Modeling a Wave on Mild Sloping Bottom Topography and Its Dispersion Relation Approximation

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Abstract

Linear wave theory is a simple theory that researchers and engineers often use to study a wave in deep, intermediate, and shallow water regions. Many researchers mostly used it over the horizontal flat seabed, but in actual conditions, sloping seabed always exists, although mild. In this research, we try to model a wave over a mild sloping seabed by linear wave theory and analyze the influence of the seabed's slope on the solution of the model. The model is constructed from Laplace and Bernoulli equations together with kinematic and dynamic boundary conditions. We used the result of the analytical solution to find the relation between propagation speed, wavelength, and bed slope through the dispersion relation. Because of the difference in fluid dispersive character for each water region, we also determined dispersion relation approximation by modifying the hyperbolic tangent form into hyperbolic sine-cosine and exponential form, then approximated it with Padé approximant. As the final result, exponential form modification with Padé approximant had the best agreement to exact dispersion relation equation then direct hyperbolic tangent form.
Keywords: linear wave, mild sloping seabed, dispersion relation approximation, velocity potential

Introduction

The study of waves in oceanic and coastal waters has fascinated many researchers until nowadays. Numerous physical phenomena can be observed along the foreshore and seashore, such as shoreline movement[1], breakwater[2], tsunami, harbor seiches, wave run-up, and many others. These can help the engineer in designing harbors and modeling coastal areas.

Those studies have employed many mathematical models, which depend on fluid characteristics, bottom topography, and some forces involved. Natural conditions can be modeled by a nonlinear partial differential equation but remain difficult to solve analytically or numerically[3]. Many researchers try to make the most straightforward way by linearizing the model[4]–[6] but can keep representing the natural physical characteristic. The fluid is often assumed to be an ideal fluid that is inviscid, incompressible, and irrotational. Many models have been developed to study this ideal fluid, and linear wave theory is one of them. For about 150 years, it has been the basic theory for oceanic waves [7]. It is developed for surface gravity waves. It is also known as the Airy wave or small amplitude wave theory. The equations in this wave theory are relatively simple, but we can use them for wave and coastal studies. Some research using linear wave theory can be found in [8]–[10]. It is also used to plan the bottom protection of the shore [11]. Other researchers have used it for many cases, but most discuss a wave over horizontal flatbed bathymetry. Actually, in natural conditions, a sloping seabed always exists even though it is mild [12].

In this research, we try to govern a model for a wave in a mild sloping bed using linear wave theory, then analyze the influence of the bed's slope on the solution. Next, we try to determine the relationship between propagation speed, wavelength, and bed slope through dispersion relation. Because of the fluid's dispersive character, we also determine an approximation to the dispersion relation for all water regions. Many different approaches were studied by researchers, such as the Padé approximation by Hunt, the Taylor expansion approach by Nielsen, and two different root-finding methods by You [12]. Padé approximation is often used, for example, to find an approximation to Green's function [13]. So, in this research, we try to modify the dispersion relation with other transcendental functions before applying the Padé approximation.

Methods

The method used in this research is the descriptive method through literature, supported by an analytical study. We use linear wave theory to conduct the equation with an ideal fluid assumption. In this theory, Laplace and Bernoulli equations are employed. This analytical result is used to study dispersion relation and its approximation with the Padé approximation. Padé approximation is a good approximation. It is a polynomial approximation that is governed by Taylor series expansion. Padé approximation is governed by Taylor series expansion. It denotes $P_{M,N}(x)$, where M and N are the highest degrees of numerator and denominator polynomial terms. It is expressed in the equation below.

$$P_{M,N}(x) = \frac{\sum_{i=0}^{M} A_i x^i}{\sum_{j=0}^{N} B_j x^j} \tag{1}$$

We can find all the coefficients A_i and B_j of the Padé approximant of the given power series [14], such as equaling $P_{M,N}(x)$ with Taylor series expansion.

Results and Discussion

1. Problem Formulation

In this section, we govern the equations for a wave on a mild sloping bed illustrated in Figure 1, using linear wave theory. Laplace and Bernoulli equations are employed for continuity and momentum equations. Assuming that the fluid is ideal, which has incompressible and inviscid character, the bottom topography is impermeable and has a mild slope. Also assumed that the motion of the fluid is irrotational. For the continuity equation, we use the following Laplace equation.

$$\nabla \cdot \mathbf{u} = \nabla \cdot (\nabla\phi) = \nabla^2\phi = 0 \tag{2}$$

where $\mathbf{u} = \langle u, w\rangle$ denotes velocity vector, and ϕ is velocity potential function $\phi = \phi(x, z, t)$.A velocity potential function is a mathematical form from the irrotational motion assumption [15]. In the governing momentum equation, we consider the following Bernoulli equation for unsteady flow.

$$\frac{\partial\vec{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \frac{\nabla p}{\rho} + g\nabla\vec{z} = 0$$

where p is pressure, ρ is density, and g is the gravitational acceleration.Substituting $\nabla\phi = \vec{u}$ into (2) and integrating it with ∇, we will get the Bernoulli equation below

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi \cdot \nabla\phi + \frac{p}{\rho} + g\vec{z} = C(t) \tag{3}$$

where $C(t)$ is an arbitrary function of time t. We keep (3) and go forward to find boundary conditions. We have to consider kinematic and dynamic conditions at the fluid's bottom and surface.

Let $h(x)$ is water depth from steel water level to mild slope bottom, then $h(x) = h_0 + \beta x$ where β is its slope. The kinematic bottom boundary condition is considered in the relation between the fluid's motion with the fluid velocity at the boundary [16]. At $z = -h(x)$, the kinematic bottom boundary can be expressed by the following equation.

$$h_t + w + uh_x + vh_y = 0 \tag{4}$$

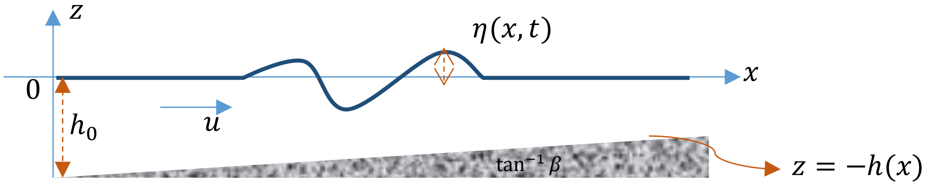

Figure 1. The illustration of a wave on a mild sloping bed

Because of an impermeable and mild slope bottom topography, the normal velocity must be zero. Defining $\nabla_h \equiv \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j}$ makes (4) become the Bottom Boundary Condition (BBC) as follows

$$\nabla_h\phi \cdot \nabla_h h = -\phi_z, \quad at\ z = -h(x) \tag{5}$$

Now consider the kinematic surface boundary condition at $z = \eta(x,t)$. Because a particle at the surface remains at the surface, we will get the equation below for Kinematic Free Surface Boundary Condition (KFSBC)

$$w - \frac{\partial\eta}{\partial t} - u\frac{\partial\eta}{\partial x} - v\frac{\partial\eta}{\partial y} = 0 \tag{6}$$

In the term of the velocity potential equation, (6) will become

$$\eta_t + \nabla_h\phi\nabla_h\eta = \phi_z, \quad at\ z = \eta \tag{7}$$

For DynamicFree Surface Boundary Condition (DFSBC), we get it from (3). It is related to the stress forces at the boundary [10]. Remember that the pressure is constant at the surface. Hence, along the surface, $\sigma_1 = \sigma_2$. See the illustration in Figure 2. So, the pressure along the surface must be equal to the atmospheric pressure$p_0 = p_0(t)$. Because $\nabla_h p \ll 1$ along the L,applying (3) at the surface, we get

$$\frac{p}{\rho} + g\eta + \frac{1}{2}\nabla\phi\nabla\phi + \phi_t = C(t) = \frac{p_0(t)}{\rho}, \quad at\ z = \eta$$

$$g\eta + \frac{1}{2}\nabla\phi\nabla\phi + \phi_t = 0, \quad at\ z = \eta \tag{8}$$

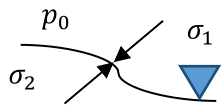

Figure 2. Pressure at the surface

In order to linearize Equations(2),(3) and BCs (5),(7), and (8), we remove nonlinear terms and justify linearization. We define the non dimensional variables as follows.

$$x' = kx \to x = 1/kx'$$
$$z' = kz \to z = 1/kz' \tag{9}$$
$$t' = \frac{1}{T}t = \sqrt{gk}\, t \to t = Tt' = \frac{1}{\sqrt{gk}}t'$$
$$\eta' = \frac{1}{a}\eta \to \eta = a\eta' \tag{10}$$
$$u' = \frac{u}{a/T} \to u = \frac{a}{T}u' = \frac{a}{T}\phi'_x$$
$$\phi' = \frac{\phi}{a/Tk} \to \phi = \frac{a}{Tk}\phi' \tag{11}$$

where $k = \frac{2\pi}{L}$ is wave number, T is wave period and a is the amplitude of the wave.

Plugging (11) into (2) and (5) will show the following equation for Laplace equation.

$$\phi'_{x'x'} + \phi'_{y'y'} + \phi'_{z'z'} = 0, \quad -kh \le z' \le ka\eta'$$
$$\nabla_{h'}\phi' \cdot \nabla_{h'}h' = -\phi'_{z'} \quad at\ z' = -h'$$

Substituting (9),(10),(11) into (7) we will get

$$\sqrt{gk}\, a\eta'_{t'} + \left(\frac{a}{Tk}\right)k^2 a\nabla_{h'}\phi' \cdot \nabla_{h'}\eta' = \frac{a}{Tk}k\phi'_{z'}$$
$$\eta'_{t'} + ka\nabla_{h'}\phi' \cdot \nabla_{h'}\eta' = \phi'_{z'} \quad at\ z' = ka\eta' \tag{12}$$

Let $\epsilon = ka$, (12) becomes

$$\eta'_{t'} + \epsilon\, \nabla_{h'}\phi' \cdot \nabla_{h'}\eta' = \phi'_{z'}, \qquad at\ z' = \epsilon\eta'$$

Substituting (9),(10),(11) into (8) we will get

$$ga\eta' + \frac{a}{k}\left(\sqrt{gk}\right)^2\phi'_{t'} + \frac{1}{2}\frac{a^2}{k^2}(gk)k^2(\nabla'\phi' \cdot \nabla'\phi') = 0$$
$$\eta' + \phi'_{t'} + \frac{1}{2}\epsilon(\nabla'\phi' \cdot \nabla'\phi') = 0, \quad at\ z' = \epsilon\eta'$$

Here, we look into $\epsilon = ka = \frac{2\pi}{L}a$. For example $L = 100\ m \to \epsilon \approx 0{,}01$

Take ϵ as $\epsilon \to 0$, then we get

$$\nabla'^2\phi' = 0, \qquad -kh \le z' \le\ ka\eta'$$
$$\nabla_{h'}\phi' \cdot \nabla_{h'}h' = -\phi'_{z'} \quad at\ z' = -h'$$
$$\eta'_{t'} = \phi'_{z'}, \qquad z' = ka\eta' \tag{13}$$
$$\eta' + \phi'_{t'} = 0, \qquad z' = ka\eta' \tag{14}$$

Take $\frac{\partial}{\partial t'}$ of (14), then

$$\eta'^{\prime}_{t'} + \phi'_{t't'} = 0 \tag{15}$$

Substituting (15) into (13) will lead us to the following equation

$$\phi'_{z'} + \phi'_{t't'} = 0$$

Finally, we put back the dimensions into the equations and the boundary condition as follows.

$$\nabla^2\phi = 0, \qquad -h \le z \le \eta \tag{16}$$

$$\phi_z + \nabla\phi \cdot \nabla h = 0, \qquad z = -h(x) \tag{17}$$

$$\phi_{tt} + g\,\phi_z = 0, \qquad z = \eta \tag{18}$$

$$\eta = -\frac{1}{g}\phi_t, \qquad z = \eta \tag{19}$$

The equation set above consists of continuity equation (16), kinematic bottom boundary (17), kinematic free surface boundary (18), and dynamic free surface boundary (19).

2. Analytical Solution

We solve (16)-(19) using the separable variable. Let the solution for velocity potential is in the form of

$$\phi(x,z,t) = F(z,h)A(x)e^{i\omega t} \tag{20}$$

where $F(z,h)$ is a weak function of $h. A(x)$ is a function of the x, and ω is the angular frequency. Under the assumption of a small slope in the bottom boundary, $h(x)$ can be expressed as $h(x) = h(\beta x), \beta \ll 1$ (small parameter). Substituting (20) into Laplace equation (1) yields

$$\frac{1}{F}\left(\frac{\partial^2 F}{\partial z^2} + \beta^2\frac{\partial^2 F}{\partial x^2}\right) = -\frac{1}{A}\frac{\partial^2 A}{\partial x^2}$$

Neglecting terms of $O(\beta^2)$ leads us to the following equation

$$\frac{1}{F}\frac{\partial^2 F}{\partial z^2} = -\frac{1}{A}\frac{\partial^2 A}{\partial x^2} = k^2 \tag{21}$$

Where k is the separation variable and is supposed to be a constant, the differential equations for F and A are to be solved. First, we will solve (21) for A by the characteristic equation.

$$A'' + Ak^2 = 0 \tag{22}$$

The solution of (22) is given by

$$A = C_1e^{ikx} + C_2e^{-ikx} \tag{23}$$

If we assume the wave is in $the\ +x$ direction only, then $C_2 = 0$ from (23). Hence, we get

$$A = C_1e^{ikx} \tag{24}$$

For F in (21), we have the bottom condition in the form

$$F'' - k^2F = 0 \tag{25}$$

For a progressive wave, we will use case: $k^2 > 0$, then the general solution of (25) is given by

$$F(z,h) = C_3\cosh k(h+z) + C_4\sinh k(h+z) \tag{26}$$

C_3 and C_4 are any constants that need to be found. Applying (20) into (17), we can get C_4 as the following equation

$$\frac{1}{F}\frac{\partial F}{\partial z} = -\frac{1}{A}\frac{\partial A}{\partial x}\frac{\partial h}{\partial x}, \quad at\ z = -h$$

$$\frac{F'(-h)}{F(-h)} = -\frac{A'}{A}h_x$$

$$\frac{kC_4}{C_3} = -h_x \frac{\left(C_1 ike^{ikx}\right)}{C_1 e^{ikx}}$$
$$C_4 = -ih_x C_3 \tag{27}$$

Let η is a progressive wave with $\eta = a\, e^{i((k_r+ik_i)x-\omega t)}$. Applying (20) into (19), we get C_3 as the following equation.

$$C_3 = \frac{ga}{\omega i} \frac{e^{-k_i x}}{(\cosh kh - ih_x \sinh kh)} \tag{28}$$

Finally, by substituting function $A(x)$(24), $F(z,h)$ (26) and some resulted coefficients (27), (28), the form of $\phi(x,z,t)$ is given by the following equation

$$\phi(x,z,t) = \frac{ga}{\omega i} \frac{(\cosh k(h+z) - ih_x \sinh k(h+z))}{(\cosh kh - ih_x \sinh kh)} e^{-k_i x} e^{i(k_r x - \omega t)}$$
$$= \frac{gae^{-k_i x}}{\omega(i\cosh kh + h_x \sinh kh)}(\cosh k(h+z) - ih_x \sinh k(h+z))\,(\cos(k_r x - \omega t) - i\sin(k_r x - \omega t)) \tag{29}$$

For the final form of velocity potential, we take the real part of (29) and substitute $h_x = \beta$ as follows:

$$\phi(x,z,t) = \frac{gae^{-k_i x}}{\omega\beta \sinh kh}(\cosh k(h+z)\cos(k_r x - \omega t) - \beta \sinh k(h+z)\sin(k_r x - \omega t))$$

3. Dispersion relation Approximation

The dispersion relation is obtained from a combination of the two free-surface conditions by substituting the representation for ϕ and the vertical structure (18) as follows.

$$-\omega^2 FAe^{-i\omega t} + gF'Ae^{-i\omega t} = 0$$
$$\omega^2 = g\frac{F'(0)}{F(0)} = gk\frac{\tanh kh - ih_x}{1 - ih_x \tanh kh} \tag{30}$$

Remember that $h(x) = h_0 + \beta x$. The resulting dispersion relation is derived from the real part of (30).

$$\omega^2 = gk\tanh(kh) \tag{31}$$

The wave number k is an important parameter that has to be calculated. Now, modify (31) as a function of kh, as the equation below

$$\frac{c^2}{gh} = \frac{\tanh(kh)}{kh} \tag{32}$$

Shallow water, intermediate, and deep water are defined as $kh \le 0.1\pi, 0.1\pi < kh < \pi$, and $kh \ge \pi$, respectively [6].

The approximations for shallow, deep water and common Padé approximant for $\frac{\tanh(kh)}{kh}$can be seen in Table 1. The result comparison of them can be seen in Figure 3. The result is good enough, but just in shallow water regions.

Table 1. Some dispersion relation approximations formula

	Approximation

Shallow water	$\frac{c^2}{gh} \approx 1$
Padé	$\frac{c^2}{gh} \approx \left(\frac{1 + \frac{1}{15}(kh)^2}{1 + \frac{2}{5}(kh)^2} \right)$
Deepwater	$\frac{c^2}{gh} \approx \frac{1}{kh}$

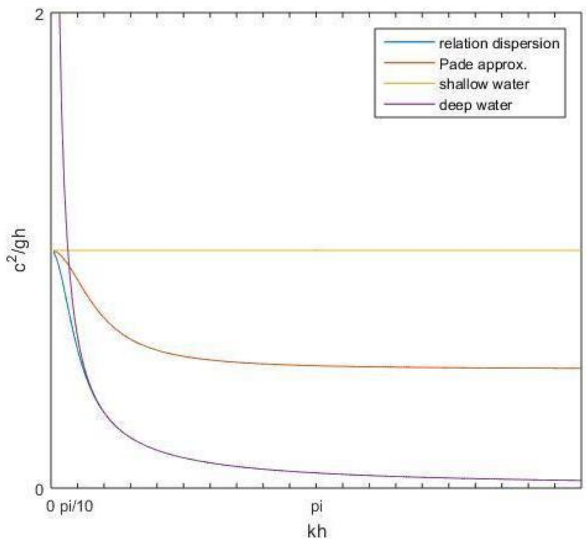

Figure 3. The plot of dispersion relation with shallow, deep water and common Padé approximant

For deep water $\tanh kh \to 1$ for $kh \to \infty$, the angular frequency will approach $\omega = \sqrt{gk}$. When phase speed $c = \frac{\omega}{k} = \frac{L}{T}$, It depends on wavelength or frequency. Hence, the wave in deep water is called a dispersive wave. Besides, for very shallow water $\tanh kh \to kh$ for $kh \to 0$, the dispersion relation will approach $\omega = k\sqrt{gh}$. The phase speed becomes $c = \sqrt{gh}$. It is not influenced by wavelength or frequency, so it is called non-dispersive.

Now, let modify (32) with characteristic equations to find other approximations with Padé approximant, see the following equations

$$\frac{c^2}{gh} = \frac{\sinh(kh)}{kh\cosh(kh)} \tag{33}$$

$$\frac{c^2}{gh} = \frac{e^{2kh} - 1}{kh(e^{2kh} + 1)} \tag{34}$$

Remember that the Taylor series expansions for $\sinh(kh)$ and $\cosh(kh)$ are the following equation

$$\sinh(kh) \ = kh + \frac{(kh)^3}{3!} + \frac{(kh)^5}{5!} + O((kh)^7) \tag{35}$$

$$\cosh(kh) \ = 1 + \frac{(kh)^2}{2!} + \frac{(kh)^4}{4!} + O((kh)^6) \tag{36}$$

$$e^{2kh} = 1 + kh + \frac{(2kh)^2}{2!} + \frac{(2kh)^3}{3!} + \frac{(2kh)^4}{4!} + \frac{(2kh)^5}{5!} + O((2kh)^6) \tag{37}$$

We use Padéapproximation in (1) and Taylor series expansion (35),(36), and (37) to get Padé approximation for $\sinh(kh)$, $\cosh(kh)$, and e^{kh}, respectively.

$$\frac{A_0 + A_1(kh) + A_2(kh)^2 + A_3(kh)^3}{1 + B_1(kh) + B_2(kh)^2} = kh + \frac{(kh)^3}{3!} + \frac{(kh)^5}{5!} \quad (38)$$

$$\frac{A_0 + A_1(kh) + A_2(kh)^2}{1 + B_1(kh) + B_2(kh)^2} = 1 + \frac{(kh)^2}{2!} + \frac{(kh)^4}{4!} \quad (39)$$

$$\frac{A_0 + A_1(kh) + A_2(kh)^2 + A_3(kh)^3}{1 + B_1(kh) + B_2(kh)^2} = 1 + kh + \frac{(2kh)^2}{2!} + \frac{(2kh)^3}{3!} + \frac{(2kh)^4}{4!} + \frac{(2kh)^5}{5!} \quad (40)$$

From (38), we got $A_0 = A_2 = B_1 = 0, A_1 = 1, A_3 = B_1 = 0, B_2 = -\frac{1}{20}$. Besides, from (39) we got $A_0 = 1, A_1 = B_1 = 0, A_2 = \frac{5}{12}, B_2 = -\frac{1}{12}$. Using all these coefficients, we can get Padé approximation to equation (33). To avoid vertical asymptote, we neglect some terms, so we had

$$\frac{c^2}{gh} \approx \frac{\left(1 + \frac{7}{60}(kh)^2\right)}{\left(1 + \frac{5}{12}(kh)^2\right)} \quad (41)$$

With some similar steps, we got $A_0 = 1, A_1 = \frac{4}{3}, A_2 = B_2 = 0, A_3 = \frac{2}{3}, B_1 = -\frac{2}{3}$ for (40). Then, we get the following Padé approximation for (34).

$$\frac{c^2}{gh} \approx \frac{\left(1 + \frac{1}{3}kh\right)}{\left(1 + \frac{1}{3}kh + \frac{1}{3}(kh)^2\right)} \quad (42)$$

As a result, we plotted the exact dispersion relation equation (32), common Padé approximation, Padé approximation (41),(42), see Figure 4. This figure shows that Padé approximation with exponential form is more accurate than common Padé approximation. Here, we found that the number of terms of the Taylor series expansion effect the Padé approximant.

