

## Means End Analysis (MEA) Learning Model in Developing Algebraic Reasoning Ability: a Literature Study

**Aflich Yusnita Fitrianna<sup>1\*</sup> and Jarnawi Afgani Dahlan<sup>2</sup>**

<sup>1</sup> Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia,

Jl. Dr. Setiabudhi No. 229 Bandung, Indonesia

<sup>2</sup> Prodi Pendidikan Matematika, IKIP Siliwangi,

Jl. Terusan Jenderal Sudirman, Cimahi, Indonesia

\*aflichyf@upi.edu

Received: 22 April 2021 ; Accepted: 11 April 2022 ; Published: 30 Juni 2022

Doi: 10.15575/ja.v8i1.10677

### Abstrak

Penelitian ini merupakan studi literatur mendeskripsikan model pembelajaran *Means Ends Analysis* (MEA) dalam mengembangkan kemampuan penalaran aljabar. Kemampuan penalaran aljabar adalah suatu kemampuan berpikir matematika dengan melibatkan proses representasi, penalaran, analisis dalam sebuah situasi sehingga diperoleh sebuah pola yang menuju pada generalisasi. Kemampuan ini penting untuk dimiliki siswa karena penalaran aljabar dalam penggunaannya dapat mengeksplorasi struktur matematika. Salah satu model pembelajaran yang dapat diimplementasikan untuk mengembangkan kemampuan penalaran aljabar adalah model pembelajaran *Means End Analysis* (MEA). Pada penelitian mengkaji sumber data berupa 1 buku dan 22 artikel yang relevan dengan kemampuan penalaran aljabar dan model pembelajaran MEA. Teknik analisis data yang digunakan adalah analisis isi. Berdasarkan hasil kajian diperoleh bahwa Model MEA mendukung dalam mengembangkan kemampuan penalaran aljabar. Langkah-langkah pada pembelajaran MEA mendukung untuk setiap indikator dari kemampuan penalaran aljabar.

**Kata kunci:** Pembelajaran Matematika, Kemampuan Penalaran Aljabar, Model Pembelajaran *Means Ends Analysis*

### Abstract

*This research is a literature study to describe the Means Ends Analysis (MEA) learning model to develop algebraic reasoning ability. Algebraic reasoning ability is the ability to think mathematics by involving the process of representation, reasoning, and analysis in a situation so that a pattern that leads to generalization is obtained. This ability is important for students to have because it can explore mathematical structures. One of the learning models that can be implemented to develop algebraic reasoning ability is the Means End Analysis (MEA) learning model. In this study, the data sources were examined in the form of 1 book and 22 articles relevant to algebraic reasoning ability and the MEA learning model. The data analysis technique used is content analysis. Based on the results of the study, it was found that the MEA Model supports the development of algebraic reasoning ability. The steps in the MEA learning support for each indicator of the algebraic reasoning ability.*

**Keywords:** Mathematics Learning, Algebraic Reasoning Ability, Means Ends Analysis (MEA) Learning Model

## **1. INTRODUCTION**

Algebra is one of the mandatory materials studied in the 2013 Curriculum. (Freudenthal, 1975), gives the opinion that algebra is not only focused on symbols or commonly called variables but also includes 'relationships'. Algebra is one of the most important themes in developed countries. (Katz, 2001), states that Algebra learning in schools is based on the history of algebra, namely paying attention to the following points: a) the subject must have a clear focus on solving equations, for example general rules for manipulating expressions must be developed because we need them to convert complex problems into equations of the form relatively simple; b) symbolism should be introduced gradually, as needed. Students will eventually discover for themselves the advantages of symbols over words in problem solving contexts, especially when verbal procedures become very complicated, c) geometric ideas have always been part of algebra and should be used whenever possible, both in developing algebraic procedures and in developing understanding analytical geometry.

The attachment of Permendikbud No. 24 of 2016 students are introduced to algebra in grade 7. Before studying algebra, the knowledge or requirement that must be possessed by students is integer operations. The material for integer operations is a link for grade VII students in experiencing the transition from arithmetic to algebra. This transition may or may not work. From several studies on the transition from arithmetic to algebra, it was concluded that: 1) students still find it difficult to understand the equal sign (=) (Nurrahman, 2015; Pratiwi & Kurniadi, 2018; Wijaya, 2016), 2) emphasis on ignoring the use of variables and the use of arithmetic, 3) generalization, 4) abstraction, 5) giving meaning to the graph (Wijaya, 2016). Students consider the sign "=" not an equality, but a hint to do something or write down the final result (Kieran, 1995). This is the beginning of a problem in algebra learning.

