_{ss}(t), \quad (14)$$

where

$$\alpha = \frac{R}{2L}, \omega_d = \sqrt{\frac{1}{LC} - \alpha^2}, \quad (15)$$

and $i_{ss}(t)$ is the steady-state sinusoidal current determined by the input source. The transient part represents the natural oscillation with exponential decay, while the steady-state part follows the frequency of the excitation signal. This analytical solution is well-documented in standard circuit analysis literature [5, 11] and is used as the analytical solution y^{An} . The numerical solutions, y^{Num} , obtained from Euler, Heun, and RK4 are compared point-by-point with this analytical response to compute the RMSE and relative errors defined in previous section.

Results and Discussion

Comparison of Computational Cost

Figure 2 shows the relationship between computational time and the number of time steps for the two fixed-step numerical methods, namely RK4 and ABM4. For both methods, the number of time steps is given explicitly, allowing a direct and fair comparison of computational efficiency under identical temporal resolutions. As shown in Figure 2, the computational time increases monotonically with the number of time steps for both RK4 and ABM4, which indicates that the computational time increases linearly with the number of time steps. Using a finer time step increases the number of computations and results in longer runtime.

For the same number of time steps, ABM4 consistently needs less computational time than RK4. This efficiency advantage can be credited to the multi-step predictor-corrector of ABM4, because it reuses information from previous steps and requires fewer repeated computations than the single-step RK4 method. In contrast, RK4 is a single-step method that recomputes four intermediate slopes at every time step, resulting in higher per-step computational overhead.

Overall, these results indicate that while both methods exhibit numerical computational cost growth with increasing time resolution, ABM4 provides a more computationally efficient compared to RK4 for the same fixed-step configuration without compromising numerical accuracy.

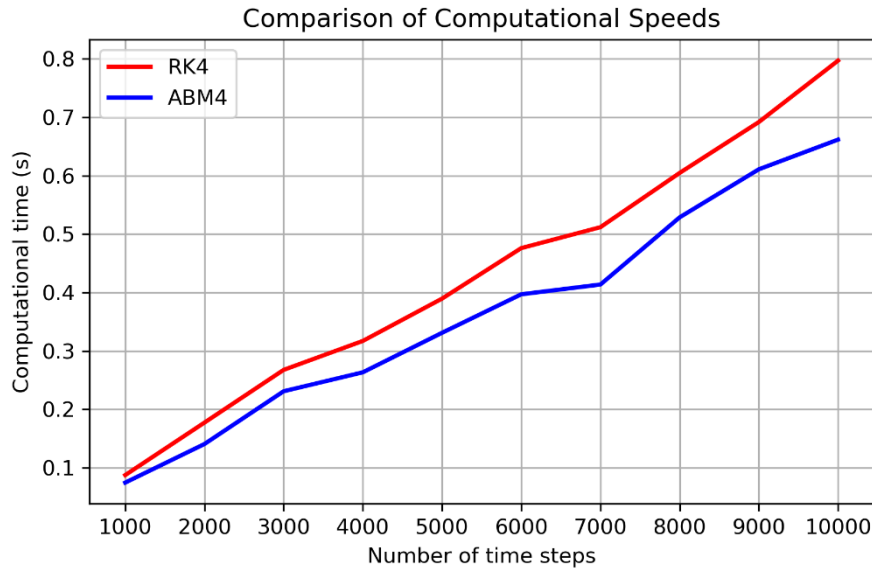


Figure 2. Comparison of computational speeds of the RK4 and ABM4 methods for different numbers of time steps.

Comparison of Global Accuracy (RMSE)

The global accuracy of the numerical solutions is evaluated using the Root Mean Square Error (RMSE) with respect to the analytical reference solution. Figure 3 shows the RMSE as a function of the number of time steps for the RK4 and ABM4 methods. For both RK4 and ABM4, the RMSE consistently reduces as the number of time steps increases. This indicates that refining time discretization increases the overall accuracy of the numerical solutions. For larger time steps, both methods yield relatively large errors that decrease as the step size is reduced. Conversely, using large number of steps results in very small RMSE values, indicating close agreement between the numerical and analytical solutions.