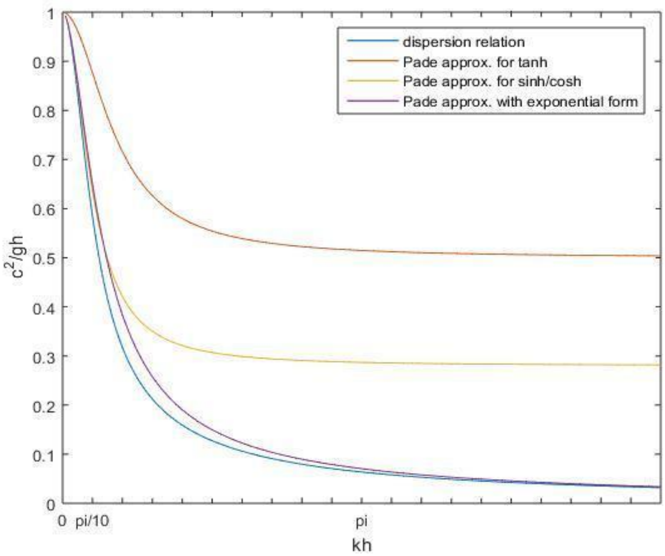

Figure 4. The plot of Padé approximation for some modified dispersion relations

Conclusion

In this research, we have governed a wave on a mild sloping beach using linear wave theory. We solve the governing equation by using the separable variable method. Under a progressive wave, we found a solution for velocity potential. As the result, the slope of the bed affects the solution. An approximation of dispersion relation has also been found for all water regions. The modified hyperbolic tangent function in dispersion relation results in a better approximation to the exact one in using the Padé approximation. Modifying hyperbolic tangent form into an exponential gave a better agreement in approximating dispersion relation using Padé approximation.

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Distribution-Based Fuzzy Time Series Markov Chain Models for forecasting Inflation in Bandung

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Abstract

This study discusses the application of the Fuzzy Time Series Markov Chain method which was developed by determining the length of the interval using the distribution method. In the fuzzy forecasting method, the determination of the length of the interval is an important thing that will affect the accuracy of the forecasting results. The development of this forecasting model aims to get better forecasting accuracy results. In this study, general inflation data for the city of Bandung is used for the period January 2016 – June 2021. The data is divided into two groups, namely in sample data and out sample data with a ratio of 90: 10. In the data processing process, the Python programming language is used. Based on the accuracy test using the MAPE method, it can be concluded that this method provides better forecasting results with a MAPE value of 1.16%.

Keywords: Fuzzy Time Series Markov Chain, Distribution Based Interval, MAPE, Inflation, Python.

Introduction

Time series forecasting according to Makridakis, et.al. [1] is a forecasting process based on historical data observations. The time series forecasting method is divided into two, forecasting techniques based on statistics and mathematical models and forecasting techniques based on artificial intelligence[2]. Conventional methods such as ARIMA require many conditions that must be met so that this method can be applied properly, while not all data can meet these conditions, therefore fuzzy forecasting techniques are a solution for this problem[3].

Fuzzy Time Series (FTS) is a forecasting concept that was first proposed by Song and Chissom in 1993 [4]. The development of fuzzy forecasting models generally focuses on achieving a high level of forecasting accuracy by increasing the three main stages, namely, fuzzification, defuzification, and fuzzy inference [5]. The development of the fuzzy method was carried out by several researchers including the development of methods by adding fuzzy logic relation tables [6], development by adding weight to the fuzzy relation process and repetition of relations [7][8], development by induction of Markov chains [9]. Determination of the length of the interval in the fuzzy method is crucial in the forecasting process because it will affect the accuracy of forecasting results. Among the methods for determining the length of the interval are intervals based on the mean and distribution, ratio, and granular computational approaches and entropy methods [1][10][11][12][13]. In several studies evaluating fuzzy forecasting models by comparing the Markov Chain method with the Chen and Weighted Markov Chain methods, from these studies the Markov Chain method gives the best results [14][15].

Fuzzy time series forecasting is widely used for solving forecasting problems. However, the fuzzy method specifically in the Markov Chain method has deficiency in determining arbitrary intervals. Some improve it with the struges formula method [15][16], the average method [17] distribution method. Based on this explanation, this study focuses on the application of the Distribution Based Fuzzy Time Series Markov Chain method in forecasting inflation.

Method

1. Fuzzy Time Series (FTS)

FTS is defined as follows : Let U be universe of discourse, where $U = \{u_1, u_2, \dots, u_n\}$, then fuzzy set A_i of u_i is defined as $A_i = \left\{\frac{f_A(u_1)}{u_1} + \frac{f_A(u_2)}{u_2} + \cdots + \frac{f_A(u_n)}{u_n}\right\}$, where $f_A(u_i)$ is the membership function u_i of A with $f_A(u_i) \in [0,1]$ and $1 \le i \le n$.

Definition 1. Let the universe of discourse $Y(t)$ with $(t = \dots, 0,1,2, \dots, n, \dots)$ is a subset of real number defined by fuzzy set A_i $(i = 1,2, \dots)$. If $F(t)$ consisted of A_i, $F(t)$ is defined as a fuzzy time series on $Y(t)$ $(t = \dots, 0,1,2, \dots, n, \dots)$.

Definition 2. Suppose $F(t)$ is caused by $F(t-1)$, then the relation between $F(t)$ and $F(t-1)$ can be expressed as $F(t) = F(t-1) \circ R(t-1, t)$ where $R(t, t-1)$ is the relation matrix to describe the fuzzy relationship between $F(t-1)$ and $F(t)$, where '' is the operator.

Definition 3. Suppose $F(t) = A_i$ is caused by $F(t-1) = A_j$ and the relation between $F(t)$ and $F(t-1)$ can be denoted $F(t-1) \to F(t)$ $(t = \dots, 0,1,2, \dots, n, \dots)$, then Fuzzy Logical Relationship (FLR) is defined as $A_i \to A_j$. Where A_i is called the current stage and A_j is the next stage.

2. Distribution Based

Distribution based is an algorithm in FTS which is used to determine the length of intervals. Distribution based length can be determined by the following algorithm [10]:

1) Calculate all the absolute differences between A_{i+1} and A_i $(i = 1, \dots, n-1)$ as the first differences and the average of the first differences.
2) According to the average, determine the base for length of intervals based by following Table 1.
3) Plot the cumulative distribution of the first differences. The base in step 2 is used as interval on the plot.
4) Choose the largest interval length that has a data value less than at least half the amount of the differences data.

Table 1. Base Mapping Table

Range	Base
0,1-1,0	0,1
1,1-10	1
11-100	10
101-1000	100

3. Distribution Based Fuzzy Time Series Markov Chain (DFTSMC)

1) Determine the universe of discourse U:

$$U = [D_{min} - B_1, D_{max} + B_2]$$

Where B_1, B_2: positive real number, D_{min} : minimum data, D_{max} : maximum data

2) Partition interval U with the length according to the distribution algorithm.
3) Define the fuzzy set for each partition interval U with the following equation:

$$A_i = \left\{ \sum_{j=1}^{n} \frac{\mu_{ij}}{u_i} \right\} \quad (1)$$

$$\mu_{ij} = \{1, 0{,}5, 0, i = j \quad j = i - 1 \; or \; i = j - 1 \; other$$

Where μ_{ij} : membership function

4) Fuzzified the data.
5) Determine Fuzzy Logical Relationship (FLR) and Fuzzy Logical Relationship Group (FLRG).
6) Determine the Markov transition probability matrix based on FLRG. The Markov transition probability matrix has dimension of $n \, x \, n$, where n is the number of fuzzy sets. The state transition probability is formulated as follows:

$$P_{ij} = \frac{M_{ij}}{M_i}, i, j = 1,2, \dots, n \quad (2)$$

Where P_{ij} : Probability of transition from state A_i to A_j, M_{ij} : Number of transitions from state A_i to A_j, M_i : Number of data included in state A_i.

The transition probability matrix **R** can be written as [9]:

$$R = [P_{11} \; P_{12} \; \cdots \; P_{1n} \; P_{21} \; P_{22} \; \dots \; P_{2n} \; \vdots \; P_{n1} \; \vdots \; P_{n2} \; \ddots \; \dots \; \vdots \; P_{nn} \;]$$

Definition 1. If $P_{ij} \geq 0$ then state A_j is accessible from state A_i.

Definition 2. If state A_i and A_j are accessible to each other, then A_i communicates to A_j.

7) Calculate the initial forecast value with the following rules:
   a. If FLRG A_i is the empty set ($A_i \rightarrow \emptyset$), then $F(t) = m_i$, where m_i is midpoint of interval U_i.
   b. If FLRG A_i is one to one relation, then $F(t) = m_k P_{ik}$, where m_k is midpoint of interval U_k.
   c. If FLRG A_i is one to many relation, where data retrieved by $Y(t-1)$ at time $(t-1)$ is in state A_j, then $F(t) = m_1 P_{j1} + m_2 P_{j2} + \cdots + m_{j-1} P_{j(j-1)} + Y(t-1) P_{jj} + m_{j+1} P_{j(j+1)} + \cdots + m_n P_{jn}$, with $m_1, m_2, \dots, m_{j-1}, m_{j+1}, \dots, m_n$ are midpoint of interval $U_1, U_2, \dots, U_{j-1}, U_{j+1}, \dots, U_n$.
8) Calculate the value of the forecast adjustment with the following rules:
   a. If state A_i communicates to A_j, with $F(t-1) = A_i$ and makes an increasing transition to state A_j at time where $(i < j)$, then $D_{t1} = \frac{l}{2}$, where l : the length of interval.
   b. If state A_i communicates to A_j, with $F(t-1) = A_i$ and makes a decreasing transition to state A_j at time t where $(i > j)$, then $D_{t1} = -\frac{l}{2}$.
   c. If transition of state A_i to $F(t-1) = A_i$ and make a jump forward transition to state A_{i+s} at time t where $(1 \leq s \leq n - i)$ then $D_{t2} = \left(\frac{l}{2}\right) s, (1 \leq s \leq n - i)$, s the number of forward jumps.
   d. If transition of state A_i to $F(t-1) = A_i$ and make a jump backward transition to state A_{i-v} at time t where $(1 \leq v \leq i)$, then $D_{t2} = -\left(\frac{l}{2}\right) v, (1 \leq v \leq i)$, v the number of backward jumps.

9) Calculate the final forecast value with the following equation:

$$F'(t) = F(t) \pm D_{t1} \pm D_{t2} \tag{3}$$

4. Forecast accuracy

Determine forecasting accuracy using MAPE with the following formula:

$$MAPE = \frac{\sum_{t=1}^{n} \left|\left(\frac{Y_t - F_t}{Y_t}\right) x100\%\right|}{n} \tag{4}$$

Where F_t : forecast value, Y_t : actual data at time t, and n : number of data.

5. Research Design

In this study, secondary data for general inflation in Bandung is used from January 2016 – June 2021. The data is divided into two categories, in sample data or data to be used in the model and out sample data to be used as forecasting accuracy calculations. In the data processing, the Python programming language is used. The flow in the study is described in Figure 1.

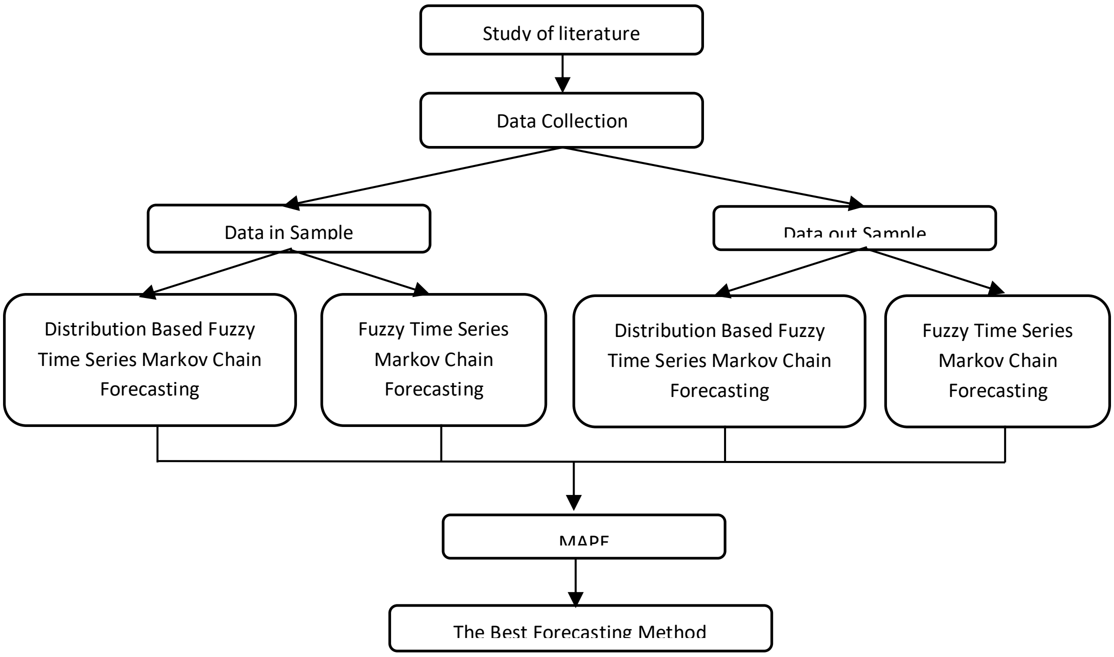

Figure 1. Research Design

Result and discussion

In this study, general inflation data in Bandung from January 2016 to December 2020 was applied as the in-sample data. The data are presented in Table 2.

Table 2. In Sample data of General Inflation in Bandung

Month	2016	2017	2018	2019	2020
January	0,53	0,49	0,83	0,09	0,38
February	-0,15	0,38	0,22	-0,08	0,35
March	0,2	-0,02	0,21	0,03	0,25
April	-0,17	0,1	0,27	0,43	0,16

Month	2016	2017	2018	2019	2020
May	0,24	0,47	0,22	0,84	-0,25
June	0,6	0,99	0,48	0,1	0,41
July	0,71	-0,27	0,17	0,55	-0,14
August	-0,49	0,06	-0,02	0,61	-0,1
September	0,14	0,11	-0,24	-0,28	-0,05
October	0,14	-0,03	0,5	-0,13	0,08
November	0,52	0,39	0,36	0,14	0,26
December	0,63	0,73	0,71	0,45	0,39

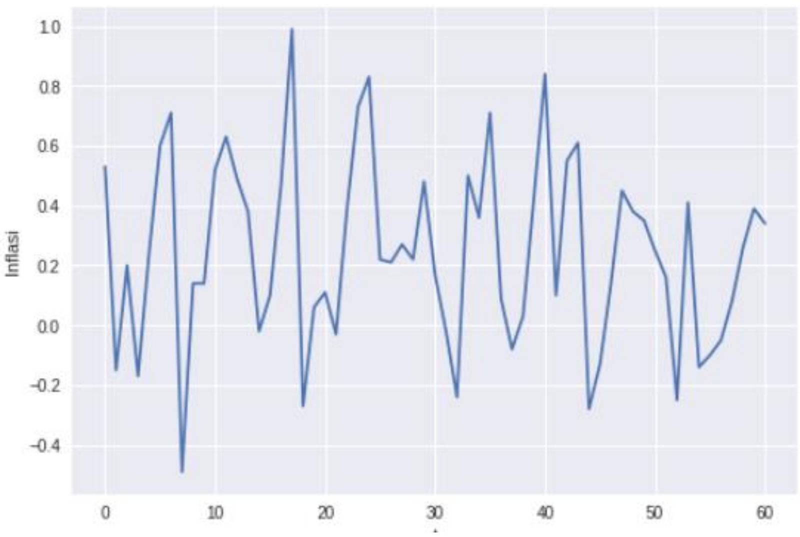

Figure 2. Training Data Plot of General Inflation in Bandung

Based on data in Table 2, the universe of discourse is $U = [D_{min} - B_1; D_{max} + B_2] = [-0,5 ; 1,1]$. Next, determine the length of interval according to distribution algorithm.

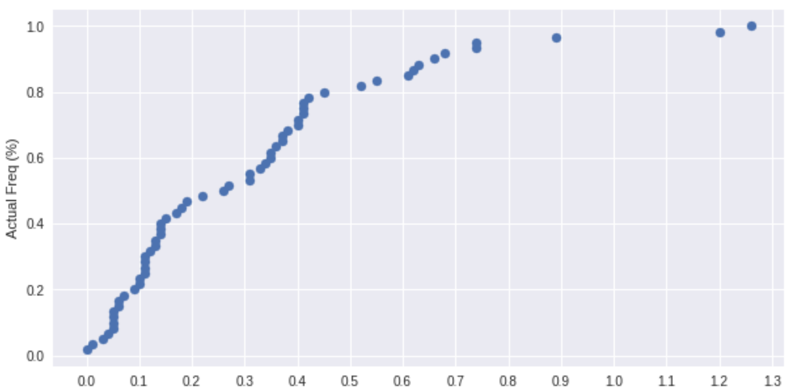

Figure 3. Cumulative Distribution Plot

Based on Figure 3, the largest interval length is 0.2. By using equation 1, the fuzzy set is obtained, then continue the process of fuzzification, FLR, and FLRG. After obtaining FLRG, a Markov transition probability matrix is made where each element is a probability value obtained from equation 2. So that a Markov transition probability matrix of order 8 x 8 is obtained as follows:

$$R = \begin{bmatrix} 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0{,}22\ 0{,}22\ 0{,}33\ 0{,}22\ 0\ 0\ 0\ 0\ 0{,}09\ 0{,}36\ 0{,}18\ 0{,}27\ 0{,}09\ 0\ 0\ 0\ 0{,}14\ 0{,}14\ 0{,}36\ 0{,}21\ 0{,}14 \\ 0\ 0\ 0\ 0{,}08\ 0{,}83\ 0{,}16\ 0{,}33\ 0\ 0{,}25\ 0{,}08\ 0\ 0{,}33\ 0\ 0\ 0{,}16\ 0{,}33\ 0{,}16\ 0\ 0{,}2\ 0\ 0{,}4\ 0{,}2\ 0\ 0\ 0{,}2\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0 \end{bmatrix}$$

The process of calculating the initial forecast, the forecast adjustment value, and the final forecast value as described in the rules in the DSMC method. The results of the in-sample data forecasting are presented in Table 3.

Table 3. In Sample Data Forecasting Result

t	Actual Data	Initial Forecast Value	Adjustment Value	Final Value
1	0,53	-	-	-
2	-0,15	0,31	-0,4	-0,09
3	0,2	0,122222222	0,2	0,322222222
…	…	…	…	…
59	0,26	0,210909091	0,1	0,310909091
60	0,39	0,235714286	0,1	0,335714286

After doing the forecasting process on the in-sample data with t = 60, a looping process will be carried out by substituting the last forecast result as data to t = 61 as much as the number of out sample data. Forecasting results are presented in Table 4.

Table 4. Out Sample Data Forecasting Result

t	Actual Data	Initial Forecast Value	Adjustment Value `	Final Value
60	0,39	0,235714286	0,1	0,335714286
61	0,34	0,372777778	0	0,372777778
62	0,43	0,347777778	0	0,347777778
63	0,4	0,392777778	0	0,392777778
64	0,44	0,377777778	0	0,377777778
65	0,43	0,397777778	0	0,397777778
66	-0,45	0,392777778	-0,4	-0,007222222

The result of the test is the MAPE value calculated based on equation 4 of the DFTSMC method compared to the MAPE value of the FTS Markov Chain method. The MAPE value of all data carried out by the DFTSMC method is 1.16%, the MAPE value of the out-sample data from the DFTSMC method is 2.2%. The MAPE value of all data carried out by the FTS Markov Chain method is 1.24% while the MAPE value of the out-sample data from the FTS Markov Chain method is 1.84%. The comparison of the two methods is presented in Figure 4.