One of the mathematical abilities that support algebra learning is algebraic reasoning. According to Lins (Watson, 2010), states that algebraic reasoning is a deliberate process from a context to a structure. The context referred to here can be in the form of a particular mathematical problem. Algebraic reasoning ability includes all mathematical thinking because in its use it can explore mathematical structures. Algebraic reasoning ability is the basis for mathematical thinking processes which include arithmetic because it can explore mathematical structures. Everyone has the basic ability to think algebraically because basically this ability is a human way of simplifying problems in everyday life. (Ontario Ministry of Education, 2014), states that algebraic reasoning is important because it encourages students' mathematical understanding beyond the results of special calculations and procedural application formulas. Through algebraic reasoning ability, students can show their activities in analyzing, generalizing, solving problems, building patterns shown in the form of tables, words, pictures, diagrams, and mathematical expressions.

Class VII students are usually in the age range of 12 years and over. According to Piaget's Theory (Mu'min, 2013) aged 12 years and over students are in the formal operational stage where students can use concrete operations to form more complex operations. Students do not need the help of objects or concrete events because they already have the ability to think abstractly. Mason & Hewit (Twohill, 2013) very young children already have skills consisting of algebraic reasoning. Challenging mathematics learning can provide a new learning atmosphere for students (Kariadinata et al., 2017; Susilawati et al., 2019). However, several research results related to algebraic reasoning show that algebraic reasoning abilities are not optimal to be developed. (Herutomo & Saputro, 2014), provide research results that students fail in the transition process from arithmetic to algebra, and as a result students' reasoning is only limited to inductive patterns. (Cahyaningtyas et al., 2018), obtained the results of research that students often have difficulty in finding information from the questions given so that students have difficulty predicting patterns and relating information. In algebra learning students memorize procedures that students know as operations on a series of symbols, solve artificial problems that have no meaning in students' daily lives, and are assessed by the teacher not

based on understanding the mathematical concepts and reasoning involved, but on the ability to generate answers. appropriate (Kaput & Blanton, 2005). The difficulties experienced by students in solving math or algebra problems can be caused by the low ability of the sense of symbols and the sense of structure (Sugilar et al., 2019)

Classrooms that are built to promote algebraic reasoning, are one of the appropriate professional support to be provided and influence changes in learning activities. Teachers' algebra knowledge and algebra teaching are important elements in an effort to support students' algebraic reasoning (Blanton & Kaput, 2005). Developing algebraic reasoning at the secondary school level requires more than simply moving the algebra learning curriculum to the primary school level (Kieran, 2004). The results of the research by (Glassmeyer & Edwards, 2016), concluded that the teacher who was the subject of the study described algebraic reasoning in a way that only required procedural knowledge to solve problems with a single solution, more than one solution or representation. Therefore, in teaching and learning activities, it is necessary to use a learning model that supports student activities in developing students' algebraic reasoning ability.

Mathematics learning activities in developing algebraic reasoning ability according to (Kieran et al., 2016), students can start by learning to communicate in algebraic language from its meaning, and through collective discussion, verbalization, and argumentation, gradually become proficient in generalization. Interaction between students and teachers is needed in considering, evaluating, opposing, and justifying the resolution of a problem. Students can provide different pieces of information and develop other students' explanations to jointly create a complete idea or solution. Students can use their own language and work together to make statements that are clear enough that others can understand. One of the learning models that support these activities is the Means End Analysis (MEA) learning model.

The MEA learning model is one of the innovative learning models and variations of problem solving. (Suherman, 2008), argues that the MEA learning model is a learning model that presents material using a heuristic-based problem-solving approach. This learning model emphasizes that students are not only judged from the final result but also in the learning process. Students are required to know the goal to be achieved or the problem to be solved by breaking the problem into parts (sub) objectives.