In general, RK4 gives smaller RMSE values than ABM4 for the same number of time steps, especially when the time step is large. As the number of time steps increases, the difference in accuracy becomes smaller, and both methods produce very small RMSE values. Overall, increasing the number of time steps improves the accuracy of both methods, with RK4 being slightly more accurate and ABM4 achieving comparable accuracy at finer resolutions.

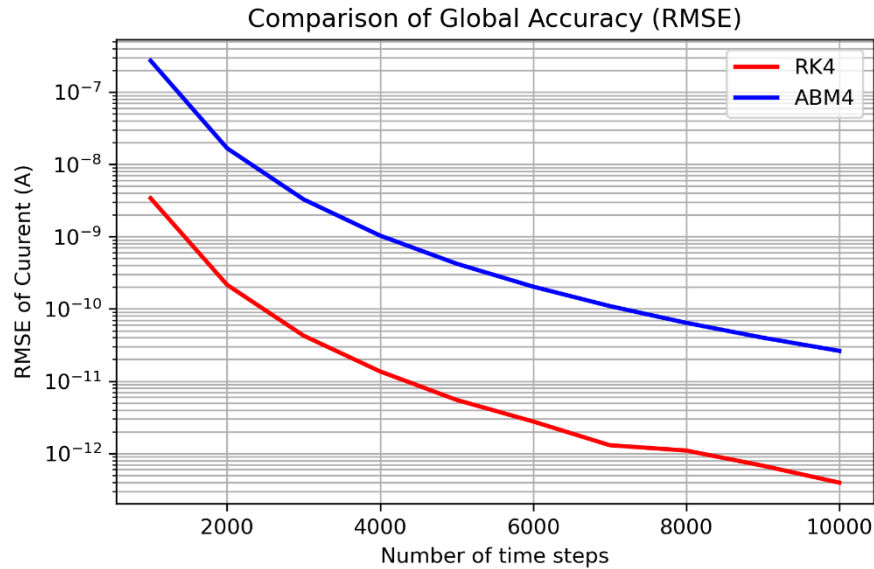


Figure 3. RMSE versus number of time steps for RK4 and ABM4 methods (logarithmic scale on the vertical axis).

Peakwise Relative Error

The peakwise relative error, to further evaluate the numerical performance, is assessed at the first positive peak of the current response. This error measure focuses on the accuracy of the numerical solution at a particular point of the transient response, where small numerical inaccuracies can lead to noticeable deviations in the peak magnitude.

Figure 4 presents a comparison of the peakwise relative error for the RK4 and ABM4 methods. The left plot illustrates the current response around the first peak for $N = 5000$, highlighting the reference peak obtained from the analytical solution. The plot shows the peakwise relative error as a function of the number of time steps.

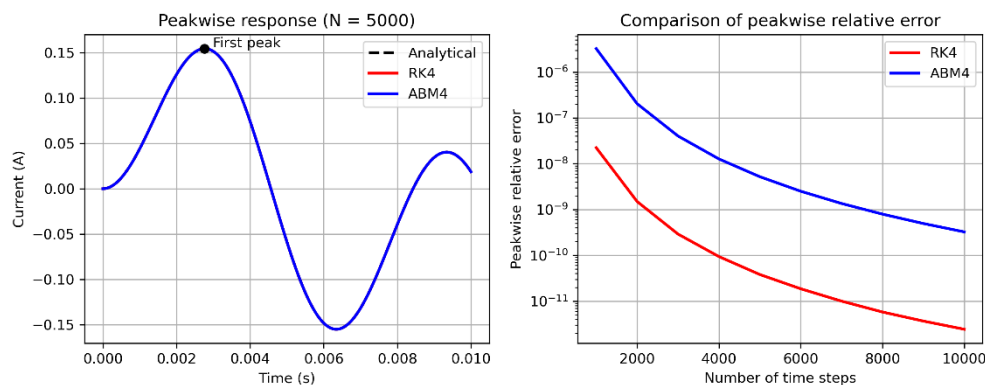


Figure 4. (Left) Current response around the first peak for $N = 5000$. (Right) Peakwise RE as a function of the number of time steps for RK4 and ABM4

For both RK4 and ABM4, the peakwise relative error decreases as the number of time steps increases. This indicates that refining the time discretization improves the accuracy of the numerical solution at the first peak. At smaller time steps, the peakwise error is relatively large, conversely, larger time steps become very small.