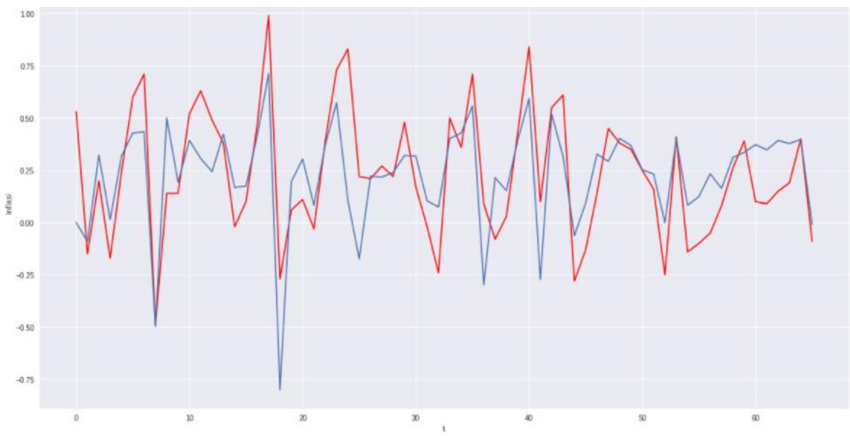

Figure 4. MAPE Plot

Based on Figure 4, forecasting using the DFTSMC method results in a data pattern following the actual data. However, the DFTSMC method produces forecasting values that tend to be constant while the original data has a fluctuating pattern.

Conclusions

Based on the results and discussion of the conclusions obtained from this study are the application of the DFTSMC method to forecasting inflation in the city of Bandung produces a data pattern that is close to the actual data pattern. The DFTSMC method has a better accuracy rate than the FTS Markov Chain with a MAPE value of 1.16%. The results of inflation forecasting in Bandung using the DFTSMC method provide the same data pattern as the previous data pattern.

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The Optimization Problem of Batik Cloth Production with Fuzzy Multi-Objective Linear Programming and Application of Branch and Bound Method

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Abstract

This study discussed fuzzy multi-objective linear programming (FMOLP) and its application. This research was conducted in Rumah Batik Mentari Jambi, which produces five batik motifs typical of the Jambi. In this research, the tolerance for additional raw material capacity is included in the model. This research aims to find out the number of tolerances needed, the maximum number of batik needed to be produced, and the minimum production time so that the producer can earn the maximum profit. The decision variables in FMOLP are the number of pieces of batik measuring in $2m^2$, which means the decision variables must be an integer. Therefore, after obtaining the optimal solution from FMOLP, then proceed with the branch and bound method to obtain the integer solution. The result of this research is that the addition of raw materials needed to earn optimal solutions is as much as 50% of the tolerance assumed in the model. Thus, owner can earn the optimal profit of Rp. 5,675,800.00/week by producing as many as 67 pieces of batik with the design of angso duo, 18 pieces with the design of gentala, and 50 pieces with the design of batang hari, and the minimum production time is 270 hours/week.

Keywords: Branch and bound, fuzzy linear programming, linear programming, multi-objective linear programming, optimization

Introduction

Along with the development of the industrial world and technological advances, operations research is increasingly being applied, especially in the production field. It can solve problems related to the production of a product. Linear Programming (LP) is a model in operations research that deals with allocating possible limited resources to obtain optimal results. In LP, the objective function is linear, and the constraints are also in the form of linear equation or inequation. One of the methods to find the optimal solution in LP is the simplex method.

A good linear programming requires precisely defined data, but the certainty, reliability, and data accuracy are hard to obtain, so the data becomes uncertain. Those uncertainties can be solved by developing a linear programming model in a fuzzy environment and becoming an alternative as a decision-making model, where the resulting value is reasonable to choose. If fuzzy environment is applied in optimization model, then the mathematical model will be Fuzzy Linear Programming (FLP) [1]. In fuzzy linear programming (FLP), the constraints, which have the form of equations or inequations, can involve fuzzy numbers for the coefficients of decision variables in the left-hand side and for constant in the right-hand side. In the FLP model, the fuzzy numbers reflect the tolerance value which is allowed and subjectively estimated for additional of raw material capacity. Usually, when additional of raw material capacity is applied, it will imply the additional time for production. Furthermore, if there is more than one objective in the FLP model, the model formed is called as Fuzzy Multi-Objective Linear Programming (FMOLP).

The related paper about FLP was written by S.H. Nasseri [2] in which discussed about a method to solve FLP problem with fuzzy coefficeint in the constraints and objective functions based on multi objetive model. El-Saeed Ammar [3] wrote paper about mathematical model for solving fuzzy integer linear programming. Then Weijie Wang et all [4] modeling the problem of global supply chain network in fuzzy multi-objective mixed integer programming which aimed to maximize the delivery quality and manufacturer's profit. Badhotiya [5] modeling the integrated production and distribution planning problem in the form of FMOLP. Tai-Sheng Su [6] used FMOLP model to solve remanufacturing planning problems with multiple products, Esra Cakir et al [7] used FMOLP approach for nuclear power plant installation. Legiani, et al [8] used FMOLP to maximize profits and minimize waste costs by adding the tolerance to constraints, and applied the integer value for decision variables. Erfianti and Muhaijir [9] used FMOLP to maximize profit, and minimize the processing time, and also applied the integer-valued optimal solution. Then, Chunquan Li [10] also wrote a paper about FMOLP in which the coefficients of decision variables in objective function and in the constraints are triangular fuzzy number. In this research, we built a mathematical model in the form of FMOLP with tow objectives, i.e. to maximize the profits and minimize the production time, in which the constant in the right-hand side in the constraint applied fuzzy number. The optimal solution of the FMOLP model is built with the simplex method. However, if the optimal solution is not integer-valued, the branch and bound method are used to obtain an integer-valued optimal solution.

Methods

1. Linear Programming (LP)

Linear programming is the most basic mechanism in OR for formulating various problems with simple efforts characterized by linear objective functions and constraints. Problems in LP can be solved with the simplex method [11][12][13][14]. Linear programming is the most basic mechanism for formulating various problems with simple efforts characterized by linear objective functions and constraints [15]. LP problems can be found in various fields and used as an appropriate alternative in decision-making so that the best solution is obtained [16].

The model formulation is the most decisive step in LP which includes the identification of matters relating to the objectives and constraints of the problem. Some basic elements in formulating the LP model are: [14][17]

1) Decision variables (X_j): variables that we seek to identify its value;
2) The objective function: a function that describes the goals or objectives in LP problems related to the optimal utilization of resources to obtain maximum profit or use minimum cost.
3) The constraint function: a formulation of the availability of resources in achieving the goal.

In general, the mathematical model of the LP problem is

$$\begin{aligned} &\text{Maximize} && f(\mathbf{x}) = \mathbf{c}^T\mathbf{x} \\ &\text{with constraints} && \\ & && A\mathbf{x} \leq \mathbf{b} \\ & && \mathbf{x} \geq \mathbf{0}, \end{aligned} \tag{1}$$

with $\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_n \end{bmatrix} \in \boldsymbol{R}^{\boldsymbol{n}}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} \in \boldsymbol{R}^{\boldsymbol{n}}, \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_m \end{bmatrix} \in \boldsymbol{R}^{\boldsymbol{m}}, A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \in \boldsymbol{R}^{\boldsymbol{m}\times\boldsymbol{n}}, \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \end{bmatrix}$
is a zeros vector in $\boldsymbol{R}^{\boldsymbol{n}}$ [18] .

2. Simplex Method

The steps in the simplex method are as follows [19], [20], [21]:

1) Identify decision variables and formulate them in mathematical symbols;
2) Identify the objective function and constraints;
3) Formulate the objective function and constraints in a mathematical model;
4) Convert the inequality in the constraint into an equation;
5) Input the coefficients on the objective and constraint functions into the simplex table. The coefficients of the objective function are written in the top row;
6) Identify the pivot column, i.e. the most negative entry in the top row;
7) Calculate the quotients which are computed by dividing the value on the right column (the value of $\mathbf{b}$), with the value in pivot column. If denominator is zero or negative, then quotient is ignored. The smallest quotients will be the pivot row;
8) State the pivot element, i.e. the number in intersection of pivot column and pivot row;
9) Make all other entries in pivot column to be zero and perform pivoting. This is just like the method of Gauss-Jordan;
10) The simplex method is finished or stopped if there are no more negative values in the top row. Otherwise, repeat the algorithm from step 6.
11) When the simplex method is considered finished, then state or identify the basic solution for optimal condition, i.e. look at the columns which have 1 and all other elements zeros. The maximum value appears in the top right hand corner.

3. Fuzzy Linear Programming (FLP)

The definitions of some of the terms used in Fuzzy Linear Programming (FLP) are as follows [1]:

Definition 1: Assume X is a set of objects, then the set containing ordered pairs

$$\bar{A} = \{(x, \mu_{\bar{A}}(x)) : x \in X\}$$

where $\mu_{\bar{A}}: X \to [0,1]$, is called the fuzzy set in X and $\mu_{\overline{A}}(x)$ is called the membership function.

Definition 2: Assume $\bar{A}$ is a fuzzy set in X and $\lambda \in [0,1]$ is a real number. Then, a set

$$\overline{A}^{\lambda} = \{x \in X : \mu_{\overline{A}}(x) \geq \lambda\}$$

is called $\lambda -$ cut of $\overline{A}$.

There are various types of FLP depend on the models and it’s solutions that have been summarized by Reza Ghanbari et all in [22]. The formation of the FLP model is derived from the classical LP model in optimization model (1). Let say the value of the objective function $f(\mathbf{x})$ in model (1) is denoted by z, and each constraint is modeled as a fuzzy set, then the FLP model for such LP problem (for maximization objective) is generally written as follows

Find $\mathbf{x}$ such that

$$\begin{aligned} \mathbf{c}^T\mathbf{x} &\gtrsim z \\ A\mathbf{x} &\lesssim \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0}, \end{aligned} \tag{2}$$

with $\mathbf{c} \in \boldsymbol{R}^{\boldsymbol{n}}, \mathbf{x} \in \boldsymbol{R}^{\boldsymbol{n}}, \mathbf{b} \in \boldsymbol{R}^{\boldsymbol{m}}, A \in \boldsymbol{R}^{\boldsymbol{m}\times\boldsymbol{n}}$ [23].

However, if model (1) has a minimization objective, then the FPL model is generally written as

Find $\mathbf{x}$ such that

$$\begin{aligned} \mathbf{c}^T\mathbf{x} &\lesssim z \\ A\mathbf{x} &\gtrsim \mathbf{b} \\ \mathbf{x} &\geq \mathbf{0}. \end{aligned}$$

Notation $\gtrsim$ is the fuzzy version of $\geq$ and implies "essentially greater than or equal to". While $\lesssim$ is the fuzzy version of $\leq$ and means "essentially less than or equal to" [24].

Model (2) can be written into a new problem, i.e. to find $\mathbf{x}$ of the model

$$\begin{aligned} B\mathbf{x} &\lesssim \mathbf{d} \\ \mathbf{x} &\geq \mathbf{0} \end{aligned} \tag{3}$$

with

$$\begin{pmatrix} -\mathbf{c}^T \\ A \end{pmatrix} = B \in \boldsymbol{R}^{(\boldsymbol{m}+\mathbf{1})\times\boldsymbol{n}}, \text{ and } \begin{pmatrix} -z \\ \mathbf{b} \end{pmatrix} = \mathbf{d} \in \boldsymbol{R}^{(m+1)\times 1} \text{ for maximization problem,}$$

or

$$\begin{pmatrix} \mathbf{c}^T \\ -A \end{pmatrix} = B \in \boldsymbol{R}^{(\boldsymbol{m}+\mathbf{1})\times\boldsymbol{n}}, \text{ and } \begin{pmatrix} z \\ -\mathbf{b} \end{pmatrix} = \mathbf{d} \in \boldsymbol{R}^{(m+1)\times 1} \text{ for the minimization problem.}$$

Thus model (3) contains $(m+1)$ rows [18]. Each inequality in model (3) is represented as a fuzzy set. The membership function of the fuzzy "decision" set of model (3) is

$$\boldsymbol{\mu_D}(\mathbf{x}) = \min_{\boldsymbol{i}}\{\boldsymbol{\mu_i}(\mathbf{x})\}.$$

$\boldsymbol{\mu_i}(\mathbf{x})$ is the membership function of the row i-th and can be interpreted as the degree to which $\mathbf{x}$ satisfies the inequality $B_{\boldsymbol{i}}\mathbf{x} \lesssim \mathbf{d_i}$, with $\boldsymbol{B_i}$ is row i-th of matrix B and $\boldsymbol{d_i}$ is row $\boldsymbol{i}$-th of vector $\boldsymbol{d}$ [18].

A solution that has the largest membership value is the best solution, so the true optimal solution will be

$$\max_{\boldsymbol{x}\geq\mathbf{0}} \boldsymbol{\mu_D}(\mathbf{x}) = \max_{\boldsymbol{x}\geq\mathbf{0}} \min_{\boldsymbol{i}}\{\boldsymbol{\mu_i}(\mathbf{x})\}. \tag{4}$$

If the boundaries/constraints and objective function are not satisfied, then $\mu_{\boldsymbol{i}}(\mathbf{x}) = 0$. Conversely, if the constraints and objective function are completely fulfilled, then $\mu_{\boldsymbol{i}}(\mathbf{x}) = 1$. One of the simple membership functions that can be used in this situation is

$$\mu_i(\boldsymbol{x}) = \begin{cases} 1 & ; \boldsymbol{B_i}\mathbf{x} \leq d_i \\ 1 - \dfrac{\boldsymbol{B_i}\mathbf{x} - d_i}{p_i} & ; d_i < \boldsymbol{B_i}\mathbf{x} \leq d_i + p_i \\ 0 & \boldsymbol{B_i}\mathbf{x} > d_i + p_i \end{cases} \tag{5}$$

for $i = 1,2,\dots,m+1$, with p_i is a constant that can be chosen subjectively from tolerable violations of the constraints and objective function [18], see Figure 1.

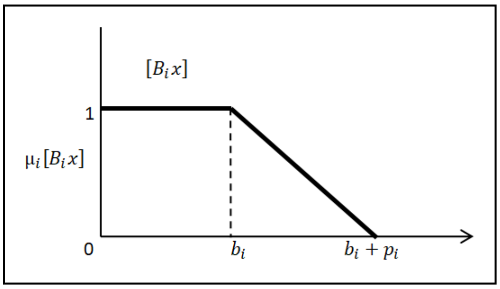

Figure 1. Illustration of Membership Function [23]

If formula (5) is substituted into (4) and with some additional assumptions, we get

$$\max_{\mathbf{x}\geq\mathbf{0}} \boldsymbol{\mu_D}(\mathbf{x}) = \max_{\mathbf{x}\geq\mathbf{0}} \min_{\boldsymbol{i}} \left\{\mathbf{1} - \frac{\boldsymbol{B_i}\mathbf{x} - \boldsymbol{d_i}}{\boldsymbol{p_i}}\right\}. \quad (6)$$

By introducing a new variable, named λ, which is basically related to equations (4), (5) and (6), i.e. by defining

$$\lambda = 1 - \frac{\boldsymbol{B_i}\mathbf{x} - d_i}{p_i} \Longrightarrow \lambda p_i = p_i - (\boldsymbol{B_i}\mathbf{x} - d_i) \Longrightarrow \lambda p_i + \boldsymbol{B_i}\mathbf{x} = d_i + p_i,$$

so that the model becomes [25]

$$\begin{array}{ll} \text{maximize} & \lambda \\ \text{with constraint} & \\ & \boldsymbol{\lambda p_i} + \boldsymbol{B_i}\mathbf{x} \leq \boldsymbol{d_i} + \boldsymbol{p_i} \\ & \mathbf{x} \geq \mathbf{0}, \end{array} \quad (7)$$

By solving the optimization problem in model (7) and assuming that the optimal solution of model (7) is $(\lambda, \mathbf{x}^*)$, then we can say that $\mathbf{x}^*$ is the optimal solution for the model (2) based on the assumption of membership function (5). Furthermore, $\lambda - cut$ can be found from equation $\lambda = 1 - t$, with $d_i + tp_i$ is the constant in right-hand side of the i^{th} constraint [18].

4. Fuzzy Multi-objective Linear Programming (FMOLP)

There are many fields of science where optimal decisions depend on two or more objectives, so that they apply multiobjective optimization. An optimization problem which involves more than one objective functions are called a multiobjective optimization problem [26]. Fuzzy multi-objective linear programming (FMOLP) is an optimization method that has more than one objective function subject to multiple constraints. The solution of an FMOLP problem can be obtained using the same method as an LP problem that has one objective function [23]. Other methods which also can be used to solve FMOLP problem are epsilon-constraint method [27], by using fuzzy dominant degrees [28], by using game theory approach [29], and the two-stage method which in the first stage, the FMOLP is transformed into the interval FMOLP, and then transformed again into the crisp multi-objective LP. Then in the second stages, the crisp multi-objective LP is converted into mono-objective program [30], and other methods as explained in [31] and [32] .

5. Branch and Bound Method

The Branch and Bound method is a well-known technique applied to solve Integer Programming problems, which is a problem that requires the decision variable to be an integer. The basic concept of this method is the branching stage and the stage of determining the boundaries for the existence of feasible solutions (bounding) [13]. Here is the algorithm of Branch and Bound method for maximization problem [14].

1) Set the initial lower bound, i.e. $z = -\infty$ and $i = 0$.
2) Fathomed/bounding stage.
   Choose a LP_i that is a sub-problem to be tested and found a solution. Next, try to achieve a fathomed condition through one of the following conditions:
   a) The value of z, which is the optimal value of LP_i, cannot produce an objective value that is better than the current lower bound value,
   b) LP_i produces an integer solution that is feasible and better than the current lower bound,
   c) LP_i has no feasible solution.

   As a result, there will be two possibilities:
   a) If LP_i fathomed and a better feasible solution is found, then the lower bound should be updated. z must be updated. If all sub-problems have reached the fathomed state, then stop branching, the lower bound is said to have provided the optimal solution. If this is not the case, then set $i = i + 1$. And repeat step 2).
   b) If L Pi is not fathomed, then go to step 3) for branching.
3) Branching stage
   Choose a variable x_j (whose constraint is integer) whose optimal value is x_j^* in the solution of the LP_i is *non integer*, such that

$$x_j \leq [\![x_j^*]\!] \text{ and } x_j \geq [\![x_j^*]\!] + 1.$$

   Then assign $i = i + 1$ and repeat step 2).

Results and Discussion

In this research, there are five decision variables for modelling the optimization problem. They express the number of batik cloths to be produced with 5 different motifs, namely
X_1 = production number of batik motif tampuk manggis per week
X_2 = production number of batik motif duren pecah per week
X_3 = production number of batik motif angso duo per week
X_4 = production number of batik motif gentala per week
X_5 = production number of batik motif per week
The length of each piece of batik cloth is 2m^2.

The data used in this research are primary data from Rumah Batik Mentari, which produces five batik motifs. The main raw materials to produce batiks are fabric, wax and fabric dye. The use or consumption of raw materials for each batik motifs is presented in Table 1.

Table 1. Data on consumption of raw materials, production time and profit

Raw Materials, profit and production time	Tampuk Manggis	Duren Pecah	Angso Duo	Gentala	Batanghari	Capacity	Tolerance
Fabric (meter)	2	2	2	2	2	240	60
Wax (ounce)	3	3.3	2.2	1.8	2.8	300	40
Fabric dye (gram)	40	55	40	45	50	5800	400
Profit per piece (IDR)	43,000	44,300	41,200	40,300	43,800		
Production time per piece (hour)	2.5	2.5	2	2	2		

Based on the data in Table 1, we want to analyze the optimal solutions if we want to maximize the profit and minimize the production time to avoid overtime. To achieve this goals, then we build a mathematical model in the form of multi-objective linear programming where the first objective is maximizing profit (Z_1), while the second objective is minimizing production time (Z_2). For this research, the owner of Rumah Batik Mentari did not state the minimum production for each motives. Therefore, the multi-objective linear programming is as follow:

$$\begin{aligned}
&\text{Maximize:} && Z_1 = 43{,}000X_1 + 44{,}300X_2 + 41{,}200X_3 + 40{,}300X_4 + 43{,}800X_5 \\
&\text{Minimize:} && Z_2 = 2.5X_1 + 2.5X_2 + 2X_3 + 2X_4 + 2X_5 \\
&\text{with constraints:} && \\
&\text{(a):} && 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 \leq 240 \\
&\text{(b):} && 3X_1 + 3.3X_2 + 2.2X_3 + 1.8X_4 + 2.8X_5 \leq 300 \\
&\text{(c):} && 40X_1 + 55X_2 + 40X_3 + 45X_4 + 50X_5 \leq 5{,}800 \\
&\text{(d):} && X_1, X_2, X_3, X_4, X_5 \geq 0
\end{aligned} \tag{8}$$

Constraint (a) in model (8) is related to usage and capacity of fabric, constraint (b) is related to usage and capacity of wax, and constraint (c) is related to usage and capacity of fabric dye, which coefficients in the left-hand side and constant in the right-hand side are referred to the data in Table 1. Based on model (8), then inequality (3) can be formed with

$$B = \begin{pmatrix} -43{,}000 & -44{,}300 & -41{,}200 & -40{,}300 & -43{,}800 \\ 2.5 & 2.5 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3.3 & 2.2 & 1.8 & 2.8 \\ 40 & 55 & 40 & 45 & 50 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}, \mathbf{d} = \begin{pmatrix} -Z_1 \\ Z_2 \\ 240 \\ 300 \\ 5{,}800 \end{pmatrix}.$$

Let B_i means the entry in i^{th}-rows in matrix B. B_3 comes from the coefficient of the decision variables in constraint (a) in model (8), B_4 is the coefficient in constraint (b) in model (8) and B_5 is the coefficient in constraint (c) in model (8). Furthermore, if a tolerance is added to each constraint, namely an additional maximum of 60 meters for fabric capacity, an additional maximum of 40 ounces for wax capacity, and an additional 400 grams for dye capacity, then model (8) is written as follows

$$\begin{aligned}
&\text{Maximize:} && Z_1 = 43{,}000X_1 + 44{,}300X_2 + 41{,}200X_3 + 40{,}300X_4 + 43{,}800X_5 \\
&\text{Minimize:} && Z_2 = 2.5X_1 + 2.5X_2 + 2X_3 + 2X_4 + 2X_5 \\
&\text{with constraints:} && \\
&\text{(a):} && 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 \leq 240 + 60t \\
&\text{(b):} && 3X_1 + 3.3X_2 + 2.2X_3 + 1.8X_4 + 2.8X_5 \leq 300 + 40t \\
&\text{(c):} && 40X_1 + 55X_2 + 40X_3 + 45X_4 + 50X_5 \leq 5{,}800 + 400t \\
&\text{(d):} && X_1, X_2, X_3, X_4, X_5 \geq 0 \\
&\text{(e):} && 0 \leq t \leq 1
\end{aligned} \tag{9}$$

From model (9), for the sake of consistency of the index for constraints, it can be said that $p_3 = 60$, $p_4 = 40$ and $p_5 = 400$. The state $t = 0$ indicates no addition of raw materials and $t = 1$ indicates the addition of raw materials. The optimal solution of the model (9) in the state $t = 0$ and $t = 1$ can be found by the simplex method, which is shown in Table 2.