(Iba, 2006), explains that the basic idea of MEA depends on two notions, namely 1) the domain of problem solving can be represented and explored in various circumstances. In particular, the state at the start of the problem-solving exercise, the objective state in which the problem has been 'solved', and any intermediate states, can all be represented. 2) MEA makes use of the correspondence between the representation and definition of operators that change from one state to another. One can solve problems by representing them as propositions and relations. That is, important aspects are aspects that are relevant to the problem at hand and are captured as a set of objects, predicates that describe the characteristics of these objects, and the relationships that exist between these objects.

Several research results state that the MEA learning model can improve mathematical critical thinking skills (Hanifiah & Prabawati, 2019; Nurafifah et al., 2013; Taubah et al., 2018), reasoning abilities (Hidayat et al., 2020), problem solving abilities (Juanda et al., 2014; Palupi et al., 2016), and the ability to understand concepts (Supendi et al., 2017). Based on this description, the authors want to dig deeper into the characteristics of the MEA learning model so that it can develop algebraic reasoning ability in junior high school students. This literature/library study is to describe the MEA learning model that can develop algebraic reasoning ability for junior high school students so that it can be used as a reference for teachers and future researchers so that it can be applied in the learning process.

## **2. METHOD**

The research that has been done is in the form of a literature study. The literature study method is a series of process activities related to the method of collecting library data through reading and note-taking activities to processing research materials (Zed, 2014). (Danandjaja, 2014), said that library research is a method of research that uses scientifically designed references or references by collecting reference materials related to research objectives, data collection techniques, integrating data to presenting data.

This literature study was chosen to develop concepts related to the MEA learning model in developing algebraic reasoning ability of junior high school students. The steps taken are: 1) determining topics that are in line with the problem, namely developing algebraic reasoning ability, 2) determining the focus of research, namely describing how the MEA learning model can develop students' algebraic reasoning ability, 3) looking for data sources in the form of books or articles from mathematics education journals. related to algebraic reasoning ability and MEA learning models, 4) reviewing articles or books obtained, 5) presenting study results in the form of descriptions, 6) compiling reports on literature studies.

The data analysis technique used in this research is a content analysis method. In this analysis, the process of selecting, combining and sorting various characteristics of the algebraic reasoning ability and MEA learning models is carried out until the relevant ones are found.

## **3. RESULT AND DISCUSSION**

The data sources studied in this study were 1 book and 22 published articles.

### **A. Learning Model Means-Ends Analysis**

Means Ends Analysis or abbreviated as MEA consists of three words, namely means, goals and analysis. Means-Ends leads to the process of identifying the "goals" to be achieved and the results of the "means" steps/means that must be taken to achieve the goals (Matlin in (Muin et al., 2014). MEA Learning Model is one variation of the problem-solving learning model. Glass and Holyak (Umar, 2017), explain that in the MEA learning model, students are free to use strategies in problem solving, construct their own knowledge and do it repeatedly until they find mathematical evidence. (Suherman, 2008), explains that the means ends analysis learning model is a variation of problem-solving learning with the syntax, namely:

- 1) Elaboration into simpler sub-problems
- 2) Identification of different sub-problem arrangements with the aim of generating connectivity
- 3) Choose a problem-solving strategy.

MEA learning steps according to Eysenck (Umar, 2017) are as follows:

- 1) The material is presented with a heuristic-based problem-solving approach.
- 2) The material is broken down into simpler sub-problems.
- 3) Sub-problems are organized into connectivity.
- 4) Selection of settlement strategy.

The advantages of the Means End Analysis (MEA) learning model are as follows:

- 1) Improve problem solving ability.
- 2) Students are able to think creatively, carefully and able to think analytically.

Disadvantages of the Means-Ends Analysis (MEA) Learning Model are as follows:

- 1) Creating meaningful problem-solving questions for students is not an easy thing.

- 2) Expressing problems that students can immediately understand is very difficult so that many students have difficulty how to respond to the problems given.