A comparison between the two methods shows that RK4 consistently produces lower peakwise relative error than ABM4 for the same number of time steps. However, the peakwise error of ABM4 also decreases steadily as the number of time steps increases, and the difference between the two methods becomes smaller at finer time discretization. In conclusion, the results demonstrate that increasing the number of time steps leads to improved accuracy at the first peak for both fixed-step methods, with RK4 providing slightly higher accuracy than ABM4 at sufficiently fine discretization.

Summary of Numerical Performance

Table 2 summarizes the computational time, RMSE, and peakwise relative error for representative numbers of time steps. The numerical values confirm the trends observed in the graphical results. ABM4 attains lower computational time than RK4, while RK4 provides smaller RMSE and peakwise relative error across all tested resolutions. This table highlights the trade-off between computational efficiency and numerical accuracy fundamental in the choice between single-step and multi-step fourth-order integration methods.

Table 2. Summary of computational time, RMSE, and peakwise relative error for RK4 and ABM4

N	Method	Comp.time (s)	RMSE	Peakwise RE
1000	RK4	0.08754	3.406E-09	2.231E-08
	ABM4	0.07466	2.720E-07	3.249E-06
3000	RK4	0.26758	4.266E-11	2.907E-10
	ABM4	0.23099	3.275E-09	4.020E-08
5000	RK4	0.38952	5.511E-12	3.808E-11
	ABM4	0.33104	4.221E-10	5.198E-09
7000	RK4	0.51181	1.302E-12	9.983E-12
	ABM4	0.41365	1.096E-10	1.352E-09
10000	RK4	0.79681	3.956E-13	2.430E-12
	ABM4	0.66163	2.627E-11	3.243E-10

Discussion

The numerical results demonstrate a clear understanding between computational efficiency and accuracy in the simulation of underdamped RLC transient responses. ABM4 offers excellent computational efficiency due to its multi-step predictor-corrector structure, making it well suited for simulations requiring large numbers of time steps or long integration intervals. In contrast, RK4 demonstrates higher accuracy, particularly in terms of global RMSE and peakwise relative error, because of its strong local error control as a single-step method.

In oscillatory systems such as underdamped RLC circuits, phase accuracy plays a crucial role in preserving transient dynamics. The peakwise relative error analysis reveals that RK4 maintains better phase than ABM4, especially at moderate time resolutions. This property is particularly important for applications where accurate prediction of transient peaks is critical, such as resonance analysis, signal processing, and educational demonstrations of circuit dynamics.

From a practical perspective, the choice of numerical method should depend on the specific objectives of the simulation. RK4 is more appropriate for short-time transient analysis and applications where high phase accuracy is required, while ABM4 provides a computationally efficient alternative

for long-duration simulations with sufficiently fine time discretization. These results highlight that no single method is universally superior, but the optimal solver depends on the balance between accuracy requirements and computational efficiency.

Conclusion

This study performed a comparative evaluation of the fourth-order Runge-Kutta method (RK4) and the Adams-Bashforth-Moulton predictor-corrector method (ABM4) for simulating the underdamped transient response of a series RLC circuit. The comparison was based on computational efficiency, global accuracy measured by RMSE, and local accuracy evaluated through peakwise relative error at the first oscillation peak. The results show that ABM4 consistently requires less computational time than RK4 for the same number of time steps due to its multi-step structure, while RK4 achieves higher numerical accuracy across all tested temporal resolutions.

RK4 demonstrates superior phase fidelity, as indicated by its smaller peakwise relative error, which is critical for accurately capturing oscillatory transient behavior in underdamped systems. Although ABM4 exhibits slightly larger peakwise errors, its accuracy increases steadily with finer time discretization and becomes comparable to RK4 at sufficiently small step sizes. Overall, the findings indicate that RK4 is more suitable for short-time transient analysis and applications requiring high phase accuracy, whereas ABM4 provides a computationally efficient alternative for long-duration simulations with fine time resolution. These results offer practical guidance for selecting appropriate numerical solvers in oscillatory electrical circuit analysis.

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