Table 2. Solution of Model (9) when $t = 0$ and $t = 1$

Decision Variables	$t = 0$	$t = 1$
X_1	0	0
X_2	0	0
X_3	2.86	128.57
X_4	34.29	2.86
X_5	82.86	18.57
Z_1	5,128,571.43	6,225,714.29
Z_2	240	300

Based on the value of Z_1 on the state $t = 0$ and Z_1 on the state $t = 1$, then we have

$$p_0 = 6{,}225{,}714.29 - 5{,}128{,}571.43 = 1{,}097{,}142.86.$$

Afterwards, the FMOLP model can be built which will maximize λ with both objective functions in model (8) being constraints on FMOLP. The first objective function related to maximize profit can be written as

$$\text{Minimize: } -Z_1 = -43{,}000X_1 - 44{,}300X_2 - 41{,}200X_3 - 40{,}300X_4 - 43{,}800X_5.$$

Then the FMOLP model formed is as shown in model (10).

$$\text{Maximize: } \lambda$$

with constraints:

$$\begin{array}{llr}
\text{(a):} & 43{,}000\,X_1 + 44{,}300X_2 + 41{,}200X_3 + 40{,}300X_4 + 43{,}800X_5 - 1{,}097{,}142.86\lambda & \geq 5{,}128{,}571.43 \\
\text{(b):} & 2.5X_1 + 2.5X_2 + 2X_3 + 2X_4 + 2X_5 + 60\lambda & \leq 300 \\
\text{(c):} & 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 + 60\lambda & \leq 300 \\
\text{(d):} & 3X_1 + 3.3X_2 + 2.2X_3 + 1.8X_4 + 2.8X_5 + 40\lambda & \leq 340 \\
\text{(e):} & 40X_1 + 55X_2 + 40X_3 + 45X_4 + 50X_5 + 400\lambda & \leq 6{,}200 \\
 & \lambda, X_1, X_2, X_3, X_4, X_5 & \geq 0
\end{array} \quad (10)$$

The constraint (a) in model (10) is taken from the maximization objective in model (9) and the constrain (b) is taken from the minimization objective in model (9). While the constraints (c), (d) and (e) in model (10) are respectively derived from the constraints (a), (b) and (c) in model (9), respectively. The optimal solution of model (10) is found using the two-phase technique because there is an inequality in the form of greater than equal (≥) in the constraints. The solution of model (10) is shown in Table 3

Table 3. Solution of model (10)

Solution	
X_1	0
X_2	0
X_3	65.71
X_4	18.57
X_5	50.71
λ	0.5

Since $\lambda = 0.5$, then we obtain $t = 1 - 0.5 = 0.5$. By substituting $t = 0.5$ into model (9), then the total of inventory capacity required is

$$\text{Fabric capacity: } B_3\mathbf{x} = 2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 = 270.$$

Wax capacity: $B_4\mathbf{x} = 3X_1 + 3.3X_2 + 2.2X_3 + 1.8X_4 + 2.8X_5 = 320.$

Fabric dye capacity: $B_5\mathbf{x} = 40X_1 + 55X_2 + 40X_3 + 45X_4 + 50X_5 = 6{,}000.$

Thus, the inventory of raw materials needed to produce five batik motifs is 270 meters of fabric/ material, 320 ounces of wax and 6,000 grams of fabric dye.

The membership function on the constraints is used to determine the degree of membership value for each constraint. Based on the membership function formula in equation (5), for this case, the membership function value for each constraint of model (8) is obtained as follows

$$\mu_3[\mathbf{x}] = 1 - \frac{B_3\mathbf{x} - d_3}{p_3} = 1 - \frac{270 - 240}{60} = 0.5.$$

$$\mu_4[\mathbf{x}] = 1 - \frac{B_4\mathbf{x} - d_4}{p_4} = 1 - \frac{320 - 300}{40} = 0.5.$$

$$\mu_5[\mathbf{x}] = 1 - \frac{B_5\mathbf{x} - d_5}{p_5} = 1 - \frac{6{,}000 - 5{,}800}{400} = 0.5.$$

The membership degree value for each constraint of 0.5 indicates that, the addition of raw materials needed to obtain the optimum solution is 0.5 times the tolerance of each constraint. That is, for fabric capacity an additional $0.5 \times 60 = 30$ meters is needed. For the wax capacity, it takes an additional $0.5 \times 40 = 20$ ounces of wax. And for the dye capacity, it takes an additional $0.5 \times 400 = 200$ grams of dye.

Thus, if the constants in right-hand side in the first constraint, the second constraint and the third constraint respectively are applied 270, 320 and 6,000 to model (9), then using the simplex method, the optimal solution can be obtained, namely $X_1 = 0, X_2 = 0, X_3 = 65.71, X_4 = 18.57, X_5 = 50.71$ with optimum values $Z_1 = 5{,}677{,}142.86$ and $Z_2 = 270$.

The decision variables $X_i; i = 1,2,3,4,5$ in this optimization problem is the number of fabric in pieces (2m^2), so an integer solution is required in this problem. Unfortunately, FMOLP gives a non-integer solutions, so it is necessary to apply the Branch and Bound method to the model by adding the integer constraints X_1, X_2, X_3, X_4 and X_5. The Branch and Bound chart to find the integer solution is shown in Figure 2 with $t = 0.5$. Through the Branch and Bound method, for the purpose of maximizing Z_1 and minimizing Z_2, the optimum solution is finally obtained in sub-problem 36 because in this sub-problem, the value of Z_1 is the highest, while the value of Z_2 is the lowest, with condition that all decision variables $(X_1, X_2, X_3, X_4, X_5)$ have integer value, i.e

$$X_1 = 0, X_2 = 0, X_3 = 67, X_4 = 18, X_5 = 50, Z_1 = 5{,}675{,}800, Z_2 = 270.$$

Conclusion

The optimization problem of batik cloth production at Rumah Batik Mentari is analyzed through the Fuzzy Multi Objective Linear Programming (FMOLP) model by considering two objective functions, namely maximizing profit and minimizing production time. The optimum solution of the FMOLP is to produce batik cloth with a total of 135 pieces of batik cloth consisting of 67 pieces of angso duo motif, 18 pieces of gentala motif and 50 pieces of batang hari motif. Meanwhile, the motifs of tampuk mangosteen and durian pecah do not contribute to gain maximum profit based on current data. With this result, maximum profit can be obtained is IDR 5,675,800 / week with the required production time of 270 hours / week. Eventhough current result might be unsatisfactory for the owner because there are two motifs that do not contribute to gain maximum profit based on this research analysis, but due to some people's interest in such two motifs, the owner still produce batik with motifs of tampuk mangosteen and durian pecah in few number of production. While in the meantime, the owner needs to review the capacity comparing to materials consumptions such that the analysis will give result of optimal solution for all motifs.

Acknowledgments

The author would like to thank the business owner of Rumah Batik Mentari Jambi who is willing to be interviewed and conducted research at the business location regarding the production process and information provided for the smooth running of this research

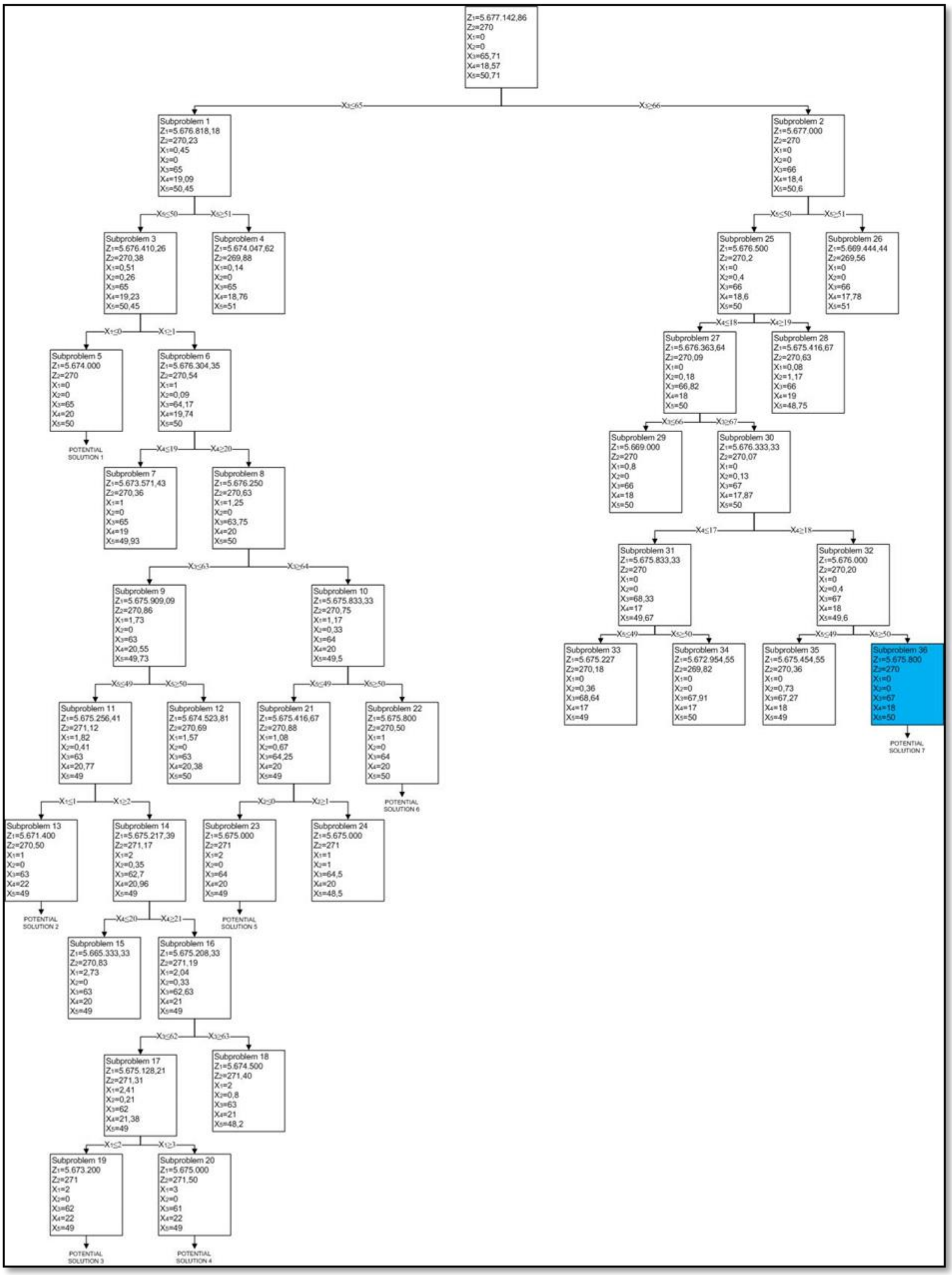

Figure 2. Chart of Branch and Bound Method

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On The Edge Irregularity Strength of Firecracker Graphs $F_{2,m}$

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Abstract

Let $G = (V, E)$ be a graph and k be a positive integer. A vertex k-labeling $f : V(G) \rightarrow \{1,2,\cdots,k\}$ is called an edge irregular labeling if there are no two edges with the same weight, where the weight of an edge *uv* is $f(u) + f(v)$. The edge irregularity strength of *G*, denoted by *es*(*G*), is the minimum *k* such that G has an edge irregular *k*-labeling. This labeling was introduced by Ahmad, Al-Mushayt, and Bacˇa in 2014. An (n,k)-firecracker is a graph obtained by the concatenation of n k-stars by linking one leaf from each. In this paper, we determine the edge irregularity strength of fireworks graphs $F_{2,m}$.

Keywords: edge irregular labeling, firecracker, the edge irregularity strength

Introduction

Graph labeling was first introduced by Sadlàčk (1964), then Stewart (1966), Kotzig and Rosa (1970). The process of labeling a graph includes assigning values (labels), represented by a set of positive integers to vertices, edges, or both. These numbers are called labels [1]. There are several types of labeling on graphs, including gracefull labeling, harmony labeling, total irregular labeling, magic labeling, and anti-magic labeling. The concept of irregular labeling on a graph was first introduced by Chartrand et al. in 1986 [2].

In 2014, Ahmad et al. [3] introduced edge irregular labeling of graphs, namely edge irregular labeling. For an integer k, a total labeling $f : V(G) \rightarrow \{1,\ 2,\ \cdots k\}$ is called an edge irregular k-labeling of G if every two distinct edges e_2 and e_2 in E satisfy $w_f(e_1) \neq w_f(e_2)$, where $w_f(e_1 = uv) = f(u) + f(v)$. As an example, we have a graph $P_2 \cup C_3$ in the Figure 1.

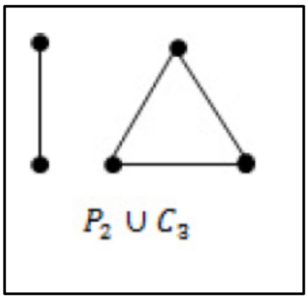

Figure 1. Given a graph $P_2 \cup C_3$

Then, we give a vertex labeling of the graph. The labeling is can be seen in the Figure 2.

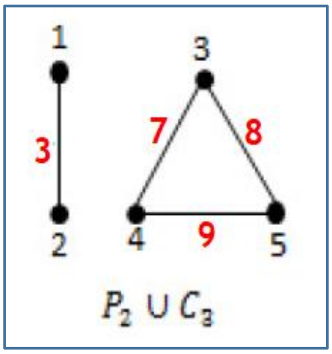

Figure 2. An edge irregular 5-labeling of $P_2 \cup C_3$

By the labeling in the Figure 2, the weight of edges of the graph are 3, 7, 8, and 9 and the maximum label used in this labeling is 5. Besides that, there are no two edges with the same weight, so the labeling is an edge irregular k-labeling of $P_2 \cup C_3$ with k=5.

The minimum k for which a graph G has an edge irregular k-labeling, denoted by $es(G)$, is called *the edge irregularity strength* of G. For example, given an edge irregular 3-labeling of $P_2 \cup C_3$ in the Figure 3.

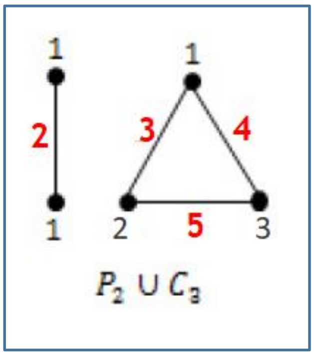

Figure 3. An edge irregular 3-labeling of $P_2 \cup C_3$

The labeling of the Figure 3 is in edge irregular 3-labeling of $P_2 \cup C_3$ because there are no two edges with the same weight and the maximum label used is 3. It is impossible to have an edge irregular k-labeling of $P_2 \cup C_3$ with maximum label 2. So, 3 is the minium k for which $P_2 \cup C_3$ has an edge irregular k-labeling. We conclude that the edge irregularity strength of $P_2 \cup C_3$ is 3, denoted by **es($P_2 \cup C_3$)=3.**

To get the exact value of es of a graph *G*, we would previously determine a lower bound and an upper bound of *es*(*G*) before. A lower bound on *es*(*G*) is obtained by using the theorem from Martin Baca and Ali Ahmad in 2014. [3] as follows.

Theorem 1 [3] : Let $G = (V, E)$ be a graph with the maximum degree Δ, then

$$es(G) \geq maks\left\{\left\lceil\frac{|E(G)|+1}{2}\right\rceil, \Delta(G)\right\}$$

Other results about computing the edge irregularity strength of graphs are given by Imran et al. in [4]. In the paper, Imran et al. determined the egde irregularity strength of caterpillars, n-star graphs, kite graphs, cycle chains and friendship graphs.

Tarawneh et al. [5], determined he edge irregularity strength of corona product of cycle with isolated vertices. In [6], Tarawneh et al. determined he exact value of edge irregularity strength for triangular grid graph , zigzag graph and Cartesian product $P_n x P_m x P_2$.

In 2017, Ahmad et al determined the edge irregularity strengths of some chain graphs and the join of two graphs. They also introduced a conjecture and open problems for researchers for further research [7]. Ahmad et al. [6], gave computing of the edge irregularity strength of bipartite graphs and wheel related graphs. In [8] , Asim et al. gained an edge irregular k-labeling for several classes of trees. Asim et al also gained the edge irregularity strength of disjoint union of star graph and subdivision of star graph [9].

In 2020, Ahmad et al performed a computer based experiment dealing with the edge irregularity strength of complete bipartite graphs. They also presented some bounds on this parameter for wheel related graphs [6]. In [10], Tarawneh et al. gave the edge irregularity strength of some classes of plane graphs.

In this paper, we determined the exact value of es of firecracker graphs $F_{2,m}$ with arbitrary m.

A firecracker is a graph obtained by the concatenation of stars by linking one of leaf from each. If the number of stars is n and the number of leaves in each star is m, then the firecracker is denoted by $F_{n,m}$.

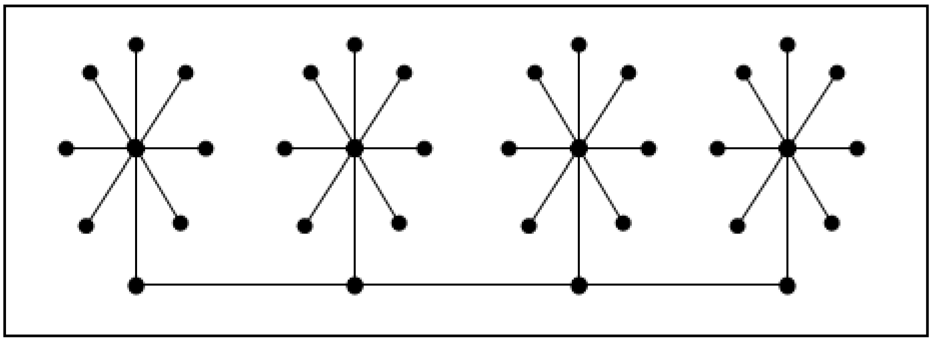

Figure 4. Firecracker graph $F_{4,8}$

Methods

The method we use in this research is analytical method. To get the exact value of es of firecracker graph, we consider a lower bound and an upper bound of $es(F_{n,m})$. Theorem 1 is used to have a lower bound of $es(F_{n,m})$. Besides that, an upper bound of $es(F_{n,m})$, we construct an edge irregular-k labeling with minimum k.

Result and Discussion

The main result of our research is the edge irregularity strength of firecracker graphs $\boldsymbol{F_{2,m}}$ is $\boldsymbol{m+1}$. The result written in Theorem 2.

Theorem 2 : Let firecracker graphs $\boldsymbol{F_{2,m}}$, for $m \geq 2$, we have edge irregularity strength by

$$es(F_{2,m}) = m + 1$$

Proof.

We consider

$$es(F_{2,m}) \geq m + 1 \qquad (1)$$

and

$$es(F_{2,m}) \leq m + 1 \qquad (2)$$

To prove inequality (1), we use Theorem 1.