## **B. Algebraic Reasoning Ability**

Algebraic reasoning is a process in which students generalize mathematical ideas from a particular set of examples, establish generalizations through argumentation, and express them in a more formal and age-appropriate way. Algebraic reasoning ability is one of the mathematical thinking abilities (Kaput & Blanton, 2005). According to (Kieran, 2004), algebraic reasoning ability is the ability to think that involves the development of mathematical thinking by building definitions/understanding of symbols and algebraic operations. Algebraic reasoning is related to a child's ability to think logically about quantities (known or unknown) and the relationships between them (Kaput, Blanton & Moreno in (Twohill, 2013)). (Carpenter & Levi, 2000) define algebraic reasoning by identifying two main topics, namely making generalizations and using symbols to represent mathematical ideas and solve problems.

There are several levels of development of the algebraic reasoning ability which are known as Growth Points. This growth point can be used to see the development of students' algebraic reasoning abilities starting from elementary school age.

**Table 1.** Growth Points in Algebraic Reasoning Ability

<i><b>Growth Point</b></i>	<i><b>Characteristics</b></i>
<b>GP 0: Pre Formal Pattern</b>	Children have no formal understanding of "patterns". Children cannot identify terms that are repeated in a pattern.
<b>GP 1: Informal Pattern</b>	Children can identify similarities and demonstrate understanding of patterns by copying, expanding, inserting missing terms, in visual, numerical, repeating and growing spatial patterns.
<b>GP 2: Generalization</b>	Children can correctly identify in no time. Children can describe a pattern explicitly Children can offer other possibilities by reasoning.
<b>GP 3: Abstract Generalization</b>	Children can describe patterns explicitly, describe rules as expressions in symbolic notation and use those expressions to generate other terms.

Sumber: (Twohill, 2013)

Apart from being seen from the Growth Point, (Herbert & Brown, 1997) explained that the components of algebraic reasoning consist of three activities, namely: 1) looking for patterns, 2) recognizing patterns, 3) generalizing. (Lannin, 2005), states that the emphasis on patterns as an introduction to algebra has a role in the representation of variables. Furthermore, generalization through pattern activities can create a bridge between students' arithmetic knowledge and their understanding of symbolic representations.

(Blanton et al., 2018), describe 4 important aspects of early algebraic reasoning, namely:

1) Generalization

Generalization is the essence of algebraic thinking and the essence of mathematical activity. Generalization is characterized as a mental process in which a person draws conclusions from several examples into a single, unified form. For example, in simple computations, a child may notice that after a few times he has added an even number and an odd number the result is an odd number. In this case, the child begins to sort all examples of the addition of certain even

numbers and odd numbers to get odd numbers, so the generalization is that the sum of any even numbers and any odd numbers is odd.

2) Representing generalizations

The activity of representing not only gives expression to the generalizations made by students in problem situations, but also shapes the nature of students' understanding of these concepts. Representing generalizations is one of the activities that can build an understanding of a general trait. In the previous example students could represent using their own words as "the sum of an even number and an odd number is an odd number". They may represent generalizations in other ways, such as with variable notation.

3) Justify generalizations

In this activity students develop mathematical arguments to defend or deny the truth of the proposed generalization.

4) Reasoning with generalization

Students carry out further activities based on generalizations that are considered as objects of reasoning in new problem scenarios. Returning to the even and odd example, a student might use a known generalization such as "sum of even and odd numbers" to explain the sum of three odd numbers.

(Kieran, 2004), explains that there are three activities that can be used to grow students' algebraic reasoning abilities.

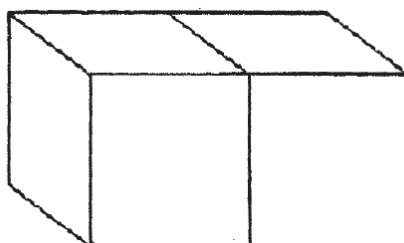
- 1) Algebraic generalization activities that involve forming expressions and equations so that they become algebraic objects. Examples: i) equations containing unknowns that represent problem situations, ii) general expressions arising from arithmetic or geometric patterns, and iii) expressions of rules governing numerical relationships. The objects that underlie expressions and equations are variables and unknown variables.
- 2) Transformational activities. For example, collecting like terms, factoring, expanding, replacing, adding and multiplying polynomial expressions, exponentiation with polynomials, solving equations, simplifying expressions, working with equivalent expressions and equations, and so on. This activity is related to changing the form of an expression or equation to maintain equality.
- 3) Global mathematical activity (meta-level). Examples are problem solving, modeling, paying attention to structure, studying change, generalizing, analyzing relationships, justifying, proving, and predicting - activities that can be done without using algebra at all.

### **C. Implementation of the Means Ends Analysis (MEA) Learning Model to Develop Algebraic Reasoning Ability**

Based on the description related to the notion of algebraic reasoning ability is a mathematical thinking ability by involving the process of representation, reasoning, analysis in a situation so that a pattern is obtained that leads to generalization. Based on this description, the indicators of algebraic reasoning ability in this study were used as follows: 1) looking for patterns, 2) recognizing patterns, 3) generalizing, 4) presenting generalizations, 5) justifying generalizations and 6) reasoning with generalization. In this sense, it refers to the ability of students to solve a problem so that students are expected to be able to understand a concept that has the same pattern and can be generalized. This is in accordance with the results of research by (Kaput & Blanton, 2005). Problem solving can also be used to develop students' algebraic reasoning.

The Means Ends Analysis (MEA) learning model is a variation of problem-solving learning that provides opportunities for students to reduce the difference between the statement of a problem and the goal to be achieved. Based on the MEA learning steps according to Eysenck (Umar, 2017) the material step can be translated into simpler sub-problems into an activity that can support algebraic reasoning ability indicators, namely looking for patterns. In the sub-problems step, the connectivity can support algebraic reasoning ability indicators, namely recognizing patterns and connecting them with others knowledge. Then in the step of choosing a settlement strategy, it supports algebraic reasoning ability indicator, namely generalization. This is in accordance with the results of research by (Indraswari et al., 2018), shows that in the pattern search indicator, the subject identifies things that are known and asked based on observations. Students can represent known information in graphs or tables. In addition, they find the elements that make up the pattern and make a relationship between the two quantities. In pattern recognition indicators, they make conjectures about the relationship between two quantities and prove it. In generalization indicators, they determine the general rules of the pattern contained in each pattern element by using algebraic symbols and creating mathematical models.

Here is an example problem from (Lannin, 2005), that develops students' algebraic reasoning through a problem-solving problem: A company builds cubes in sequence and uses a sticker machine to attach "smiley" stickers to the cube's surface. The machine places exactly one sticker on each surface of each cube. Each surface of the cube must have a sticker, if a rod with a length of 2 rods as in the picture below requires 10 stickers. Explain how you can find the number of stickers needed for a rod of any length. Write a formula you can use to determine this.



**Figure 1. Cubes in succession**

In this situation, neither the explicit nor the recursive relationship is declared directly. Students have to make a relation to count the number of stickers on the cube. The questions above test several competencies and abstract thinking skills. The problem can be solved by first making a pattern for one stick, two sticks, etc. how many stickers are needed so that it can be made into a more general generalization and formal formula. With the application of the MEA learning model, students can start by creating a sub-goal which will lead to the final goal. This sub-objective is useful for students in recognizing patterns and generalizing them. This is in accordance with the research of (Hidayat et al., 2020) that this increase in reasoning ability can be found in several indicators with the highest indicator being in determining patterns. Students are guided to develop their reasoning abilities so that they can solve reasoning problems in various ways.

#### **4. CONCLUSION**

Based on the theoretical study that has been described and the discussion, it can be concluded that the MEA learning model can support students' algebraic reasoning ability development, gradually in the material step it can be translated into simpler sub-problems into an activity that can support algebraic reasoning ability indicators, namely looking for patterns. In the sub-problems step, the

connectivity can support the algebraic reasoning ability indicators, namely recognizing patterns and connecting them with others knowledge. Then in the step of choosing a settlement strategy, it supports the algebraic reasoning ability indicator, namely generalization. Getting used to the application of the MEA learning model, can train students to solve problems and explore students' thinking skills in general and in particular in algebraic reasoning abilities.