In the Figure 5, we can see an illustration of firecracker graph $F_{2,m}$

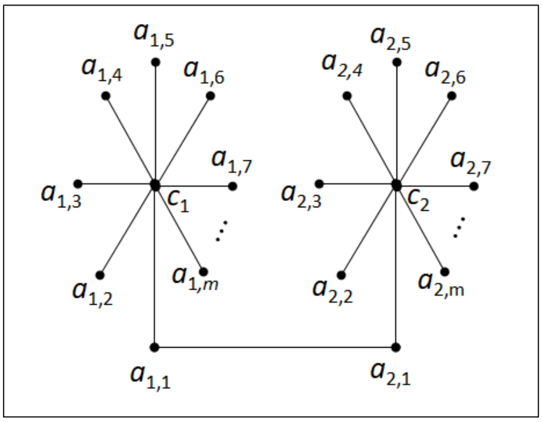

Figure 5. Firecracker graph $F_{2,m}$

From the illustration of Figure 5, we have the graph $F_{2,m}$ has $2m+1$ edges and the maximum degree $\Delta = m$. By using Theorem 1, we have

$$es(F_{2,m}) \geq \left\{\lceil\frac{|E(F_{2,m})|+1}{2}\rceil, \Delta(F_{2,m})\right\} = maks\left\{\lceil\frac{2m+2}{2}\rceil, m\right\} = maks\{m+1, m\} = m + 1.$$

So, we have

$$es(F_{2,m}) \geq m + 1$$

Next, we give an edge irregular k –labeling with $k = m+1$ to get $es(F_{2,m}) \leq m + 1$ as follows.

For $m \geq 2$.

$$f(c_i) = \begin{cases} 1 & for\ \ i = 1 \\ m+1 & for\ i = 2 \end{cases}$$

$$f(a_{i,j}) = \begin{cases} f(c_i), & for\ j = 1 \\ j & for\ i = 1\ and\ 1 < j \leq m \\ f(a_{1,1}) + (j-1), & for\ i = 2\ and\ 1 < j \leq m \end{cases} \qquad (3)$$

From the labeling formula (3), we have the weight of edges of firecracker graphs $F_{2,m}$ as follows :

$$w_f(c_i a_{i,j}) = \{j+1, \quad for\ i = 1\ and\ 1 \le j \le m\ ;\ mi + i, \quad for\ i = 2\ and\ j = 1\ ;\ mi + i - (m+1) + j\ for\ i = 2\ and\ 1 < j \le m\ ; \tag{4}$$

$$w_f(a_{i,1} a_{i+1,1}) = \{m+2, \quad for\ i = 1\ ;\ 2m+1, \quad for\ i = 2.$$

From the edge weight formula (4), there are no two edges with the same weight. The maximum label used in the labeling f is $m+1$. So, f is an edge irregular-$(m+1)$ of $F_{n,m}$. So, we can conclude that

$$es(F_{2,m}) \le m+1$$

From inequalities (1) and (2), we have an equality

$$es(F_{2,m}) = m+1.$$

For an illustration, in the Figure 6, we can see the edge irregular labeling f of firecracker graph $F_{2,m}$ $m = 8$.

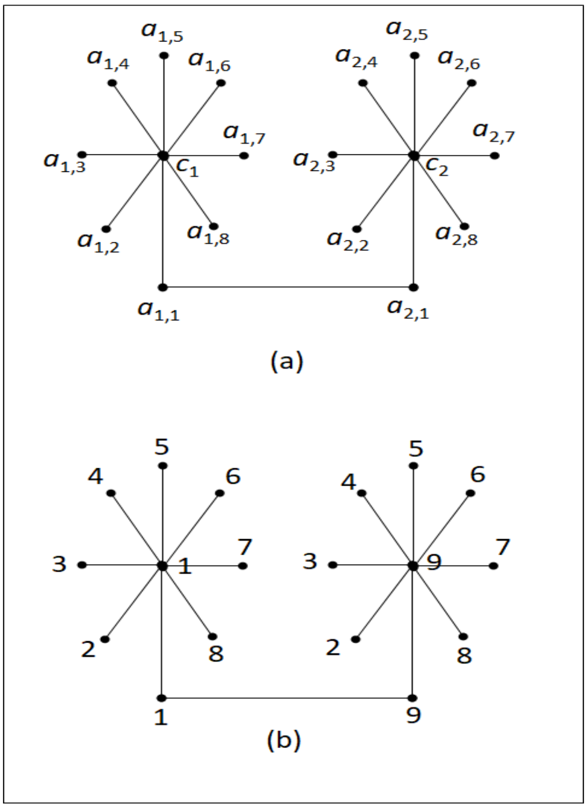

Figure 6. (a) An illustration of $F_{2,8}$; (b) An illustration of edge irregular-9 labeling f of $F_{2,8}$

In the Figure 7, we can see the weight of edges of $F_{2,8}$ under the labeling in Figure 6.

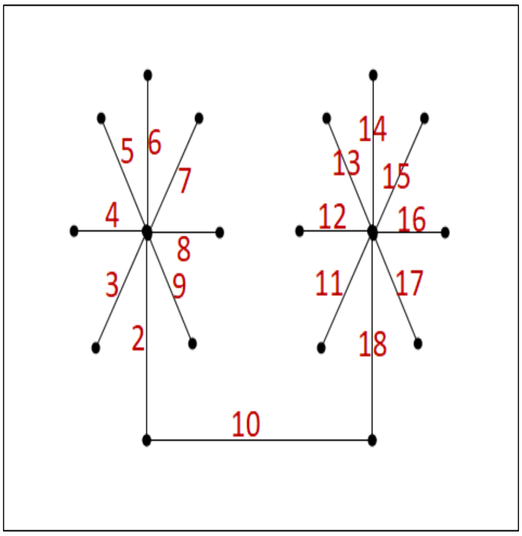

Figure 7. The weight of edges of $F_{2,8}$ under the labeling f

From the illustration of the Figure 7, we can see that there are no two edges in $F_{2,8}$ with the same weight under the labeling f.

Conclusion

By the research, we have a lower bound of $es(F_{2,m})$ is $m+1$, which is also an upper bound of $es(F_{2,m})$. So that, we can conclude the exact value of the edge irregularity strength of firecracker graphs $F_{2,m}$ is $m+1$ for $m \geq 2$.

Acknowledgement

Author acknowledges the support from UIN Sunan Gunung Djati Bandung. The author wish to thank the referees for their thoughtful suggestions.

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Algoritma *Affine Cipher* dan Modifikasi *Affine Cipher*, serta Kombinasinya dengan *Cipher* Transposisi Grup Simetri untuk Mengamankan Pesan Teks

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Abstrak

Kriptografi adalah ilmu yang mempelajari teknik-teknik matematika yang berhubungan dengan aspek keamanan informasi. Metode kriptografi yang digunakan untuk mengamankan pesan diantaranya adalah *affine cipher* dan *cipher* transposisi grup simetri. *Affine cipher* merupakan salah satu algoritma kriptografi klasik yang menggunakan metode substitusi. Modifikasi *Affine Cipher* membagi teks terlebih dahulu menjadi kelompok yang terdiri dari k karakter lalu kemudian menggunakan *Affine Cipher. Cipher* transposisi grup simetri merupakan salah satu algoritma kriptografi klasik yang menggunakan metode transposisi. *Cipher* transposisi grup simetri menggunakan grup simetri-n sebagai kunci rahasia. Semakin besar nilai n akan semakin banyak pula kemungkinan kunci dari *cipher* transposisi grup simetri. Metode substitusi dan transposisi memiliki tingkat keamanan yang cenderung lebih rendah. Untuk meningkatkan keamanan penyandian pesan maka dilakukan penggabungan algoritma *cipher* substitusi dan *cipher* transposisi yang disebut super enkripsi. Pada penelitian ini dilakukan penggabungan algoritma *affine cipher* dan *cipher* transposisi grup simetri, serta modifikasi *affine cipher* dan *cipher* transposisi grup simetri.

Kata kunci: Affine Cipher, Modifikasi Affine Cipher, Cipher Transposisi Grup Simetri, Super Enkripsi

Abstract

Cryptography is a science that studies mathematical techniques related to aspects of information security. Cryptographic methods used to secure messages include affine ciphers and symmetric group transposition ciphers. Affine cipher is one of the classical cryptographic algorithms that use the substitution method. In Modification of Affine Cipher, we first divide the text into groups of k characters and then use Affine Cipher. Cipher transposition symmetric group is one of the classical cryptographic algorithms that use the transposition method. The cipher transposition symmetric group uses the n-symmetric group as the secret key. The larger value of n, the more possible keys from the cipher transposition symmetric group. Substitution and transposition methods have a lower level of security. On the other hand, the key of the affine cipher can be found by an exhaustive key search. To increase the security of message encoding, a substitution cipher algorithm and a transposition cipher algorithm are combined which is called super encryption. In this study, the

affine cipher algorithm and cipher transposition symmetric group were combined, as well as modification of affine cipher and symmetric group transposition cipher.
Keywords: Affine Cipher, Modification of Affine Cipher, Cipher Transposition Symmetric Group, Super Encryption

Pendahuluan

Teknologi komputer semakin lama semakin berkembang dan semakin maju. Perkembangan teknologi komputer dapat meningkatkan kemudahan dalam berkomunikasi dengan menggunakan akses internet. Namun, internet memiliki jangkauan yang sangat luas dan dapat dilihat oleh banyak orang sehingga rentan terjadi penyadapan penyandian. Penyadapan penyandian dapat mengakibatkan orang lain mengetahui pesan yang kita kirimkan. Hal ini membuat aspek keamanan pada pertukaran informasi menjadi sangat penting. Untuk itu perlu adanya usaha dalam mengamankan pesan informasi yang akan dikomunikasikan [1]. Kriptografi adalah ilmu yang mempelajari teknik-teknik matematika yang berhubungan dengan aspek keamanan informasi seperti kerahasiaan, integritas data, serta otentikasi [2],[3],[4],[5]. Pengamanan pesan menggunakan kriptografi tidak lepas dari metode enkripsi dan dekripsi. Enkripsi merupakan proses menyandikan pesan asli (plainteks) menjadi pesan tersandi (cipherteks). Sedangkan dekripsi merupakan proses mengembalikan cipherteks menjadi plainteks semula [6],[7],[8].

Affine cipher adalah perluasan dari algoritma *caesar cipher* yang diperoleh dengan mengalikan plainteks dengan suatu bilangan m yang relatif prima dengan nilai pergeseran b, kemudian hasilnya dijumlahkan dengan nilai pergeseran b [6]. Berdasarkan Nurjamiyah [9], algoritma *affine cipher* dapat digunakan untuk menyembunyikan pesan rahasia ke dalam teks dengan efektif. *Affine cipher* adalah salah satu metode penyandian pesan menggunakan algoritma kriptografi klasik. Kriptografi klasik merupakan algoritma kriptografi yang berbasis karakter dimana enkripsi dan dekripsi dilakukan pada setiap karakter pesan [10]. Algoritma Modifikasi Affine Cipher merupakan metode Affine Cipher dengan menerapkan modifikasi pada plainteks yang dikelompokan menjadi k karakter setiap kelompoknya, kemudian disusun ulang dengan posisi terbalik. Secara umum algoritma kriptografi klasik dikategorikan menjadi dua, yaitu *cipher* substitusi dan *cipher* transposisi. *Cipher* substitusi merupakan proses penyandian pesan dengan mengganti huruf dari plainteks menjadi huruf, angka, atau simbol lainnya [11]. Sedangkan *cipher* transposisi merupakan proses penyandian pesan dengan cara mengubah susunan huruf-huruf [6].

Cipher transposisi grup simetri merupakan teknik transposisi menggunakan permutasi karakter di mana pengirim dan penerima menyepakati kunci rahasia menggunakan grup simetri-n, kemudian membagi teks asli (plainteks) menjadi blok-blok yang memuat beberapa karakter [12]. Penyandian pesan menggunakan grup simetri S_n untuk mengamankan informasi akan menghasilkan cipherteks yang tidak dapat dimengerti. Dengan menggunakan grup simetri-n maka terdapat kemungkinan sebanyak $n!$ Kunci. Semakin besar nilai n akan semakin banyak pula kemungkinan kunci dari algoritma *cipher* transposisi grup simetri [12].

Metode substitusi dan transposisi memiliki tingkat keamanan yang cenderung lebih rendah [13]. Disisi lain, kunci dari *affine cipher* dapat ditemukan dengan *exhaustive key search*. Jika menggunakan karakter alfabet 26 huruf, maka hanya terdapat 25 kemungkinan untuk nilai b dan hanya terdapat 12 nilai m yang relatif prima dengan 26 [6]. Teknik-teknik klasik penggabungan memberikan *cipher* lebih aman dan kuat [14]. Super Enkripsi adalah metode kriptografi berbasis karakter yang mengkombinasikan dua buah cipher untuk memperoleh cipher yang lebih kuat sehingga tidak mudah untuk dipecahkan, dan juga untuk menangani penggunaan cipher tunggal yang secara komparatif lemah [15],[16],[17],[18]. Cipher substitusi dan cipher transposisi dapat dikombinasikan untuk memperoleh cipher yang lebih kuat (super) daripada hanya satu cipher saja. Proses penyandian pesan dilakukan dengan mengenkripsikan plaintext menggunakan *cipher* substitusi sederhana, kemudian hasilnya dienkripsi kembali menggunakan *cipher* transposisi (atau sebaliknya) [6]. Penelitian mengenai algoritma super enkripsi yang dilakukan oleh [14] menggabungkan *cipher* substitusi yaitu *hill cipher* dan *cipher* transposisi yaitu transposisi kolom. Penelitian tersebut menyatakan bahwa algoritma super enkripsi dapat menambah keamanan pada pesan teks.

Pada Penelitian ini, peneliti menggunakan algoritma super enkripsi yang mengkombinasikan algoritma *cipher* substitusi dan *cipher* transposisi dalam mengamankan atau menyandikan pesan teks. Adapun algoritma *cipher* substitusi yang digunakan adalah *affine cipher* dan modifikasi *affine cipher* sedangkan algoritma *cipher* transposisi yang digunakan adalah *cipher* transposisi grup simetri. Hal ini dilakukan karena penggabungan dua buah *cipher* akan lebih sulit dipecahkan daripada hanya menggunakan satu *cipher* saja. Selain itu, banyaknya kemungkinan kunci pada *cipher* transposisi grup simetri dapat menguatkan tingkat keamanan dari *affine cipher* maupun modifikasi *affine cipher*.

Metode

Metode penelitian yang digunakan oleh penulis menggabungkan dua buah algoritma yang telah ada untuk memperoleh proteksi ganda untuk pengamanan teks yaitu *affine cipher* dan *cipher* transposisi grup simetri, serta modifikasi *affine cipher* dan *cipher* transposisi grup simetri.

1. Kriptografi

Kriptografi (*cryptography*) berasal dari Bahasa Yunani: "*cryptós*" artinya "*secret*" (rahasia), sedangkan "*gráphein*" artinya "*writing*" (tulisan). Jadi, kriptografi secara harfiah berarti "*secret writing*" (tulisan rahasia) [6]. Kriptografi merupakan ilmu menulis pesan rahasia yang bertujuan untuk menyembunyikan makna pesan tersebut [19]. Kriptografi berkembang sesuai dengan masalah yang dihadapi sehingga muncul beberapa istilah yang digunakan untuk menandai aktifitas-aktifitas rahasia untuk mengirim pesan. Proses mengacak pesan disebut enkripsi dan ketika merapikan pesan teracak disebut dekripsi. Pesan awal yang belum diacak atau yang sudah dirapikan disebut plainteks dan pesan yang sudah diacak disebut cipherteks [20].

2. Algoritma *Affine Cipher*

Proses enkripsi plainteks menggunakan algoritma *affine cipher* dapat dilakukan dengan persamaan berikut [6]:

$$C = (mP + b) \; mod \; n \tag{1}$$

keterangan:
$C =$ cipherteks
$P =$ plainteks
$n =$ banyaknya alfabet
$m =$ bilangan bulat yang relatif prima dengan n
$b =$ jumlah pergeseran

Proses dekripsi menggunakan algoritma *Affine cipher* dapat dilakukan jika ada balikan dari $m \; (mod \; n)$ yang dinyatakan dengan $m^{-1} \; (mod \; n)$. Berdasarkan Munir [6], Persamaan yang digunakan saat proses dekripsi adalah

$$P = m^{-1} \; (C - b) \; mod \; n \tag{2}$$

3. Algoritma Modifikasi Affine Cipher

Algoritma ini merupakan metode Affine Cipher dengan menerapkan modifikasi pada plainteks yang dikelompokan menjadi k karakter setiap kelompoknya, kemudian disusun ulang dengan posisi terbalik [1]. Sebagai contoh jika kata MATEMATIKA akan dibagi kedalam kelompok dengan empat karakter maka menjadi MATE MATI KA kemudian posisi dibalik sehingga menjadi ETAM ITAM AK. Setelah proses modifikasi pada plainteks dilakukan, maka proses pada Affine Cipher dapat dilakukan yaitu seperti tertera pada persamaan (1).

Proses dekripsi juga sama dengan algoritma Affine Cipher pada persamaan (2), dengan setelahnya menambahkan proses pembagian menjadi kelompok dengan empat karakter kemudian dilakukan pembalikan posisi karakter.

4. Algoritma *Cipher* Transposisi Grup Simetri

Proses enkripsi menggunakan algoritma *cipher* transposisi grup simetri yaitu, pertama pengirim dan penerima menyepakati kunci rahasia menggunakan grup simetri-n. Kemudian membagi teks asli (plainteks) menjadi blok-blok yang memuat beberapa karakter. Pesan asli tidak dapat diketahui kecuali oleh seseorang yang mempunyai kunci untuk mendekripsi cipherteks ke bentuk asal [10].

Proses dekripsi menggunakan algoritma *cipher* transposisi grup simetri pada dasamya sama saja dengan proses enkripsinya namun pada proses dekripsi penerima pesan terlebih dahulu menginverskan kunci yang telah disepakati sebelumnya [10].

5. Kombinasi Algoritma Super Enkripsi (Affine Cipher dan Cipher Transposisi Grup Simetri)

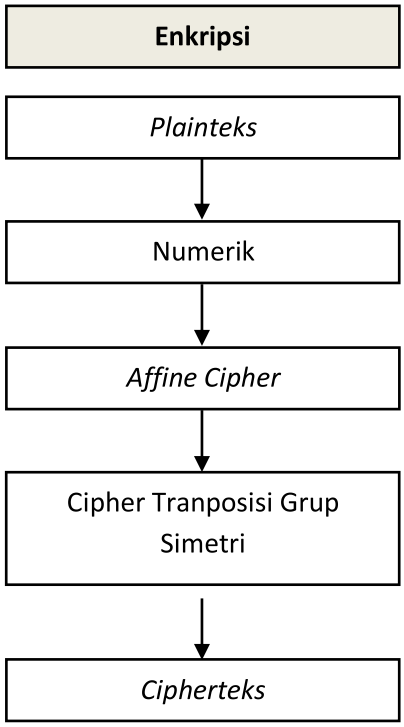
Secara matematis proses enkripsi menggunakan metode super enkripsi dapat dijelaskan sebagai berikut:

1) Menentukan plainteks.
2) Menentukan kunci m dan b di mana m relatif prima dengan n dan $1 < b < n, (n = 26)$.
3) Mengubah plainteks alfabet ke dalam bentuk numerik.
4) Mengubah hasil enkripsi bentuk numerik menjadi alfabet.
5) Menentukan kunci grup simetri K.
6) Membagi plainteks menjadi beberapa blok sesuai dengan jumlah permutasi pada K.
7) Melakukan enkripsi menggunakan algoritma cipher transposisi grup simetri.
8) Memperoleh pesan teks yang telah disandikan (cipherteks).

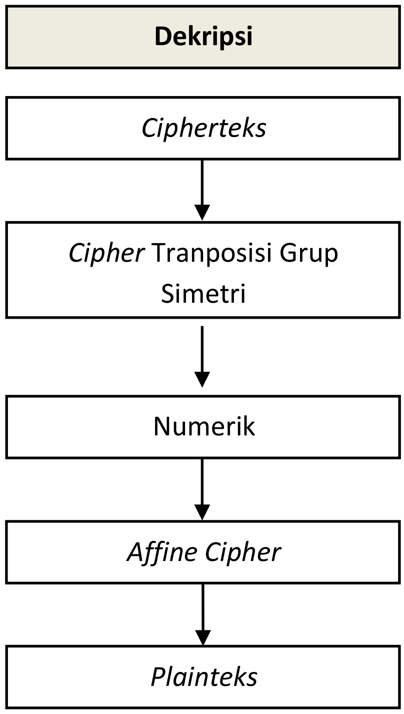
Sedangkan proses dekripsi secara matematis menggunakan metode super enkripsi dapat dijelaskan sebagai berikut:

1) Mendapatkan pesan yang telah disandikan (cipherteks)
2) Menentukan K^{-1} sebagai kunci yang akan digunakan untuk dekripsi dengan cipher transposisi grup simetri
3) Membagi plainteks menjadi beberapa blok sesuai dengan jumlah permutasi pada K.
4) Melakukan dekripsi menggunakan algoritma cipher transposisi grup simetri.
5) Mengubah cipherteks alfabet ke dalam bentuk numerik. cipherteks merupakan plainteks hasil dekripsi menggunakan cipher transposisi grup simetri
6) Menentukan m^{-1}.
7) Melakukan dekripsi menggunakan algoritma *affine cipher*
8) Mengubah hasil dekripsi bentuk numerik menjadi alfabet.
9) Memperoleh pesan teks asli (plainteks).

Proses enkripsi dan deskripsi diilustrasikan seperti pada Gambar 1 dan Gambar 2 berikut:



Gambar 1. Skema Enkripsi Algoritma Super Enkripsi



Gambar 2. Skema Dekripsi Algoritma Super Enkripsi

6. Kombinasi Algoritma Modifikasi Super Enkripsi (Modifikasi Affine Cipher dan Cipher Transposisi Grup Simetri)

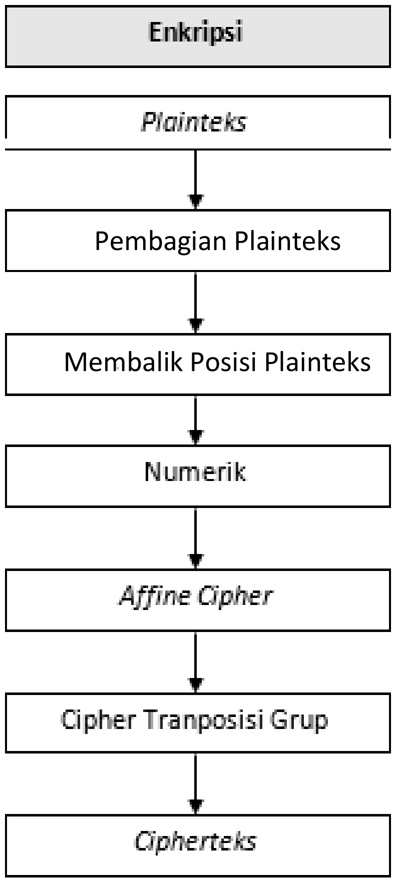
Secara matematis proses enkripsi menggunakan metode super enkripsi dapat dijelaskan sebagai berikut:

1) Menentukan plainteks.
2) Membagi plainteks ke dalam kelompok dengan k karakter.
3) Membalik posisi karakter di setiap kelompoknya
4) Menerapkan proses enkripsi kombinasi *affine cipher dan cipher* transposisi grup simetri pada bagian 5

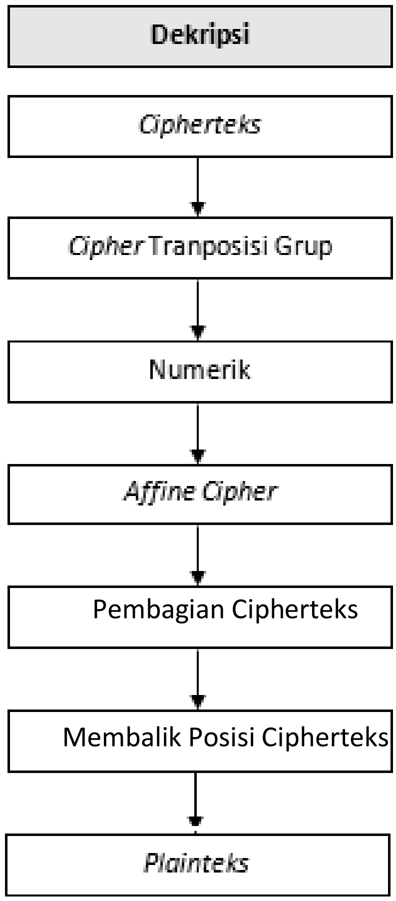
Secara matematis proses dekripsi menggunakan metode super enkripsi dapat dijelaskan sebagai berikut:

1) Mendapatkan pesan yang telah disandikan (cipherteks)
2) Menerapkan proses dekripsi kombinasi *affine cipher dan cipher* transposisi grup simetri pada bagian 5.
3) Membagi plainteks ke dalam kelompok dengan k karakter.
4) Membalik posisi karakter di setiap kelompoknya
5) Memperoleh pesan teks asli (plainteks).

Proses enkripsi dan dekripsi diilustrasikan seperti Gambar 3 dan Gambar 4 berikut:



Gambar 3. Skema Enkripsi Algoritma Modifikasi Super Enkripsi



Gambar 4. Skema Dekripsi Algoritma Modifikasi Super Enkripsi

Hasil dan Diskusi

Pada penelitian ini, penulis menggunakan plainteks "PRAMUKA UIN MALANG" untuk disandikan dengan metode *Affine Cipher* dan Cipher Transposisi Grup Simetri.

1. Enkripsi Menggunakan Algoritma *Affine Cipher* dan *Cipher* Transposisi Grup Simetri

Proses enkripsi dilakukan menggunakan algoritma *affine cipher* terlebih dahulu. Karakter alfabet yang digunakan pada algoritma *affine cipher* sejumlah 26 huruf dimana A=0, B=1, C=2,…, Z=25. Berikut merupakan proses enkripsi menggunakan *Affine Cipher*:

Plainteks: PRAMUKA UIN MALANG

Kunci $m = 11$ dan $b = 14$

Karena karakter yang digunakan sejumlah 26 huruf maka $n = 26$. Berdasarkan karakter alfabet yang digunakan, plainteks PRAMUKA UIN MALANG ekuivalen dengan 15 17 0 12 20 10 0 20 8 13 12 0 11 0 13 6. Menggunakan (1) maka proses perhitungan enkripsi menggunakan algoritma *Affine Cipher* adalah sebagai berikut:

$p_1 = 15 \rightarrow c_1 = (11 \cdot 15 + 14)\ mod\ 26 = 179\ mod\ 26 = 23$ (huruf X)

$p_2 = 17 \rightarrow c_2 = (11 \cdot 17 + 14)\ mod\ 26 = 201\ mod\ 26 = 19$ (huruf T)

$p_3 = 0 \rightarrow c_3 = (11 \cdot 0 + 14)\ mod\ 26 = 14\ mod\ 26 = 14$ (huruf O)

$p_4 = 12 \rightarrow c_4 = (11 \cdot 12 + 14)\ mod\ 26 = 146\ mod\ 26 = 16$ (huruf Q)

$p_5 = 20 \rightarrow c_5 = (11 \cdot 20 + 14)\ mod\ 26 = 234\ mod\ 26 = 0$ (huruf A)

$p_6 = 10 \rightarrow c_6 = (11 \cdot 10 + 14)\ mod\ 26 = 124\ mod\ 26 = 20$ (huruf U)
$p_7 = 0 \rightarrow c_7 = (11 \cdot 0 + 14)\ mod\ 26 = 14\ mod\ 26 = 14$ (huruf O)
$p_8 = 20 \rightarrow c_8 = (11 \cdot 20 + 14)\ mod\ 26 = 234\ mod\ 26 = 0$ (huruf A)
$p_9 = 8 \rightarrow c_9 = (11 \cdot 8 + 14)\ mod\ 26 = 102\ mod\ 26 = 24$ (huruf Y)
$p_{10} = 13 \rightarrow c_{10} = (11 \cdot 13 + 14)\ mod\ 26 = 157\ mod\ 26 = 1$ (huruf B)
$p_{11} = 12 \rightarrow c_{11} = (11 \cdot 12 + 14)\ mod\ 26 = 146\ mod\ 26 = 16$ (huruf Q)
$p_{12} = 0 \rightarrow c_{12} = (11 \cdot 0 + 14)\ mod\ 26 = 14\ mod\ 26 = 14$ (huruf O)
$p_{13} = 11 \rightarrow c_{13} = (11 \cdot 11 + 14)\ mod\ 26 = 135\ mod\ 26 = 5$ (huruf F)
$p_{14} = 0 \rightarrow c_{14} = (11 \cdot 0 + 14)\ mod\ 26 = 14\ mod\ 26 = 14$ (huruf O)
$p_{15} = 13 \rightarrow c_{15} = (11 \cdot 13 + 14)\ mod\ 26 = 157\ mod\ 26 = 1$ (huruf B)
$p_{16} = 6 \rightarrow c_{16} = (11 \cdot 6 + 14)\ mod\ 26 = 80\ mod\ 26 = 2$ (huruf C)

Jadi, cipherteks yang dihasilkan adalah XTOQAUO AYB QOFOBC.

Selanjutnya, cipherteks tersebut dienkripsi kembali menggunakan cipher transposisi grup simetri. Berikut merupakan proses enkripsi menggunakan cipher transposisi grup simetri:

Kunci: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ )$

Membagi plainteks menjadi blok-blok yang terdiri dari lima huruf dengan ketentuan jika terdapat kekurangan pada blok maka ditambahkan dengan karakter %. Sedangkan untuk spasi diganti dengan karakter #.

XTOQA	UO#AY	B#QOF	OBC%%

Kemudian, setiap blok diubah menjadi seperti di bawah ini dengan menggunakan kunci yang telah ditentukan.

Blok 1: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ ) = (1\ 2\ 3\ 4\ 5\ X\ T\ O\ Q\ A\ 4\ 1\ 5\ 3\ 2\ Q\ X\ A\ O\ T\ )$
Blok 2: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ ) = (1\ 2\ 3\ 4\ 5\ U\ O\ \#\ A\ Y\ 4\ 1\ 5\ 3\ 2\ A\ U\ Y\ \#\ O\ )$
Blok 3: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ ) = (1\ 2\ 3\ 4\ 5\ B\ \#\ Q\ O\ F\ 4\ 1\ 5\ 3\ 2\ O\ B\ F\ Q\ \#\ )$
Blok 4: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ ) = (1\ 2\ 3\ 4\ 5\ O\ B\ C\ \%\ \%\ 4\ 1\ 5\ 3\ 2\ \%\ O\ \%\ C\ B\ )$

Sehingga diperoleh cipherteks yaitu QXAOTAUY#OOBFQ#%O%CB.

2. Dekripsi Menggunakan Algoritma Super Dekripsi (*Affine Cipher* dan *Cipher* Transposisi Grup Simetri)

Pengembalian plainteks menjadi pesan teks semula (plainteks) dilakukan dengan proses dekripsi menggunakan *cipher* transposisi grup simetri terlebih dahulu. Proses dekripsi menggunakan cipher transposisi grup simetri dilakukan dengan cara yang sama seperti proses enkripsi, namun dengan menggunakan kunci invers. Berikut adalah proses dekripsi menggunakan cipher transposisi grup simetri.

Cipherteks: QXAOTAUY#OOBFQ#%O%CB

Kunci: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ )$

$K^{-1} = (4\ 1\ 5\ 3\ 2\ 1\ 2\ 3\ 4\ 5\ )$

$$K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ )$$

Membagi plainteks menjadi blok-blok yang terdiri dari lima huruf sebagai berikut:

QXAOT	AUY#O	OBFQ#	%O%CB

Selanjutnya, setiap blok diubah menjadi seperti di bawah ini dengan menggunakan kunci yang telah ditentukan.

Blok 1: $K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ ) = (1\ 2\ 3\ 4\ 5\ Q\ X\ A\ O\ T\ 2\ 5\ 4\ 1\ 3\ X\ T\ O\ Q\ A\ )$
Blok 2: $K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ ) = (1\ 2\ 3\ 4\ 5\ A\ U\ Y\ \#\ O\ 2\ 5\ 4\ 1\ 3\ U\ O\ \#\ A\ Y\ )$
Blok 3: $K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ ) = (1\ 2\ 3\ 4\ 5\ O\ B\ F\ Q\ \#\ 2\ 5\ 4\ 1\ 3\ B\ \#\ Q\ O\ F\ )$

Blok 4: $K^{-1} = (1\;2\;3\;4\;5\;2\;5\;4\;1\;3\;) = (1\;2\;3\;4\;5\;\%\;O\;\%\;C\;B\;2\;5\;4\;1\;3\;O\;B\;C\;\%\;\%\;)$.
Sehingga diperoleh plainteks yaitu XTOQAUO#AYB#QOFOBC%%. Diketahui karakter # menggantikan spasi dan karakter % menggantikan kekurangan pada blok maka plainteks yang diperoleh adalah XTOQAUO AYB QOFOBC.

Plainteks yang diperoleh dari hasil dekripsi menggunakan cipher tranposisi grup simetri akan digunakan sebagai cipherteks pada proses dekripsi menggunakan *affine cipher*.
Cipherteks: XTOQAUO AYB QOFOBC
Kunci: $m = 11$ dan $b = 14$
Berdasarkan karakter alfabet yang digunakan, maka cipherteks XTOQAUO AYB QOFOBC ekuivalen dengan 23 19 14 16 0 20 14 0 24 1 16 14 5 14 1 2. Untuk melakukan dekripsi, maka harus ditentukan $m^{-1}(mod\; n)$ terlebih dahulu. Menentukan $m^{-1}(mod\; n)$ dapat dihitung dengan menggunakan kekongruenan linier.
Misalkan $11^{-1}(mod\; 26) = x$ maka $x = 19$ karena $11 \cdot 19\; (mod\; 26) = 1$.
Berdasarkan (2) maka proses perhitungan dekripsi menggunakan algoritma *Affine Cipher* adalah sebagai berikut:
$c_1 = 23 \to p_1 = 19(23 - 14)\; mod\; 26 = 171\; mod\; 26 = 15$ (huruf P)
$c_1 = 19 \to p_1 = 19(19 - 14)\; mod\; 26 = 95\; mod\; 26 = 17$ (huruf R)
$c_1 = 14 \to p_1 = 19(14 - 14)\; mod\; 26 = 0\; mod\; 26 = 0$ (huruf A)
$c_1 = 16 \to p_1 = 19(16 - 14)\; mod\; 26 = 38\; mod\; 26 = 12$ (huruf M)
$c_1 = 0 \to p_1 = 19(0 - 14)\; mod\; 26 = -266\; mod\; 26 = 20$ (huruf U)
$c_1 = 20 \to p_1 = 19(20 - 14)\; mod\; 26 = 114\; mod\; 26 = 10$ (huruf K)
$c_1 = 14 \to p_1 = 19(14 - 14)\; mod\; 26 = 0\; mod\; 26 = 0$ (huruf A)
$c_1 = 0 \to p_1 = 19(0 - 14)\; mod\; 26 = -266\; mod\; 26 = 20$ (huruf U)
$c_1 = 24 \to p_1 = 19(24 - 14)\; mod\; 26 = 190\; mod\; 26 = 8$ (huruf I)
$c_1 = 1 \to p_1 = 19(1 - 14)\; mod\; 26 = -247\; mod\; 26 = 13$ (huruf N)
$c_1 = 16 \to p_1 = 19(16 - 14)\; mod\; 26 = 38\; mod\; 26 = 12$ (huruf M)
$c_1 = 14 \to p_1 = 19(14 - 14)\; mod\; 26 = 0\; mod\; 26 = 0$ (huruf A)
$c_1 = 5 \to p_1 = 19(5 - 14)\; mod\; 26 = -171\; mod\; 26 = 11$ (huruf L)
$c_1 = 14 \to p_1 = 19(14 - 14)\; mod\; 26 = 0\; mod\; 26 = 0$ (huruf A)
$c_1 = 1 \to p_1 = 19(1 - 14)\; mod\; 26 = -247\; mod\; 26 = 13$ (huruf N)
$c_1 = 2 \to p_1 = 19(2 - 14)\; mod\; 26 = -228\; mod\; 26 = 6$ (huruf G)
Jadi, plainteks yang dihasilkan adalah PRAMUKA UIN MALANG.

3. Enkripsi Menggunakan Algoritma Modifikasi Super Enkripsi (Modifikasi *Affine Cipher* dan *Cipher* Transposisi Grup Simetri)

Sama halnya dengan pada proses enkripsi *affine cipher,* dengan menggunakan kunci $m = 11$ dan $b = 14$, serta $k = 5$, maka diperoleh:
Plainteks : PRAMUKA UIN MALANG
Pembagian Plainteks : PRAMU-KA UI-N MAL-ANG
Pembalikan Posisi Plainteks :UMARP-IU AK-LAM N-GNA

Karakter alfabet yang digunakan pada algoritma modifikasi *affine cipher* sejumlah 26 huruf dimana A=0, B=1, C=2,…, Z=25, sehingga proses numerik akan menjadi 20 12 0 17 15-8 20 0 10- 11 0 12 13- 6 13 0 Karena kata yang digunakan beserta bilangan kunci sama dengan proses algoritma super enkripsi sehingga dengan proses modulo pada bagian 2 diperoleh 0 16 14 19 23- 24 0 14 20-5 14 16 1-2 1 14. Yang dikonversi menjadi Cipherteks AQOTX-YA OU-FOQ B-CBO

Selanjutnya, cipherteks tersebut dienkripsi kembali menggunakan cipher transposisi grup simetri. Berikut merupakan proses enkripsi menggunakan cipher transposisi grup simetri:

Kunci: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ )$

Membagi plainteks menjadi blok-blok yang terdiri dari lima huruf dengan ketentuan jika terdapat kekurangan pada blok maka ditambahkan dengan karakter %. Sedangkan untuk spasi diganti dengan karakter #.

AQOTX	YA#OU	FOQ#B	CBO%%

Kemudian, setiap blok diubah menjadi seperti di bawah ini dengan menggunakan kunci yang telah ditentukan.

Blok 1: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ ) = (1\ 2\ 3\ 4\ 5\ AQOTX\ 4\ 1\ 5\ 3\ 2\ TAXOQ\ )$

Blok 2: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ ) = (1\ 2\ 3\ 4\ 5\ YA\#OU\ 4\ 1\ 5\ 3\ 2\ OYU\#A)$

Blok 3: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ ) = (1\ 2\ 3\ 4\ 5FOQ\#B4\ 1\ 5\ 3\ 2\ \#FBQO)$

Blok 4: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ ) = (1\ 2\ 3\ 4\ 5\ CBO\%\%4\ 1\ 5\ 3\ 2\ \%C\%OB)$

Sehingga diperoleh cipherteks yaitu TAXOQOYU#A#FBQO%C%OB

4. Dekripsi Menggunakan Algoritma Super Dekripsi (*Affine Cipher* dan *Cipher* Transposisi Grup Simetri)

Pengembalian plainteks menjadi pesan teks semula (plainteks) dilakukan dengan proses dekripsi menggunakan *cipher* transposisi grup simetri terlebih dahulu. Proses dekripsi menggunakan cipher transposisi grup simetri dilakukan dengan cara yang sama seperti proses enkripsi, namun dengan menggunakan kunci invers. Berikut adalah proses dekripsi menggunakan cipher transposisi grup simetri.

Cipherteks: TAXOQOYU#A#FBQO%C%OB

Kunci: $K = (1\ 2\ 3\ 4\ 5\ 4\ 1\ 5\ 3\ 2\ )$

$K^{-1} = (4\ 1\ 5\ 3\ 2\ 1\ 2\ 3\ 4\ 5\ )$

$$K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ )$$

Membagi plainteks menjadi blok-blok yang terdiri dari lima huruf sebagai berikut:

TAXOQ	OYU#A	#FBQO	%C%OB

Selanjutnya, setiap blok diubah menjadi seperti di bawah ini dengan menggunakan kunci yang telah ditentukan.

Blok 1: $K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ ) = (1\ 2\ 3\ 4\ 5TAXOQ2\ 5\ 4\ 1\ 3AQOTX\ )$

Blok 2: $K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ ) = (1\ 2\ 3\ 4\ 5\ OYU\#A\ 2\ 5\ 4\ 1\ 3\ YA\#OU\ )$

Blok 3: $K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ ) = (1\ 2\ 3\ 4\ 5\ \#FBQO2\ 5\ 4\ 1\ 3\ FOQ\#B4\ )$

Blok 4: $K^{-1} = (1\ 2\ 3\ 4\ 5\ 2\ 5\ 4\ 1\ 3\ ) = (1\ 2\ 3\ 4\ 5\ \%C\%OB2\ 5\ 4\ 1\ 3\ CBO\%\%\ )$.

Sehingga diperoleh plainteks yaitu AQOTXYA#OUFOQ#BCBO%% Diketahui karakter # menggantikan spasi dan karakter % menggantikan kekurangan pada blok maka plainteks yang diperoleh adalah AQOTXYA OUFOQ BCBO.

Plainteks yang diperoleh dari hasil dekripsi menggunakan cipher transposisi grup simetri akan digunakan sebagai cipherteks pada proses dekripsi menggunakan *affine cipher*.

Cipherteks: AQOTXYA OUFOQ BCBO

Kunci: $m = 11$ dan $b = 14$

Berdasarkan karakter alfabet yang digunakan, maka cipherteks AQOTXYA OUFOQ BCBO ekuivalen dengan diperoleh 0 16 14 19 23 24 0 -14 20 5 14 16 - 1 2 1 14. Dengan proses yang sama pada algoritma super enkripsi maka diperoleh numerik 20 12 0 17 15 8 20- 0 10 11 0 12 -13 6 13 0. Lalu dikembalikan ke huruf abjad menjadi UMARPIU AKLAM NGNA. Kemudian dikelompokan ke dalam kata yang beranggotakan lima karakter menjadi UMARP-IU AK-LAM N-GNA. Proses terakhir adalah proses membalikan posisi karakter sehingga diperoleh PRAMUKA UIN MALANG.