## References

- Blanton, M., Brizuela, B. M., Stephens, A., Knuth, E., Isler, I., Gardiner, A. M., Stroud, R., Fonger, N. L., & Stylianou, D. (2018). Implementing a Framework for Early Algebra. In *Teaching and Learning Algebraic Thinking with 5 to 12 Years Old, ICME-13 Monographs* (pp. 27–49). [https://doi.org/10.1007/978-3-319-68351-5\\_2](https://doi.org/10.1007/978-3-319-68351-5_2)
- Blanton, M., & Kaput, J. (2005). Helping Elementary Teachers Build Mathematical Generality into Curriculum and Instruction. *ZDM - International Journal on Mathematics Education*, 37(1), 34–42. <https://doi.org/10.1007/bfo2655895>
- Cahyaningtyas, Novita, D., & Toto. (2018). Analisis Proses Berpikir Aljabar. *Jurnal Pendidikan Matematika Dan Sains*, 4(1), 50–60.
- Carpenter, T. P., & Levi, L. (2000). Developing Conceptions of Algebraic Reasoning in the Primary Grades. *Research Report: National Center for Improving Student Learning and Achievement in Mathematics and Science*, 5, 1–22.
- Danandjaja, J. (2014). *Metode Penelitian Kepustakaan*. Antropologi Indonesia.
- Freudenthal, H. (1975). What is Algebra and What Has it Been in History? *Archive for History of Exact Sciences*, 16(3), 189–200. <https://doi.org/10.1007/BF00328154>
- Glassmeyer, D., & Edwards, B. (2016). How Middle Grade Teachers Think About Algebraic Reasoning. *Mathematics Teacher Education and Development*, 18(2), 92–106.
- Hanifah, I., & Prabawati, N. (2019). Penerapan Strategi MEA (Means-Ends Analysis) dalam Meningkatkan Kemampuan Berpikir Kritis Matematik. *Prosiding Seminar Nasional Program Studi Magister Pendidikan Matematika Universitas Siliwangi*, c, 253–259.
- Herbert, K., & Brown, R. H. (1997). Patterns As Tools. *Patterns as Tools for Algebraic Reasoning*, 3(February 1997), 340–345.
- Herutomo, A. R., & Saputro, T. E. M. (2014). Analisis Kesalahan dan Miskonsepsi Siswa Kelas VIII pada Materi Aljabar. *Edusentris*, 1(2), 134. <https://doi.org/10.17509/edusentris.v1i2.140>
- Hidayat, A. W., Surya, A., Sukarno, & Poerwanti, J. I. S. (2020). The Effect of Means Ends Analysis Model on Reasoning Skills of Planes. *Advance in Social Science, Education and Humanities Research*, 397(22), 629–633. <https://doi.org/10.2991/assehr.k.200129.079>
- Iba, W. (2006). *Means – Ends Analysis* (pp. 1–4).
- Indraswari, N. F., Budayasa, I. K., & Ekawati, R. (2018). Algebraic Reasoning in Solving Mathematical Problem Based on Learning Style. *Journal of Physics: Conference Series*, 947(1). <https://doi.org/10.1088/1742-6596/947/1/012061>



- Juanda, M., Johar, R., & Ikhsan, M. (2014). Peningkatan Kemampuan Pemecahan Masalah dan Komunikasi Matematis Siswa SMP melalui Model Pembelajaran Means-ends Analysis. *Kreano*, 5(2), 105–113. <https://doi.org/10.15294/kreano.v5i2.3322>
- Kaput, J. J., & Blanton, M. L. (2005). Characterizing a Classroom Practice that Promotes Algebraic Reasoning. *Journal for Research in Mathematics Education*, 36(5), 412.
- Kariadinata, R., Sugilar, H., Farlina, E., & Kurahman, O. T. (2017). *The Sociomathematical Norms in Linear Algebra Lecture*.
- Katz, V. J. (2001). Using The History of Algebra in Teaching Algebra. *Proceedings of 12th ICMI Study Conference: The Future of the Teaching and Learning of Algebra*, 353.
- Kieran, C. (1995). A New Look at School Algebra-Past, Present, and Future. *Journal of Mathematical Behavior*, 14(1), 7–12. [https://doi.org/10.1016/0732-3123\(95\)90020-9](https://doi.org/10.1016/0732-3123(95)90020-9)
- Kieran, C. (2004). Algebraic thinking in the early grades: What is it. *The Mathematics Educator*, 8(1), 139–151.
- Kieran, C., Pang, J., Schifter, D., & Ng, S. F. (2016). Early Algebra: Research into its Nature, its Learning, its Teaching. In *Bigotry, Football and Scotland*. Springer.
- Lannin, J. K. (2005). Generalization and Justification: The Challenge of Introducing Algebraic Reasoning Through Patterning Activities. *Mathematical Thinking and Learning*, 7(3), 231–258. [https://doi.org/10.1207/s15327833mtl0703\\_3](https://doi.org/10.1207/s15327833mtl0703_3)
- Muin, A., Permata, C., & Afidah. (2014). The Achievement of Mathematical Connection Skills Based on Cognitive Level Through Means Ends Analysis (MEA) Strategy of Learning. *Proceeding of International Conference On Research, Implementation And Education Of Mathematics And Sciences 2014*, 4(May), 21–32.
- Mu'min, S. A. (2013). Teori Perkembangan Kognitif Jean Piaget. *Jurnal Al-Ta'dib*, 6(1), 89–99.
- Nurafifah, F., Nurlaelah, E., & Sispiyati, R. (2013). Perbandingan Peningkatan Kemampuan Berpikir Kritis Siswa SMP Antara yang Memperoleh Pembelajaran Means-Ends Analysis (MEA) dan Problem Based Learning (PBL). *Jurnal Pengajaran Matematika Dan Ilmu Pengetahuan Alam*, 18(1), 1. <https://doi.org/10.18269/jpmipa.v18i1.205>
- Nurrahman, A. (2015). Ada Apa Dengan Berpikir Aljabar Dalam Belajar. *Seminar Nasional Matematika Dan Pendidikan Matematika*, 105–108.
- Ontario Ministry of Education. (2014). K-12 Paying Attention To Algebraic Reasoning. In *Ontario*.
- Palupi, H. R., Suyitno, H., & Prabowo, A. (2016). Keefektifan Model Pembelajaran Means-Ends Analysis Pada Kemampuan Pemecahan Masalah Siswa Materi Segiempat. *Unnes Journal of Mathematics Education*, 5(2). <https://doi.org/10.15294/ujme.v5i2.11400>
- Pratiwi, W. D., & Kurniadi, E. (2018). Transisi Kemampuan Berpikir Aritmatika ke Kemampuan Berpikir Aljabar pada Pembelajaran Matematika. *Jurnal Gantang*, 3(1), 1–8. <https://doi.org/10.31629/jg.v3i1.388>

- Sugilar, H., Kariadinata, R., & Sobarningsih, N. (2019). Spektrum Symbol dan Structure Sense Matematika Siswa Madrasah Tsanawiyah. *Kalamatika: Jurnal Pendidikan Matematika*, 4(1), 37–48.
- Suherman, E. (2008). Model Belajar dan Pembelajaran Berorientasi Kompetensi Siswa. *Jurnal Pendidikan Dan Budaya*, 5(2), 11–12.
- Supendi, A., Jamiah, Y., & Ahmad, D. (2017). Model Means Ends Analysis dan Direct Instruction terhadap Kemampuan Pemecahan Masalah Matematis Siswa. *Jurnal Pendidikan Dan Pembelajaran Khatulistiwa*, 6(2), 1–10.
- Susilawati, W., Karyadinata, R., & Sugilar, H. (2019). *Cognitive conflict strategy to the improvement of students' lateral mathematical thinking ability*. 1175(1), 012174.
- Taubah, R., Isnarto, & Rochmad. (2018). Student Critical Thinking Viewed from Mathematical Self-efficacy in Means Ends Analysis Learning with the Realistic Mathematics Education Approach. *Unnes Journal of Mathematics Education Research*, 7(2), 189–195.
- Twohill, A. (2013). Algebraic Reasoning in Primary School: Developing a Framework of Growth Points. *Proceedings of the British Society for Research into Learning Mathematics*, 33(June), 55–60.
- Umar, W. (2017). Constructing Means Ends Analysis Instruction to Improve Students' Critical Thinking Ability and Mathematical Habits of Mind Dispositions. *International Journal of Education and Research*, 5(2), 261–272.
- Watson, A. (2010). *Key Understandings in Learning Mathematics*. 8–10.
- Wijaya, A. (2016). Aljabar: Tantangan Beserta Pembelajarannya. *Jurnal Gantang*, 1(1), 1–15. <https://doi.org/10.31629/jg.v1i1.1>
- Zed, M. (2014). *Metode Penelitian Kepustakaan*. Jakarta: Yayasan Obor Indonesia.