Kesimpulan

Berdasarkan pembahasan mengenai algoritma yang menggabungkan *affine cipher* dan *cipher* transposisi grup simetri, serta modifikasi *affine cipher* dan *cipher* transposisi grup simetri maka dapat ditarik kesimpulan bahwa proses enkripsi dilakukan melalui dua tahap sehingga dapat meningkatkan keamanan dan menghasilkan *cipher* yang sulit dipecahkan. Proses pertama adalah mengenkripsi plainteks menggunakan algoritma *Affine cipher* atau modifikasi *affine cipher*. Kemudian dilanjut dengan proses kedua, yaitu mengenkripsikan kembali cipherteks dari *Affine cipher* menggunakan algoritma *cipher* transposisi grup simetri. Sedangkan proses pengembalian cipherteks menjadi plainteks diawali dengan mendekripsikan cipherteks menggunakan algoritma cipher transposisi grup simetri. Kemudian hasil dari proses dekripsi cipher transposisi grup simetri didekripsikan kembali menjadi plainteks menggunakan algoritma *Affine cipher* atau modifikasi *affine cipher*.

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Analisis Faktor yang Mempengaruhi Risiko Gagal Bayar Debitur pada Lembaga Keuangan Mikro Menggunakan Regresi Logistik dan *Ant Colony Optimization* (ACO)

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Abstrak

Paper ini bertujuan untuk melakukan analisis faktor-faktor yang mempengaruhi risiko gagal bayar dari calon debitur. Metode yang digunakan adalah regresi logistic dan *Ant Colony Optimization* (*ACO*). Terdapat beberapa tahap dalam penelitian ini: (1) melakukan standarisasi data pada data faktor risiko calon debitur, (2) menetapkan asumsi model regresi logistik, (3) melakukan estimasi parameter model regresi logistik menggunakan algoritma *ACO*, dan (4) melakukan uji signifikansi setiap variabel. Dalam paper ini, data yang digunakan adalah data historis debitur sebuah Bank pada periode 2001-2011 pada Lembaga Keuangan Mikro (LKM) di Bandung, Indonesia. Hasilnya signifikansi koefisien regresi menunjukkan bahwa lima faktor yang dianalisis berpengaruh signifikan terhadap risiko gagal bayar, yaitu: usia, jumlah tanggungan keluarga, nilai jaminan, besarnya kredit yang diajukan, dan jangka waktu pengembalian kredit, dengan kekuatan korelasi sebesar 93.5%. Probabilitas risiko gagal bayar ditentukan oleh kelima faktor yang mempengaruhi. Mengetahui nilai probabilitas sangat berguna bagi LKM guna menentukan klasifikasi faktor kelayakan pemberian kredit berdasarkan predikat risiko calon debitur. Demikian sehingga, LKM dapat mengetahui faktor-faktor risiko gagal bayar dan mengambil keputusan pemberian kredit yang layak atau tidak layak.

Kata kunci: Ant Colony Optimization (ACO), faktor risiko debitur, Lembaga Keuangan Mikro (LKM), model regresi logistik, risiko gagal bayar

Abstract

This article is aimed to analyze the factors that influence the risk of default from prospective debtors. The methods that are used are logistic regression and *Ant Colony Optimization* (*ACO*). There are some steps in this research, such as: (1) standardizing the data of prospective debtor risk factor; (2) defining the assumptions of logistic regression model; (3) estimating the parameters of logistic regression model by using *ACO* algorithm; and (4) significance testing for each variable. In this article, the data used is debtor's historical banking data for 2001-2011 period, in microfinance institution Bandung, Indonesia. As a result, the factors are regression coefficient significance shows that five analyzed factors give impact to the risk of default. They are age, number of family dependents, value of collateral, amount of credit applied for, and credit repayment period. The correlation is 93.5%. Probability of risk of default is determined from the five factors. Finding out the value of probability is very useful for microfinance institutions to determine the classification of

credit eligibility factors based on the risk predicate of the prospective debtor. In this way, microfinance institutions can identify risk factors for default and make decisions about granting credit that is appropriate or not.
Keywords: Ant Colony Optimization (ACO), logistic regression method, microfinance institution, risk factor of debtor, risk of default

Pendahuluan

Dewasa ini mekanisme keuangan dan kredit mikro merupakan strategi penting dalam penanggulangan kemiskinan. Buktinya adalah pemerintah pusat maupun daerah menyalurkan dana bergulir kepada kelompok masyarakat melalui Lembaga Keuangan Mikro (LKM). Harapan dari mekanisme tersebut adalah membantu ekonomi masyarakat dengan menyalurkan dana kepada usaha mikro yang terus berkembang [1]. Penyaluran dana dari LKM kepada usaha-usaha mikro, salah satunya dengan transaksi kredit. Transaksi kredit ini dapat terjadi jika terdapat suatu keinginan, khususnya para pengusaha yang kekurangan modal untuk memperlancar usahanya. Pada proses penyaluran kredit, LKM sering dihadapkan pada suatu risiko yang dikenal dengan risiko gagal bayar debitur [2], [3]. Oleh karena itu, diperlukan analisis faktor-faktor yang mempengaruhi risiko gagal bayar untuk mengantisipasi terjadinya kesalahan LKM dalam menganalisis kredit terhadap debiturnya. Adapun hasil analisis faktor-faktor risiko berupa klasifikasi predikat kredit untuk debitur dalam kategori baik atau buruk sebagai acuan untuk pemberian kredit [4],[5].

Beberapa kajian terkait dalam antisipasi gagal bayar kredit dengan berbagai teknik klasifikasi calon debitur yang telah diusulkan diantaranya: pengklasifikasian menggunakan *Neural Networks* dan *SVM* digambarkan sebagai fungsi matematika yang kompleks [6]. Mempelajari pembentukan model penilaian kredit untuk lembaga keuangan di Bank Jerman menggunakan data pendekatan penambangan untuk analisis [7]. Penerapan *Ant Colony Optimasi* (*ACO*) pada bidang penambangan data untuk mengekstraksi pengklasifikasi berbasis aturan [8],[9]. Penelitian tentang performansi model *credit scoring* menggunakan metode regresi logistik biner dan teknik data mining seperti *Classification and Regression Tree* (*CART*), *Chi-squared Automatic Interaction Detection* (*CHAID*), *Neural Network*, serta *Multivariate Adaptive Regression Spline* (*MARS*) [10]. Penelitian risiko kredit di bank komersial Pakistan dilakukan dengan model penilaian kredit dengan model penilaian kredit untuk individu, regresi logistik, dan analisis diskriminan [11]. Penelitian lain tentang penilaian kredit didasarkan pada analisis diskriminan, yaitu metode statistik yang dirancang untuk mengklasifikasikan debitur ke dalam kelompok yang dapat dibedakan dengan jelas secara optimal baik atau buruk yang dilakukan pada penilaian risiko kredit Usaha Kecil dan Menengah (UKM) [12].

Penggunaan metode pendekatan algoritma *ACO* pada model regresi logistik untuk menganalisis faktor-faktor risiko gagal bayar debitur belum pernah dilakukan. Algoritma *ACO* ini menggunakan lima faktor yang berpengaruh signifikan terhadap risiko gagal bayar, yaitu usia, jumlah tanggungan keluarga, nilai jaminan, besarnya kredit yang diajukan, dan jangka waktu pengembalian kredit dengan tingkat korelasi sebesar 93,5%. Algoritma *ACO* yang dihasilkan lalu diterapkan untuk mengklasifikasikan debitur terhadap risiko gagal bayarnya. Tujuan dari paper ini adalah untuk mendapat klasifikasi subjek calon debitur menjadi beberapa interval tingkat risiko gagal bayar untuk yang baik atau buruk. Penelitian ini melakukan analisis faktor-faktor risiko gagal bayar untuk mengklasifikasikan predikat risiko debitur [13],[14]. Proses analisis dilakukan dengan regresi logistik berdasarkan faktor-faktor risiko debitur, dimana untuk menaksir parameter dilakukan menggunakan algoritma *ACO*. Selanjutnya, estimator model regresi logistik digunakan untuk menentukan probabilitas gagal bayar dari calon debitur. Taksiran probabilitas gagal bayar kemudian dicocokkan dengan interval kelayakan debitur, sehingga setiap debitur menyandang predikat kredit yang menggambarkan klasifikasi tingkat risiko gagal bayar. Berdasarkan klasifikasi tersebut, LKM mendapat analisis faktor risiko untuk mengambil sebuah keputusan pemberian kredit kepada debitur.

Metode

1. Metode Penelitian

Dalam penelitian ini secara keseluruhan akan dilakukan analisis secara kuantitatif dengan memanfaatkan matematika dalam penyelesaian perhitungan dan studi pustaka. Studi kepustakaan adalah teknik pengumpulan data dengan mengadakan studi penelaahan terhadap buku-buku, literatur-literatur yang berhubungan dengan masalah yang akan diselesaikan dalam penelitian ini. Masalah yang diteliti yaitu analisis faktor-faktor yang mempengaruhi risiko gagal bayar debitur pada LKM. Hasil dari penelitian ini adalah predikat klasifikasi risiko kredit berdasarkan faktor risiko gagal bayar debitur sebagai acuan pemberian kredit yang tepat sasaran untuk LKM.

2. Data Penelitian

Data yang digunakan pada penelitian ini diperoleh dari data kredit konsumtif (non ritel) debitur Bank Z periode 2001-2011. Data terdiri dari 100 sampel yang terbagi dalam 2 kategori, yaitu kategori 0 sebagai kredit tidak bermasalah atau dikatakan layak berjumlah $\mathrm{n}_0 = 84$ dan kategori 1 yang dikatakan kredit bermasalah atau tidak layak berjumlah $\mathrm{n}_1 = 16$. Variabel-variabel yang berpengaruh pada analisis penilaian kredit yang dilakukan dalam penelitian ini, yaitu terdiri dari lima faktor (variabel bebas), antara lain usia calon debitur (X_1), jumlah tanggungan keluarga (X_2), nilai jaminan (X_3), besarnya kredit yang diajukan (X_4), dan jangka waktu pengembalian kredit (X_5).

Lima faktor sebagai parameter analisis pada penelitian ini berdasarkan kriteria variabel debitur, sebagai berikut: (1) Usia (X_1): Bank Indonesia membatasi usia 21 tahun hingga 65 tahun yang dapat memiliki akses kredit, aturan pembatasan usia ini masuk dalam revisi peraturan Bank Indonesia Nomor 11 Tahun 2009; (2) Jumlah Tanggungan Keluarga (X_2): Jumlah anggota keluarga yang ditanggung oleh pihak debitur merupakan jumlah tanggungan keluarga. Semakin sedikit jumlah tanggungan keluarga, semakin kecil kemungkinan terjadinya gagal bayar; (3) Nilai Jaminan (X_3): Nilai jaminan merupakan sejumlah nilai dari barang yang dijaminkan debitur sebagai syarat pengajuan kredit, yang diukur dalam satuan rupiah. Nilai jaminan yang tinggi menjadi bahan pertimbangan yang baik untuk memberikan pinjaman kredit kepada debitur; (4) Besarnya Kredit yang Diajukan (X_4): Besarnya kredit yang diajukan debitur akan menjadi bahan pertimbangan dalam mengambil keputusan. Lembaga keuangan akan menyeleksi calon debitur berdasarkan riwayat finansial dan aset yang dimiliki sekarang. Hal ini dilakukan untuk meminimalisir terjadinya gagal bayar; (5) Jangka Waktu Pengembalian Kredit (X_5): Jangka waktu pengembalian kredit bergantung pada besarnya pinjaman dan kemampuan bayar debitur. Jangka waktu ini dinyatakan dalam satuan tahun atau bulan. Pada penelitian ini analisis faktor risiko yang digunakan yaitu analisis regresi logistik biner.

3. Model Regresi Logistik Biner

Analisis regresi logistik merupakan alat analisis data yang digunakan pada penelitian dengan tujuan untuk mengidentifikasi pengaruh variabel bebas (X) terhadap variabel terikat (Y), dimana variabel bebas dalam penelitian bersifat kategorik. Prinsip-prinsip yang digunakan dalam analisis regresi logistik, pada dasarnya sama dengan prinsip dalam analisis regresi linier secara umum.

Perbedaannya hanya dalam hal skala pengukuran dari variabel bebas (Y), sehingga teknik-teknik yang digunakan dalam analisis regresi linier juga dapat digunakan dalam analisis regresi logistik [15],[16].

Analisis regresi logistik biner digunakan untuk menaksir pengaruh dari beberapa variabel penjelas (X), terhadap variabel respon (Y), yang bersifat biner atau dikotomi. Variabel ini dikatakan biner atau dikotomi karena memiliki dua nilai kemungkinan, yaitu 0 dikatakan berhasil dan 1 dikatakan gagal. Bentuk persamaan regresi logistik biner yang digunakan adalah:

$$\pi(x_i) = \frac{\exp(\beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki})}{1 + \exp(\beta_0 + \beta_1 x_{1i} + \ldots + \beta_k x_{ki})} \qquad i = 1,2,\ldots,N \tag{1}$$

Fungsi $\pi(X)$ berbentuk non linier sehingga perlu ditransformasikan dengan menggunakan transformasi logit, sehingga diperoleh fungsi $g(X)$ yang linier dalam parameter-parameternya [16]. Fungsi logitnya adalah:

$$g(X) = \log\left[\frac{\pi(X)}{1 - \pi(X)}\right] = \sum_{k=0}^{K} x_{ik}\beta_k \tag{2}$$

4. Penaksiran Parameter Model Regresi Logistik

Tujuan dari penaksiran model regresi logistik adalah untuk menaksir parameter-parameter β_k, yang berkontribusi pada persamaan (2). Andaikan terdapat k variabel bebas $X_1, X_2, \ldots, X_k$, merujuk fungsi densitas peluang bersyarat Y terhadap β mengikuti distribusi bernoulli adalah [17]:

$$f(y|\beta) = \prod_{i=1}^{N} \pi_i^{y_i}(1 - \pi_i)^{1-y_i} \qquad y_i = 0,1 \tag{3}$$

Y diberi kode dengan 0 dan 1 untuk setiap pasangan x_i, y_i. Jika $y_i = 1$, maka kontribusi untuk fungsi Likelihood adalah $\pi(x_i)$, dan jika $y_i = 0$ maka kontribusi untuk fungsi Likelihood adalah $1 - \pi(x_i)$, di mana $\pi(x_i)$ menuliskan nilai dari $\pi(x)$ pada x_i [18]. Demikian sehingga kontribusi untuk fungsi likelihood dari pasangan (x_i, y_i) adalah:

$$l(\beta) = \sum_{i=1}^{N} \left\{ y_i \sum_{j=0}^{J} x_{ij}\beta_j - In\left(1 + e^{\sum_{j=0}^{J} x_{ij}\beta_j}\right) \right\} \tag{4}$$

5. Ant Colony Optimization (ACO)

Algoritma *Ant Colony Optimization* (ACO) diusulkan oleh Marc Dorigo pada tahun 1992, sebagai algoritma pencarian metaheuristik yang terinspirasi dari kelompok semut yang biasa dilakukan untuk mencari jalur terpendek dalam mencari makanan. Semut menggunakan komunikasi tidak langsung yang terjadi di antara semut ketika melakukan perjalanan ke sumber makanan, dan kemudian membawa kembali ke sarang mereka. Algoritma ACO adalah sebuah model yang dikembangkan dengan melihat semut sebagai objek utama pembentuk algoritmanya [18],[19].

Sesuai dengan algoritma ACO yang terinspirasi oleh perilaku koloni semut mencari makanan. Semut menemukan jarak terpendek antara sarang semut dan sumber makanan. Untuk menandai jalan yang mereka lalui ditandai dengan feromon. Feromon adalah bahan kimia yang berasal dari kelenjar endokrin. Semut dapat mencium bau feromon, dan cenderung memilih jalur yang telah ditandai dengan feromon. Jika semut telah menemukan jalur terpendek, semut melanjutkan perjalanannya melalui jalur itu. Jalur terpendek memiliki ketebalan feromon dengan probabilitas tinggi [20],[21],[22],[23]. dalam algoritma ACO, semut dapat berpindah dari vertex i ke vertex j dengan probabilitas yang dihitung sebagai berikut [24]:

Pemilihan Jalur: Semut berjalan dari titik i ke titik j dengan probabilitas seperti diberikan dengan persamaan berikut:

$$p_{i,j} = \frac{\left(\tau_{i,j}^{\alpha}\right)\left(\eta_{i,j}^{\beta}\right)}{\sum \left(\tau_{i,j}^{\alpha}\right)\left(\eta_{i,j}^{\beta}\right)} \quad (5)$$

di mana $\tau_{i,j}$: jumlah feromon di samping i, j; α : parameter pengontrol pengaruh $\tau_{i,j}$; $\eta_{i,j}$: sisi diinginkan i, j (umumnya $1/d_{i,j}$, di mana d adalah jarak); β : parameter pengontrol pengaruh $\eta_{i,j}$.

Penambahan dan penguapan feromon, secara matematis diberikan sebagai persamaan berikut ini:

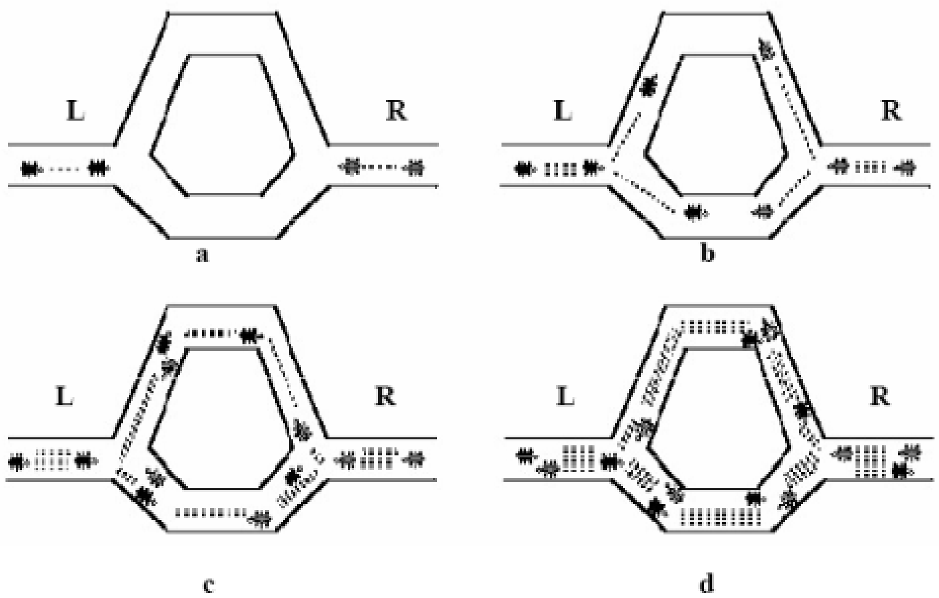
$$\tau_{i,j} = (1-\rho)\tau_{i,j} + \Delta\tau_{i,j} \quad (6)$$

dimana ρ: tingkat penguapan feromon; dan $\Delta\tau_{i,j}$: jumlah feromon yang dihasilkan.

Selanjutnya, memperbarui feromon secara algoritma matematis diberikan seperti pada persamaan sebagai berikut:

$$\tau(t, v) \leftarrow (1-\alpha).\tau(t, v) + \alpha.\Delta\tau(t, v) \quad (7)$$

Pola rute perjalanan semut-semut dari sarang menuju sumber makanan, dan sebaliknya dari sumber makanan kembali ke sarangnya, dapat diilustrasikan seperti pada Gambar 1.



Gambar 1. Perjalanan semut dari sarang ke sumber makanan

Memperhatikan Gambar 1, dapat diceritakan secara singkat sebagai berikut: (a) Pada awalnya semut-semut melakukan perjalanan secara acak dari sarang ke sumber makanan, ada yang melalui jalur panjang dan melalui jalur pendek. (b) Baik semut-semut yang melalui jalur panjang dan jalur pendek ditandai menggunakan feromon. (c) Semut-semut telah menemukan jalur terpendek, dan memperbarui feromon. (d) Akhirnya semua semut-semut berlalu-lalang dari sarang ke sumber makanan, dan sebaliknya melalui jalur terpendek.

Prinsip perjalanan semut menemukan jalur terpendek inilah, selanjutnya digunakan untuk estimasi parameter model regresi logistik, dengan menggunakan suatu algoritma yang dinamakan Ant Colony Optimization (ACO).

1) Uji Signifikansi Parameter

Uji signifikansi parameter di sini, dimaksudkan untuk menguji tingkat signifikansi dari estimator parameter-parameter yang telah diperoleh dari estimasi yang dilakukan menggunakan ACO. Pada uji signifikansi parameter logistik dilakukan dengan beberapa pengujian diantaranya yaitu, uji Likelihood ratio, uji Wald, uji Hosmer dan Lemeshow, dan R^2 dengan $\alpha = 5\%$.

- Uji Likelihood Ratio

Pada bagian ini dibahas metode pengujian serempak menggunakan statistik uji Likelihood ratio G. langkah-langkah uji Likelihood ratio yaitu perumusan hipotesis: $H_0 : \beta_1 = \beta_2 = \cdots \beta_k = 0$, artinya secara simultan variabel bebas tidak mempunyai pengaruh terhadap $\pi(X)$; melawan alternatif $H_1 : \beta_1 \neq \beta_2 \neq \cdots \beta_k \neq 0$, artinya secara simultan variabel bebas mempunyai pengaruh terhadap $\pi(X)$ [15]. Selanjutnya, menentukan statistik Likelihood ratio G hasil perhitungan berdasarkan persamaan:

$$G = -2 \frac{\left(\frac{n_1}{n}\right)^{n_0}}{\prod_{\pi_i}^{y_i} \quad (1-\pi)^{1-y_i}} ; i = 1,2, \dots, k, \tag{8}$$

dengan $i = 1, 2, \dots, k$ dengan k banyaknya variabel bebas. Menggunakan tingkat signifikansi α, nilai kritis statistik ${\chi^2}_{(1-\alpha)(1)}$ diperoleh dari Tabel χ^2 (Chi-Square). Kemudian membandingkan statistik G dan ${\chi^2}_{(1-\alpha)(1)}$, dengan kriteria sebagai berikut: Jika $\hat{G} < {\chi^2}_{(1-\alpha)(1)}$ maka H_0 diterima dan H_1 ditolak, dan Jika $\hat{G} \geq {\chi^2}_{(1-\alpha)(1)}$ maka H_0 ditolak dan H_1 diterima.

- Uji Wald

Merujuk Hosmer et al. (1989), langkah-langkah uji Wald yaitu perumusan hipotesis: $H_0 : \beta_i = 0$, ($\mathrm{i} = 1, 2, \dots, \mathrm{k}$ dengan k banyaknya variabel bebas), artinya variabel X tidak mempunyai pengaruh terhadap $\pi(X)$; melawan alternatif $\mathrm{H_1}: \beta_\mathrm{i} \neq 0$, artinya variabel X mempunyai pengaruh terhadap $\pi(X)$ [15]. Selanjutnya, menentukan statistik Z hitung, dan nilai kritis statistik $\mathrm{Z}_{(1-\frac{1}{2}\alpha)}$ diperoleh dari tabel distribusi normal standar. Kemudian membandingkan statistik Z dan $\mathrm{Z}_{(1-\frac{1}{2}\alpha)}$, dengan kriteria: Jika $-\mathrm{Z}_{\frac{1}{2}\alpha} < \mathrm{Z} < \mathrm{Z}_{(1-\frac{1}{2}\alpha)}$ maka $\mathrm{H_0}$ diterima dan $\mathrm{H_1}$ ditolak. Jika $\mathrm{Z} > \mathrm{Z}_{(1-\frac{1}{2}\alpha)}$ atau $\mathrm{Z} < -\mathrm{Z}_{\frac{1}{2}\alpha}$ maka $\mathrm{H_0}$ ditolak dan $\mathrm{H_1}$ diterima.

- Uji Kecocokan Model Regresi Logistik

Merujuk Hosmer et al. (1989), langkah-langkah pengujian kecocokan model regresi logistik dapat dilakukan menggunakan uji Hosmer dan Lemeshow [15]. Perumusan hipotesis yang digunakan adalah $\mathrm{H_0}$: Tidak terdapat perbedaan antara hasil pengamatan dengan hasil penaksiran; melawan alternatif $\mathrm{H_1}$: Terdapat perbedaan antara hasil pengamatan dengan hasil penaksiran. Statistik uji yang digunakan adalah sebagai berikut:

$$C = \sum_{j=1}^{n} \frac{(o_k - n_k \underline{\pi}_k)}{n_k \underline{\pi}_k (1 - \underline{\pi}_k)}, \tag{9}$$

dengan $o_k = \sum_{j=1}^{C_k} y_j$, $\underline{\pi}_k = \sum_{j=1}^{C_k} \frac{m_j \hat{\pi}_j}{n_k}$, dan n_k banyaknya pengamatan grup k. Selanjutnya, membandingkan antara nilai statistik C dengan nilai kritis statistik Chi Squared ${\chi^2}_{(\alpha)(v-1)}$ yang diperoleh dari tabel χ^2 (Chi-Square), dimana α tingkat signifikansi dan v derajat kebebasan. Jika nilai $C > {\chi^2}_{(\alpha)(v-1)}$ atau $P - value < \alpha$ maka H_0 ditolak dan H_1 diterima.

- Menentukan nilai R-kuadrat (R^2)

Nilai R-kuadrat R^2 adalah digunakan untuk mengukur kekuatan korelasi antara variabel bebas dengan variabel tak bebas. Besarnya nilai statistik R^2 dapat ditentukan dengan menggunakan persamaan:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \pi_i)^2}{\sum_{i=1}^{n}(y_i - \underline{y})^2}, \quad (10)$$

dengan y_i adalah nilai biner hasil observasi ke-i (misalnya 1 atau 0), $\underline{\pi}_i$ adalah prediksi nilai probabilitas ke-i, $\underline{y} = \sum_{i=1}^{n} y_i / n$, dan n banyak data yang diobservasi. Nilai $0 \leq R^2 \leq 1$, dimana jika nilai R^2 mendekati 0 maka berarti korelasinya lemah, dan jika nilai R^2 mendekati 1 maka berarti korelasinya kuat.

2) Probabilitas Risiko Gagal Bayar

Probabilitas risiko gagal bayar (problem loans) adalah merupakan salah satu dampak negatif dari kegiatan pemberian kredit. Hubungan antara peluang gagal bayar dengan $\Lambda\ (score_i)$, yaitu: (a) $Prob(Default_i) = \Lambda(score_i)$ jika terjadi *problem loans*; (b) $Prob(No\ Default_i) = 1 - \Lambda(score_i)$ jika tidak terjadi *problem loans* [17], [25].

Peluang gagal bayar (problem loans) adalah $0 \leq \Lambda(\mathrm{score_i}) \leq 1$. Debitur dengan $\Lambda\ (\mathrm{score_i})$ kredit yang rendah memiliki peluang gagal bayar (problem loans) rendah, sedangkan debitur $\Lambda\ (\mathrm{score_i})$ kredit yang tinggi memiliki peluang gagal bayar (problem loans) tinggi. Oleh karena itu, sebuah $\Lambda\ (\mathrm{score_i})$ kredit yang tinggi dapat mengakibatkan penolakan kredit.

Probabilitas risiko gagal bayar pada tahapan ini adalah digunakan untuk referensi pengambilan keputusan pemberian kredit. Keputusan ini diambil oleh pihak-pihak bank berdasarkan kriteria-kriteria tertentu, yaitu: (1) Jika diambil $0 \leq \pi(\mathrm{X}) \leq 0.40$ maka permohonan kredit diterima; (2) Jika diambil $0.41 \leq \pi(\mathrm{X}) \leq 0.70$ maka permohonan kredit dipertimbangkan; (3) Jika diambil $0.71 \leq \pi(\mathrm{X}) \leq 1$ maka permohonan kredit ditolak.

Hasil dan Diskusi

Dalam bagian ini dilakukan pembahasan tentang standarisasi data, penaksiran parameter, pengujian signifikansi parameter, dan diskusi ilustrasi keputusan pemberian kredit calon debitur.

1. Standarisasi Data

Standarisasi data merupakan proses standarisasi terhadap data yang memiliki perbedaan nilai yang cukup besar. Standarisasi data dilakukan menggunakan Z-score, dan prosesnya dapat dilakukan dengan menggunakan software SPSS versi 23.00 agar lebih efisien. Sebagai ilustrasi, sebagai contoh digunakan sebagian data dari penelitian seperti diberikan pada Tabel 1.

Tabel 1. Data kredit

Usia Kreditor	Tanggungan Keluarga	Jaminan Kredit	Kredit Diajukan	Waktu Kredit
30	3	5.000.000	6.000.000	2
32	4	6.400.000	5.000.000	2
41	4	7.560.000	9.000.000	3

42	5	71.625.000	80.000.000	7

Seperti dapat dilihat pada Tabel 1, di mana setiap variabel memiliki satuan yang sangat jauh berbeda. Bila data tersebut langsung dianalisis maka hasil output yang diperoleh tidak menggambarkan kekuatan pengaruh masing-masing variabel independen. Oleh karena itu, perlu dilakukan transformasi data menjadi Z-score. Hasil nilai Z-score menggunakan bantuan software SPSS versi 23.00, diperoleh hasil standarisasi seperti yang diberikan pada Tabel 2.

Tabel 2. Hasil standarisasi data

Usia Kreditor	Tanggungan Keluarga	Jaminan Kredit	Kredit Diajukan	Waktu Kredit
-0.625	-0.569	-0.317	-0.236	-0.792
-0.365	0.364	-0.210	-0.305	-0.792
0.804	0.364	-0.121	-0.030	-0.126
0.934	0.130	4.776	4.841	2.536

Seperti tampak pada Tabel 2, jika nilai standarisasi data dibatasi nilainya antara -3 sampai +3, pada hasil yang diperoleh terdapat nilai Z-score melebihi +3. Misalnya pada baris keempat pada kolom jaminan dan kredit yang diajukan, nilai Z-score yang diperoleh adalah 4.776 dan 4.841, sehingga data tersebut adalah outliers. Data outlier demikian dalam penelitian ini tidak diikutkan dalam analisis penaksiran parameter, yang dilakukan pada bagian berikut ini.

2. Penaksiran Parameter

Prosedur penaksiran parameter dilakukan dengan merujuk pada persamaan (4), dengan menggunakan algoritma ACO yang dibahas pada bagian 2.2.3, dan dilakukan menggunakan bantuan software Matlab R2015A [26]. Hasil taksiran parameter dan nilai ratio likelihood diberikan pada Tabel 3.

Berdasarkan taksiran parameter yang disajikan pada Tabel 3, merujuk pada persamaan (1) estimator model regresi logistik yang diperoleh adalah sebagai berikut:

$$\hat{\pi}(x) = \frac{exp\ exp\ (3.0010 + 0.8386X_1 + 0.3017X_2 + 0.3876X_3 - 0.3113X_4 + 0.3784X_5)}{1 + exp\ exp\ (3.0010 + 0.8386X_1 + 0.3017X_2 + 0.3876X_3 - 0.3113X_4 + 0.3784X_5)}. \quad (11)$$

Selanjutnya, dengan merujuk ke persamaan (2), diperoleh fungsi logit dari model regresi pada persamaan (8) yaitu:

$$\hat{g}(X) = 3.0010 + 0.8386X_1 + 0.3017X_2 + 0.3876X_3 - 0.3113X_4 + 0.3784X_5 \quad (12)$$

Tabel 3. Taksiran parameter model regresi logistik

Koefisien Parameter	Penaksir Parameter ACO (β)	$SE(\beta)$	$Z_{Wald} = \frac{\beta}{SE(\beta)}$	*P-Value*	Keputusan
Usia Debitur X_1	0.8386	0.4818	1.7406	0.0000	Significance
Tanggungan Keluarga X_2	0.3017	0.5621	0.5367	0.0211	Significance
Nilai Jaminan X_3	0.3876	0.8190	0.4733	0.0013	Significance
Jumlah Kredit X_4	0.3113	0.2134	1,4588	0.0001	Significance
Jangka Waktu	0.3784	0.6542	0.5784	0.0015	Significance

Pinjaman X_5	
Log *likelihood ratio*	3.0010

3. Pengujian Signifikansi Model

Pada bagian ini dilakukan pengujian signifikansi penaksir model yang diperoleh dari proses penaksiran seperti diberikan dengan persamaan (11). Pengujian signifikansi penaksir model meliputi: pengujian signifikansi serempak, pengujian signifikansi parsial, pengujian kecocokan model, dan penentuan nilai R-kuadrat.

1) Pengujian Serempak

Pada bagian ini dilakukan pengujian penaksir koefisien parameter secara serempak, dengan merujuk pembahasan pada bagian 2.2.4.(a). Hipotesis yang digunakan dalam pengujian ini adalah H_0: $\hat{\beta}_1 = \hat{\beta}_2 = \cdots = \hat{\beta}_5 = 0$, artinya semua variabel bebas tidak mempunyai pengaruh terhadap $\pi(X)$; dan H_1: $\exists\hat{\beta}_1 \neq \hat{\beta}_2 \neq \cdots \neq \hat{\beta}_5 \neq 0$, artinya terdapat variabel bebas mempunyai pengaruh terhadap $\pi(X)$. Berdasarkan hasil perhitungan yang diberikan pada Tabel 3, diperoleh nilai log likelihood G= 3.000, adalah nilai maksimum log likelihood yang diperoleh dengan algoritma ACO. Menggunakan tingkat signifikansi α=0.05, dari tabel distribusi χ^2 (Chi-Square) diperoleh nilai kritis statistik ${\chi^2}_{(0.05)(5)}$=1.145476226. Oleh karena nilai statistik log likelihood ratio G lebih besar dari nilai kritis statistik ${\chi^2}_{(0.05)(5)}$, berarti hipotesis H_0 ditolak atau H_1 diterima. Demikian sehingga terdapat variabel bebas yang secara signifikan mempunyai pengaruh terhadap $\pi(X)$.

2) Pengujian Parsial

Pada bagian ini dilakukan uji signifikansi masing-masing penaksir koefisien parameter yang dihasilkan dari proses penaksiran, yang hasilnya diberikan pada persamaan (11) atau (12). Pengujian signifikansi dari masing-masing penaksir koefisien parameter dilakukan menggunakan statistik uji Wald, dengan merujuk pembahasan pada bagian 2.2.4.(b) adalah seperti sebagai berikut.

Untuk penaksir koefisien parameter Usia X_1 , yaitu $\hat{\beta}_1$=0.08386. Hipotesis yang digunakan adalah:

$H_1: \hat{\beta}_1 = 0$, artinya variabel X_1 tidak mempunyai pengaruh terhadap $\pi(x)$

$H_1: \hat{\beta}_1 \neq 0$, artinya variabel X_1 mempunyai pengaruh terhadap $\pi(x)$

Berdasarkan hasil yang diberikan pada Tabel 3, diperoleh bahwa untuk penaksir koefisien parameter $\hat{\beta}_1$ atau Usia Debitur X_1 nilai statistik Z_{Wald}=1.7406, sedangkan untuk tingkat signifikansi α=0.05 dari tabel distribusi normal standar diperoleh nilai kritis $Z_{1-\frac{\alpha}{2}}$ = 1.959963985. Jadi tampak jelaslah bahwa nilai $Z > Z\ _{1-0.052}$, yaitu diperoleh keputusan H_0 diterima atau H_1 ditolak, yang artinya bahwa penaksir koefisien parameter dari variabel X_1 secara parsial mempunyai pengaruh terhadap $\pi(x)$ dengan tingkat signifikansi α=0.05. Hal ini juga diperkuat dengan nilai dari P-Value = 0.0000 yang lebih kecil daripada tingkat signifikansi α=0.05.

Menggunakan cara sama, pengujian signifikansi juga dilakukan terhadap penaksir-penaksir koefisien parameter $\hat{\beta}_2$, $\hat{\beta}_3$, $\hat{\beta}_4$ dan $\hat{\beta}_5$. Hasil pengujian secara keseluruhan dapat dilihat pada Tabel 3.

3) Menguji Kecocokan Model Regresi Logistik

Pada bagian ini dilakukan pengujian kecocokan penaksir model regresi logistik yang diberikan sebagai persamaan (8). Pengujian kecocokan ini dilakukan dengan merujuk pembahasan pada 2.2.4.(c), dimana hipotesis yang digunakan adalah H_0 : tidak ada perbedaan antara nilai observasi dengan nilai prediksi model; dan H_1 : ada perbedaan antara nilai observasi dengan nilai prediksi model. Hasil perhitungan yang dilakukan dengan merujuk persamaan (9) diperoleh nilai statistik C=5.682 dengan nilai P-Value = 0.392. Menggunakan tingkat signifikansi α=0.05, dari tabel distribusi χ^2 (Chi-Square) diperoleh nilai kritis statistik ${\chi^2}_{(1-0.05)(4)}$=9.487729037. Oleh karena itu, nampak dengan jelas bahwa nilai statistik C lebih kecil dari nilai kritis ${\chi^2}_{(1-0.05)(4)}$ atau P-Value > α, sehingga hipotesis H_0 diterima atau H_1 ditolak. Hal ini menunjukkan bahwa tidak ada perbedaan antara nilai observasi dengan nilai prediksi model.

4) Menentukan Nilai R^2

Pada bagian ini dilakukan pengukuran kekuatan korelasi antara variabel bebas dengan variabel tak bebas, dengan cara menentukan nilai R^2 merujuk pada persamaan (10). Berdasarkan hasil perhitungan diperoleh nilai R^2= 0.935, hal ini menunjukkan bahwa penaksir analisis regresi logistik memiliki hubungan yang kuat antara variabel bebas dan variabel tak bebas. Nilai $\mathrm{R}^2 = 0.935$ memberikan arti bahwa variabel bebas (independen) yang terdiri dari usia yang berhutang, jumlah tanggungan keluarga, jumlah tabungan, besarnya jaminan, besarnya kredit, jangka waktu kredit, 93,5% dapat menjelaskan pada variabel peluang gagal bayar, dan hanya 6,5% saja yang dijelaskan oleh variabel lainnya.

4. Analisis Keputusan Pemberian Kredit

Pada bagian ini dilakukan pembahasan pengambilan keputusan pemberian kredit yang didasarkan nilai probabilitas gagal bayar. Nilai probabilitas gagal bayar ditentukan dengan menggunakan penaksir model regresi logistik yang diberikan pada persamaan (11). Berdasarkan taksiran probabilitas gagal bayar, tingkat kelayakan kredit calon debitur dalam hal ini dibagi atas tiga interval kategori kelayakan debitur, seperti diberikan dalam Tabel 4.

Tabel 4. Kelayakan kredit debitur

Probabilitas Gagal Bayar (Problem Loans)	Predikat	Kategori
$\pi(X) \leq 0.49$	A	Layak
$0.50 \leq \pi(X) \leq 0.69$	B	Cukup Layak
$0.70 \leq \pi(X) \leq 1$	C	Tidak Layak

Berdasarkan Tabel 4, kelayakan kredit debitur dibagi menjadi tiga kategori yaitu: jika debitur memperoleh nilai kelayakan kurang atau sama dengan 0.49, maka seorang debitur tersebut dikatakan layak; jika debitur memperoleh nilai di antara 0.50 dan kurang atau sama dengan 0.69, maka seorang debitur dikatakan cukup layak. Cukup layak di sini berarti, seorang debitur masih memungkinkan melakukan transaksi kredit dengan beberapa pertimbangan dari pihak kreditur, misalnya dengan menambah nilai agunan yang diajukan. Adapun jika debitur yang memperoleh nilai lebih dari atau sama dengan 0,70, maka debitur tersebut dikatakan tidak layak dan memiliki problem loans yang tinggi.

Sebagai ilustrasi, misalnya seseorang calon debitur: berusia (X_1) 40 tahun, dengan tanggungan keluarga (X_2) sebanyak 4 orang. Calon debitur ini memberikan jaminan (X_3) berupa BPKB sepeda motor, yang harganya ditaksir bernilai IDR.9.375.000,00. Calon debitur ini mengajukan pinjaman sebesar (X_4) LKM sebesar IDR.11.000.000,00, untuk keperluan keluarga (konsumtif). Jangka waktu pengembalian kredit (X_5) calon debitur adalah selama 24 bulan (2 tahun). Berdasarkan informasi yang diperoleh, diprediksi nilai peluang gagal bayar (problem loans) calon debitur adalah sebesar $\hat{\pi}(X)$=0.374. Artinya peluang gagal bayar (problem loans) calon debitur sebesar 0.374 atau 37.4%. Berdasarkan tingkat kelayakan kredit debitur yang disajikan pada Tabel 4, seorang calon debitur memiliki predikat A, yaitu layak untuk memperoleh kredit sehingga penulis memberikan saran kepada pihak bank agar menerima usulan kredit tersebut. Demikian sehingga, dapat disimpulkan bahwa calon debitur akan mampu mengembalikan kredit sesuai dengan waktu yang ditentukan. Oleh karena itu, pemberian kredit dapat disetujui atau terealisasi untuk diberikan.

Kesimpulan

Berdasarkan hasil analisis dapat ditarik kesimpulan sebagai berikut: (1) Pada penelitian ini variabel-variabel yang signifikan yaitu usia, jumlah tanggungan keluarga, nilai jaminan, besarnya kredit yang diajukan, dan jangka waktu pengembalian kredit. (2) Analisis korelasi antara variabel bebas dengan variabel tak bebas diperoleh nilai R-Kuadrat sebesar 93,21%, yang menunjukkan memiliki korelasi yang kuat. Demikian sehingga estimator model regresi logistik ini dapat digunakan untuk memprediksi risiko gagal bayar calon debitur, berdasarkan lima faktor yang dianalisis disini.

Ucapan Terima Kasih

Penelitian ini didanai oleh Kementerian Riset Teknologi dan Pendidikan Tinggi Republik Indonesia dengan skema Penelitian Dosen Pemula (PDP) dengan nomor kontrak 025/SP2H/RDPKR-MONO/LL4/2021.

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